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HOPKINS AND UNDERWOOD'S

ARITHMETIC

BOOK TWO

REVISED

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HOPKINS AND UNDERWOOD'S

ARITHMETIC

BOOK TWO

REVISED

BY

JOHN W. HOPKINS

SUPERINTENDENT OF THE GALVESTON PUBLIC SCHOOLS

AND

P. H. UNDERWOOD

TEACHER OF MATHEMATICS, BALL HIGH SCHOOL
GALVESTON, TEXAS

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PREFACE

THIS book assumes that the pupils using it have a working knowledge of the four fundamental principles of arithmetic as applied to integers and to United States money. It presents the essentials of practical arithmetic, arranged by topics in conformity with the courses of study in some of the best school systems, each chapter representing a year's work, commencing with the fifth grade.

Its distinguishing features are the early introduction of decimals, the large number of problems based on the industries of our country, the clearness of illustration, and the omission of complicated problems of doubtful utility.

It aims to teach principles rather than rules. As the unitary method is the simplest for the young learner, the first two chapters give prominence to it.

The chief aim of teaching arithmetic is doubtless to develop accuracy and rapidity in the solution of problems arising in actual life and also to cultivate correct methods of reasoning. Both these aims are kept in view throughout the book. However, as students learn readily to follow processes, and, as the practice of arithmetic is of more importance to most people than its theory, especial attention is paid to the art of computation.

Chapters III and IV give a thorough review of the principles and practice of arithmetic. In these chapters

the method of ratio is brought into prominence. They contain all of general arithmetic, including mensuration, that any but specialists need to know. They also have enough of commercial arithmetic to satisfy the needs of most students.

JOHN W. HOPKINS.
P. H. UNDERWOOD.

GALVESTON, TEXAS,
January, 1912.

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ARITHMETIC

BOOK TWO

CHAPTER I

Arithmetic is the science of numbers and includes the art of computation.

The fundamental operation in arithmetic is **counting**.

The result of counting is a **number**.

A **unit** is one thing, or a group of things regarded as a single thing.

Each number has a name, as one, ten, one hundred.

Numbers may be written in words, or by the use of **symbols**, as 5, XIX. The symbols in general use among us are called figures, as 5, 6.

These figures are arranged in a regular plan or system, easy to read. This system is commonly called the Arabic System.

Writing numbers according to a system is called **Notation**.

Reading numbers so written is called **Numeration**.

NOTATION

Every number can be expressed by one or more of the following figures, sometimes called Arabic Numerals : 1, 2, 3, 4, 5, 6, 7, 8, 9, 0. The first nine of these are called **digits**, or **significant figures**, and the tenth is called **zero**, or **cipher**.

In writing a number zero is used to fill a place not named as occupied by some other figure; thus, Forty

million, five hundred six thousand, twenty-three is written: 40,506,023.

To represent numbers above nine, two or more figures are written side by side. Thus, twenty-nine is written 29; four hundred fifty-three is written 453. Notice in 29 that "9" represents "nine," and that the "2" at the left of the "9" represents "twenty;" also, in four hundred fifty-three, the "3" represents "three," the "5" represents "fifty," and the "4" represents "four hundred." The right-hand figure stands for ones, the next figure to the left stands for tens, the next to the left, hundreds, and other figures stand for different values according to the positions they occupy. These positions are sometimes called **orders.**

In numbers expressed by more than three figures, it is customary to use commas to separate the number into groups of three figures, beginning at the right. The left-hand group may contain less than three figures; thus, 79,409,700.

It is to be noticed that the figure 0 has no value in any position, except to fill it, and that the value of a significant figure depends upon the figure and the place.

The following table gives the names and positions of the **units** up to units of the fifteenth order. It also gives the first **five periods.**

Hundred-trillions	Ten-trillions	Trillions	Hundred-billions	Ten-billions	Billions	Hundred-millions	Ten-millions	Millions	Hundred-thousands	Ten-thousands	Thousands	Hundreds	Tens	Ones
TRILLIONS' PERIOD			BILLIONS' PERIOD			MILLIONS' PERIOD			THOUSANDS' PERIOD			UNITS' PERIOD		
	7	9,	4	0	9,	7	0	0,	2	9	3,	4	5	6

The period at the right is called the **units' period**: the next, the **thousands' periods**; the next, the **millions' period**; and so on.

EXERCISE 1

Write in figures:

1. Four thousand, eight hundred twenty-seven.
2. Nine thousand, seven hundred one.
3. Sixty-eight thousand, four hundred fifty-two.
4. Forty-seven thousand, three hundred eight.
5. Ninety thousand, six hundred four.
6. Eighty-seven thousand, one hundred one.
7. Twenty-two thousand, three hundred eleven.
8. Twelve thousand, fifteen.
9. Eighteen thousand, eighteen.
10. Fourteen thousand, thirty-four.
11. Thirteen thousand, five.
12. Ninety thousand, nine.
13. Fifty-four thousand, eleven.
14. Seventy-three thousand, one.
15. Six hundred four thousand, two hundred one.
16. One hundred sixty-three thousand, ten.
17. One hundred one thousand, three hundred.
18. One hundred thousand, seven.
19. Four hundred ten thousand, one hundred twenty-seven.
20. Five hundred four thousand, three hundred eight.
21. Five hundred thousand, eleven.
22. Six hundred thousand, seventeen.
23. Nine hundred ninety thousand, fifteen.

24. Two hundred one thousand, one.

25. Seventy-two thousand, four.

NUMERATION

In reading a number we read the hundreds, tens, and units of its left-hand group, and then the name of that group; then we read the other groups in like manner proceeding towards the right. The names of the units' group and the name zero are omitted in reading. To illustrate:

45 = 4 tens, 5 ones; 45 is read "forty-five."

60 = 6 tens, 0 ones; 60 is read "sixty."

345 = 3 hundreds, 5 tens, 4 ones; 345 is read "three hundred fifty-four."

2093 = 2 thousands, 0 hundreds, 9 tens, 3 ones; 2093 is read "two thousand ninety three."

43796 is read "forty-three thousand seven hundred ninety-six."

3,405,072 is read three million, four hundred five thousand, seventy-two.

65,834,217 is read sixty-five million, eight hundred thirty-four thousand, two hundred seventeen.

EXERCISE 2

Express in words:

1. 45289.
2. 90208.
3. 75307.
4. 392394.
5. 738211.
6. 328993.
7. 401012.
8. 300287.
9. 200020.
10. 910003.
11. 728000.
12. 400098.
13. 902023.
14. 630006.
15. 1000000.
16. 2227001.
17. 3456000.
18. 9287003.
19. 8307308.
20. 8000401.
21. 7000014.

22. 6100079.	**24.** 5009020.	**26.** 6203003.
23. 3927173.	**25.** 8000904.	**27.** 9000090.

28. What is the largest number expressed by two figures?

29. What is the smallest number expressed by two figures?

30. What is the largest number expressed by three figures?

31. What is the smallest number expressed by three figures?

32. Write the largest number expressed by the figures 0, 4, 5.

33. Write the smallest number expressed by the figures 0, 4, 5.

34. Write three numbers expressed by the figures 2, 3, 4.

35. Write four numbers expressed by the figures 7, 6, 8. Write three numbers expressed by the figures 9, 3, 8.

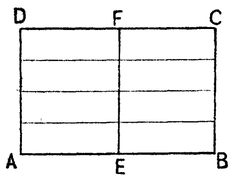
36. What is the largest number expressed by the figures 7, 3, 2, 8?

37. What is the smallest number expressed by the figures 2, 5, 3, 4?

NOTATION OF COMMON FRACTIONS

If the rectangle *ABCD* is divided into four equal parts, one of these parts is called one fourth of the rectangle; two of the parts are called two fourths of the rectangle; three of the parts are called three fourths of the rectangle; and four of the parts are called four fourths of the rectangle. In general, if any one thing is divided

into four equal parts, one of these parts is called a fourth; two of the parts are called two fourths; three of the parts, three fourths, etc. Similarly, if any thing is divided into five equal parts, one of the parts is called one fifth; two of the parts are called two fifths; three of the parts, three fifths, etc.



In the above rectangle, if the line EF is drawn so as to divide AB and DC each into two equal parts, the whole figure will be broken up into eight equal rectangles; one of these rectangles is one eighth of the whole; two of them are two eighths; three of them, three eighths, etc.

Draw a larger rectangle like the one in the book. Letter it.

Divide AE and also EB each into three equal parts and draw through the points of division lines parallel to BC. What part of $ABDC$ is one of the small rectangles? two of them? etc.

1 eighth is written $\frac{1}{8}$. 2 eighths is written $\frac{2}{8}$.
3 eighths is written $\frac{3}{8}$. 4 eighths is written $\frac{4}{8}$.
5 eighths is written $\frac{5}{8}$. 6 eighths is written $\frac{6}{8}$, etc.

How many thirds in 1 thing?
How many fourths in 1 thing?
How many sevenths in 1 thing?
How many eighths in 1 thing?
How many tenths in 1 thing?

Numbers indicated by such symbols as the above are called **fractions**.

Fractions represent one or more of the equal parts into which a number is divided.

In a fraction the number below the line is called the

denominator, meaning **namer**, because it names the parts into which the quantity is divided.

The number above the line is called the **numerator**, meaning **numberer**, because it tells the number of parts taken.

Thus, $\frac{3}{4}$ signifies that the quantity is broken into 4 equal parts, and 3 of these parts are taken; 3 is the numerator, and 4 is the denominator.

The numerator and the denominator are called the **terms of the fraction.**

A **proper fraction** is one whose numerator is less than its denominator. Thus, $\frac{4}{7}$ is a proper fraction.

An **improper fraction** is one whose numerator equals, or is greater than, its denominator. Examples: $\frac{14}{3}$, $\frac{7}{7}$.

A **mixed number** is a number made up of a whole number and a fraction. Thus, $4\frac{2}{3}$ is a mixed number.

Read the following fractions and tell what kind of fraction each is:

$\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{3}$, $\frac{3}{}$, $\frac{7}{2}$, $\frac{9}{2}$, $\frac{7}{3}$, $\frac{9}{3}$, $\frac{7}{4}$, $\frac{11}{4}$, $\frac{13}{4}$, $\frac{2}{5}$, $\frac{3}{5}$, $\frac{7}{5}$, $\frac{9}{5}$, $\frac{10}{5}$, $\frac{5}{6}$, $\frac{7}{6}$, $\frac{9}{6}$, $\frac{11}{6}$, $\frac{4}{7}$, $\frac{6}{7}$, $\frac{5}{7}$, $\frac{11}{7}$, $\frac{18}{7}$, $\frac{3}{8}$, $\frac{5}{8}$, $\frac{7}{8}$, $\frac{9}{8}$, $\frac{11}{8}$, $\frac{15}{8}$, $\frac{7}{10}$, $\frac{9}{10}$, $\frac{19}{10}$, $\frac{29}{10}$.

NOTATION OF DECIMAL FRACTIONS

The word **decimal** comes from the Latin, **decem**, meaning ten.

A fraction whose denominator is 10, 100, 1000, 10,000, etc., is called a **decimal fraction**, or simply a **decimal**; thus, $\frac{9}{10}$, $\frac{2}{100}$, $\frac{7}{1000}$, are decimal fractions.

A more convenient form for writing decimal fractions is merely an extension of our method of writing whole numbers.

In writing a whole number we named the places toward the left, beginning at the units' place. If we put a dot at the right of the units' place and name the places toward the right, as shown in the table on the next page, we have

decimal fractions. Thus, $\frac{9}{10}$ may be written .9; $\frac{2}{100}$ is written .02, $\frac{7}{1000}$ is written .007.

The dot at the right of the units' place is called the decimal point.

Figures occupying places at the right of the decimal point represent the numerator of a decimal fraction whose denominator is indicated by the place occupied by the right-hand figure. That is, .9 stands for $\frac{9}{10}$; .0023 stands for $\frac{23}{10000}$.

Zeros are used to fill places between the decimal point and the first significant figure.

Zeros at the right of a decimal may be dropped, as they do not affect its value.

The following table shows the names of the places to the right of the decimal point; thus, 5 millionths is written .000005; 23 thousandths is written .023.

				Tenths	Hundredths	Thousandths	Tenths of thousandths	Hundredths of thousands	Millionths	Tenths of millionths	Hundredths of millionths	Billionths
9	7	6	.	1	2	3	4	5	6	7	8	9

In writing a mixed number, the decimal point is placed after the whole number and then the decimal part is written; thus, twenty-five and thirty-three hundredths is written, 25.33.

The law of the decimal system of notation is: **The value of a digit in any place is always ten times the value of the same digit written in the next place to the right.**

As this law applies to figures at the left of the decimal point as well as at the right of it, the name **decimal system** includes our common method of writing whole numbers as well as fractions.

A familiar illustration of this law is the notation of United States money. For example: five dollars and fifty cents is written; $5.50. The 5 at the left of the point represents five dollars, which is ten times as much as the five dimes represented by the 5 at the right of the point.

EXERCISE 3

Write:

1. 33 hundredths.
2. 2005 thousandths.
3. 329 thousandths.
4. 101 thousandths.
5. Two hundred three thousandths.
6. Seven hundred and four thousandths.
7. Nine hundred three thousandths.
8. Nine hundred and three thousandths.
9. Six thousand seven hundred ten-thousandths.
10. Six thousand seven hundred and one ten-thousandth.
11. Five hundred ninety ten-thousandths.
12. Five hundred and ninety ten-thousandths.
13. Six thousand one ten-thousandths.
14. Seven hundred ten-thousandths.
15. Seven hundred ten thousandths.
16. Five hundred thousandths.
17. Five hundred-thousandths.
18. Two hundred seven hundred-thousandths.
19. Two hundred and seven hundred-thousandths.

NUMERATION OF DECIMALS

To read a decimal, read the number shown by the figures as if it were a whole number; then give to it the name of the place occupied by the right-hand figure; thus, .002135 is read two thousand one hundred thirty-five millionths.

If there are figures at the left of the decimal point, read them as if there were no other figures; say "and," to represent the decimal point, and then read the decimal; thus 45,132.017 is read forty-five thousand one hundred thirty-two *and* seventeen thousandths.

In general, a decimal is read as if it were a whole number and then is given the name of the place that the right-hand digit occupies. In reading decimals **and** should not be used except to connect the integral and the decimal parts of the number. For example, 500.005 is read five hundred and five thousandths. .505 is read five hundred five thousandths. 8.0379 is read eight and three hundred seventy-nine ten-thousandths. .8379 is read eight thousand three hundred seventy-nine ten-thousandths.

EXERCISE 4

Read:

1. 6.2.	**9.** .999.	**17.** 5.0067.
2. 7.9.	**10.** 99.099.	**18.** 7.0123.
3. 8.4.	**11.** 700.35.	**19.** 6.034.
4. 4.32.	**12.** .735.	**20.** 8.295.
5. .12.	**13.** 6.201.	**21.** 9.1238.
6. 5.17.	**14.** 4.027.	**22.** .0003.
7. 6.05.	**15.** 5.029.	**23.** .0054.
8. .65.	**16.** 9.001.	**24.** .4008.

ADDITION

The number obtained by combining two or more numbers into a single equivalent number is called the **sum**.

Addition is the process of finding the sum of two or more numbers.

The numbers to be added are called **addends.**

The sign of addition is +, and is read **plus.**

Thus $5 + 8 = 13$, and is read *five plus eight equals thirteen.*

Only numbers which represent objects of the same kind can be added. For example: 5 apples + 8 apples = 13 apples; but 5 apples cannot be added to 8 sticks. In this latter case, if we can give the quantities such names as will make them of the same kind, the addition can be performed; thus, apples and sticks are objects; therefore, we can say 5 objects + 8 objects = 13 objects.

In performing addition, write the numbers in a column so that the units' figures of all the numbers are in one column, the tens' figures in one column, the hundreds' figures in one column and so on; then add the right-hand column; then the next column to the left, and so on. In case the sum of one column consists of tens and units, the units are written and the tens are added with the next column.

ILLUSTRATIVE EXAMPLE.

$$\begin{array}{r} 76395 \\ 8342 \\ 9007 \\ \underline{6804} \\ 100548 \end{array}$$

We say, "4, 11, 13, 18." Write the 8 and carry the 1 to the next column. 1, 5, 14. Write the four and carry

the 1 to the next column. 1, 9, 12, 15. Write the 5 and carry the 1 to the next column. 1, 7, 16, 24, 30. Write the 0 and carry the 3 to next column. 3, 10. Write the 10. The sum is 100548.

EXERCISE 5

The following table gives the number of children of school age, the number enrolled, the average daily attendance, and the total expenditures for the public schools of the United States by states and territories for the school year 1909.

Find the totals for each of the divisions named. Also find the totals for the entire country.

State or Territory	Number of Children	Number Enrolled	Average Daily Attendance	Total Expenditures
North Atlantic Division.				
Maine	171,515	146,810	109,021	$2,772,952
New Hampshire	92,600	65,033	48,063	1,558,141
Vermont	70,296	66,174	57,483	1,512,649
Massachusetts	720,200	530,444	436,559	19,407,255
Rhode Island	118,850	78,764	61,169	2,575,693
Connecticut	251,331	187,876	144,963	5,023,152
New York	1,926,935	1,386,712	1,105,547	53,588,249
New Jersey	615,571	424,534	309,661	15,651,625
Pennsylvania	1,757,651	1,263,034	994,969	38,523,925
South Atlantic Division.				
Delaware	53,125	39,546	27,117	539,957
Maryland	358,666	239,420	147,018	3,748,021
District of Columbia	70,633	54,792	43,918	2,797,192
Virginia	644,714	394,072	257,724	4,393,562
West Virginia	323,359	276,333	193,353	4,287,606
North Carolina	670,000	521,202	335,969	2,993,045
South Carolina	517,875	334,902	230,849	1,905,236
Georgia	816,000	547,912	357,710	4,005,325
Florida	181,391	141,928	101,780	1,714,938

State or Territory	Number of Children	Number Enrolled	Average Daily Attendance	Total Expenditures
South Central Division.				
Kentucky	705,333	519,192	307,505	4,850,189
Tennessee	693,714	512,158	345,905	3,404,555
Alabama	675,818	416,390	271,648	2,498,708
Mississippi	602,143	459,981	284,366	2,674,648
Louisiana	530,125	249,067	171,801	3,607,295
Texas	1,273,823	833,631	557,356	10,289,755
Arkansas	522,375	374,154	243,232	3,110,164
Oklahoma	503,125	381,329	237,377	3,300,000
North Central Division.				
Ohio	1,059,686	792,513	656,788	25,415,053
Indiana	660,000	530,341	425,243	13,329,725
Illinois	1,344,200	997,453	783,299	33,231,345
Michigan	750,200	535,850	447,765	14,690,964
Wisconsin	685,000	466,554	322,766	10,676,214
Minnesota	599,875	435,109	308,673	11,942,449
Iowa	634,286	505,192	361,805	11,925,017
Missouri	965,800	706,690	495,995	13,151,365
North Dakota	149,430	135,203	88,139	4,479,871
South Dakota	151,000	121,979	76,694	3,645,996
Nebraska	354,833	281,375	191,076	7,171,445
Kansas	495,333	394,380	289,674	8,336,353
Western Division.				
Montana	80,300	54,627	45,798	2,070,195
Wyoming	27,000	23,182	16,595	602,293
Colorado	189,783	162,660	103,157	5,143,504
New Mexico	80,472	47,987	29,380	539,945
Arizona	34,000	28,608	17,863	889,148
Utah	118,166	84,803	65,884	2,762,580
Nevada	16,325	9,761	6,910	499,898
Idaho	86,840	74,207	51,012	1,998,189
Washington	232,000	205,566	150,925	8,541,589
Oregon	140,873	114,467	95,081	3,635,516
California	517,250	352,278	272,252	15,985,256

SUBTRACTION

Subtraction is the process of finding the **difference** of two quantities.

The number subtracted is the **subtrahend.**

The other number is the **minuend.**

The sign of subtraction is $-$, and is read **minus.**

In subtraction, as in addition, the numbers must represent things of the same kind.

In performing subtraction, the subtrahend is usually written directly below the minuend, the figures of like orders being placed in the same vertical column, as in addition. To each figure of the subtrahend is then added whatever number is required to produce the figure above it in the minuend, or if this is smaller than that of the subtrahend this figure plus 10. This "difference" is placed directly below:

ILLUSTRATIVE EXAMPLE. From 913 take 537.

$$\begin{array}{r} 913 \\ \underline{537} \\ 376 \end{array}$$

(*a*) 7 and 6 are 13 $(3+10)$. Write 6, carry 1. 1 and 3 are 4; 4 and 7 are 11 $(1+10)$. Write 7, carry 1. 1 and 5 are 6; 6 and 3 are 9. Write 3.

(*b*) The usual explanation of subtraction is: 7 from 13 leaves 6; 30 from 100 leaves 70; 500 from 800 leaves 300.

$$\begin{array}{r} 913 \\ \underline{537} \\ 376 \end{array}$$

(*c*) The work might be performed thus: 7 from 13 leaves 6. Write 6, carry 1. 4 from 11 leaves 7. Write 7, carry 1. 6 from 9 leaves 3. Write 3.

Addition gives a check on the work of subtraction; for the minuend is the sum of the difference and the subtrahend. Thus, in the above example, $913 = 376 + 537$.

Any method of subtraction may be used, but the **additive** method, here given first, is now regarded as the best.

EXERCISE 6

The population of the United States in 1900 and in 1910 by states is stated in the following table.

Find the increase in each state and territory from 1900 to 1910, and verify the answers.

	1900	1910		1900	1910
Alabama	1,828,697	2,138,093	Nebraska	1,066,300	1,192,214
Arizona	122,931	204,354	Nevada	42,335	81,875
Arkansas	1,311,564	1,574,449	New Hampshire	411,588	430,572
California	1,485,053	2,337,549	New Jersey	1,883,669	2,537,167
Colorado	539,700	799,024	New Mexico	195,310	327,301
Connecticut	908,420	1,114,756	New York	7,268,894	9,113,279
Delaware	184,835	202,322	North Carolina	1,893,810	2,206,287
District Columbia	278,718	331,069	North Dakota	319,146	577,056
Florida	528,542	751,139	Ohio	4,157,545	4,767,121
Georgia	2,216,331	2,609,121	Oklahoma	790,391	1,657,155
Idaho	161,772	225,594	Oregon	413,536	672,765
Illinois	4,821,550	5,638,591	Pennsylvania	6,302,115	7,665,111
Indiana	2,516,462	2,700,876	Rhode Island	428,556	542,610
Iowa	2,231,853	2,224,771	South Carolina	1,340,316	1,515,400
Kansas	1,470,495	1,690,949	South Dakota	401,570	583,888
Kentucky	2,147,174	2,289,905	Tennessee	2,020,616	2,184,789
Louisiana	1,381,625	1,656,388	Texas	3,048,710	3,896,542
Maine	694,466	742,371	Utah	276,749	373,351
Maryland	1,188,044	1,295,346	Vermont	343,641	355,956
Massachusetts	2,805,346	3,366,416	Virginia	1,845,184	2,061,612
Michigan	2,420,982	2,810,173	Washington	518,103	1,141,900
Minnesota	1,751,394	2,075,708	West Virginia	958,800	1,221,119
Mississippi	1,551,270	1,797,114	Wisconsin	2,069,042	2,333,860
Missouri	3,106,665	3,293,335	Wyoming	92,531	145,965
Montana	243,329	376,053			

MULTIPLICATION

Find, by adding, the value of $7 + 7 + 7 + 7 + 7$, taking 7 five times. The process can be shortened, as follows:

$7 \times 5 = 35$. Five 7's are 35.

Multiplication is thus seen to be a short method of addition when the numbers to be added are all the same. The number to be repeatedly added is called the **multiplicand.** The number which indicates how many times the multiplicand is to be added is called the **multiplier.** The result of multiplication is a **product.** The multiplicand and multiplier are **factors** of the product.

A number used with names of objects is a **concrete** number; thus, $6, 7 apples.

A number that is not concrete is called a **pure** number, or an **abstract** number; thus, 6, 7, 12.

Since the multiplier denotes "number of times," it must always be an abstract number. The multiplicand may be either concrete or abstract.

Thus, $6 can be multiplied by 7, or added to itself 7 times, but $6 cannot be multiplied by 7 apples, or added to itself 7 apples times.

The product is concrete if the multiplicand is concrete. The product is abstract if the multiplicand is abstract.

In multiplication as in addition, as the sum is of the same kind as the addends, so the product is of the same kind as the multiplicand. Thus, 5×7 apples = 35 apples.

The sign of multiplication is $\times$, and is read **multiplied by,** or **times.** Thus, $34. $\times$ 7 means $34. is to be multiplied by 7. 7 $\times$ $34. is read 7 times $34. In each case, $34. is the multiplicand, and 7 is the multiplier.

A change in the order of the factors does not change the product.

MULTIPLICATION BY NUMBERS CONTAINING MORE THAN ONE DIGIT

In multiplying by a number which contains two or more figures, we multiply each figure of the multiplicand by the units' figure of the multiplier, then by the tens' figure, and so on until all the figures are used, the work being arranged as shown in the following example. These separate results are called **partial products.** The multiplier is placed below the multiplicand, and the right-hand figure of each partial product is placed directly below the figure of the multiplier which was used in obtaining that partial product. Then the partial products are added.

Example 1. Multiply 3562 by 249.

$$\begin{array}{r} 3562 \\ 249 \\ \hline 32058 \\ 14248 \\ 7124 \\ \hline 886938 \end{array}$$

EXPLANATION. $9 \times 2 = 18$; write 8 and carry 1. $9 \times 6 = 54$, $54 + 1$ (which was carried from 18) $= 55$; write 5 and carry 5. $9 \times 5 = 45$; 45 plus $5 = 50$; write 0 and carry 5. $9 \times 3 = 27$; 27 plus $5 = 32$; write 32. Then multiply by 4; $4 \times 2 = 8$; write 8 in the tens' place because 4 is in the tens' place; $4 \times 6 = 24$; write 4 in the next place and carry 2; $4 \times 5 = 20$, $20 + 2 = 22$; write 2 and carry 2; $4 \times 3 = 12$, $12 + 2 = 14$; write 14. Then multiply by the 2 of the hundreds' place, and add the three partial products.

As the result of multiplying any figure by zero, or of multiplying zero by any number, is zero, we may omit the zeros of the multiplier, if we remember to place in their proper positions the products obtained by using the other figures of the multiplier; that is, to place each figure under the figure of its own order in the multiplier, as hundreds under hundreds, and so forth.

Example 2. Multiply 45073 by 2007.

$$\begin{array}{r} 45073 \\ 2007 \\ \hline 315511 \\ 90146 \\ \hline 90461511 \end{array}$$

EXPLANATION. 45073 multiplied by 7 = 315511, the first partial product; 45073 × 2 = 90146, the last partial product. Its right-hand figure 6 is placed under the 2 of the multiplier, just as it would be placed, if instead of ciphers there had been two digits in the multiplier.

EXERCISE 7

1. An office desk costs $25. How much will 3 such desks cost? 8 desks? 36 desks? 49 desks?

2. Eggs sell for 26¢ per dozen. Find the cost of 8 dozen; 18 dozen; 94 dozen.

3. There are 5,280 feet in a mile. How many feet in 19 miles? in 76 miles?

4. How many days in 39 weeks?

5. A contractor pays in wages $78 a day. How much will he pay in 78 days?

6. How many hours in 89 days?

7. A train travels at the rate of 34 miles an hour. How far will it run in 47 hours?

8. How many acres in a ranch containing 98 sections of land? (1 section = 640 acres.)

9. A degree on a meridian of the earth is about 69 miles. How many miles in 17 degrees?

10. A cubic foot of rock weighs 148 pounds. How many pounds do 3,297 cubic feet weigh?

11. The rent of a dwelling is $28 per month. Find the rent for 3 years.

12. A gallon of water contains 231 cubic inches. How many cubic inches in 368 gallons?

13. A book has 360 pages, each page has 32 lines, and each line averages 9 words. How many words in the book?

14. A carpenter earns $3.20 a day. At this rate, how much will he earn in 299 days?

15. A brick mason earns $5.60 a day. How much will he earn in 310 days?

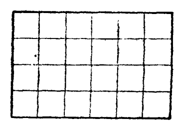
AREAS OF RECTANGLES

The word **unit** is often used to mean **measure.** Thus, in 5 feet the **unit** is one foot; in 5 yards the unit is one yard; in 3 quarts the unit is one quart.

The unit or measure of area is a square whose side is one unit long. This square may be a square inch, a square foot, a square mile, etc.

Draw, on squared paper, rectangles nine by five units, ten by six units, eight by seven inches, and find their areas.

Area of a rectangle = length × breadth.



EXERCISE 8

1. A city block is 100 yards long and 90 yards wide. Find its area.

2. Find the area of a square whose side is 84 yards.

3. Find the area of a square having 320 rods for a side.

4. Find the area of a rectangle, the length being 140 yards and the width 84 yards.

5. Find the area of a rectangle 238 yards long and 96 yards wide.

6. A farm, rectangular in shape, 440 yards long and 380 yards wide, contains how many square yards?

7. Find the area of a rectangle 75 rods long and 63 rods wide.

8. Find the area of a grass plot 240 feet by 84 feet.

9. A sheet of paper 18 inches long and 14 inches wide contains how many square inches?

10. A township is 6 miles long and 6 miles wide. How many square miles does it contain?

11. A county, having the shape of a rectangle, is 24 miles long and 18 miles wide. How many square miles in its area?

12. A street is 1760 yards long and 23 yards wide. How many square yards does it contain?

In multiplying, when there are decimal places in the multiplicand, the multiplication is performed as in the case of a whole number; but the same number of decimal places must be pointed off in the product as there are in the multiplicand.

EXERCISE 9

Find the product:

1. \$ 79.94 × 8.
2. \$ 32.20 × 7.
3. \$ 79.49 × 6.
4. \$128.29 × 7.
5. \$399.39 × 9.
6. \$454.59 × 12.
7. \$729.38 × 11.
8. \$237.38 × 9.
9. \$720.99 × 7.
10. \$285.68 × 8.
11. \$ 51.33 × 7.
12. \$ 79.29 × 7.
13. \$ 29.97 × 8.
14. \$179.38 × 6.
15. \$373.39 × 5.
16. \$799.94 × 8.
17. \$822.50 × 9.
18. \$998.78 × 11.

19. $778.75 × 12.

20. $732.75 × 9.

21. $928.34 × 7.

22. $653.82 × 5.

23. When shoes sell for $3.90 a pair, how much will 24 pairs of shoes cost?

24. If overcoats sell for $7.98, find the price of 20 overcoats.

25. Mackintoshes sell for $6.95 apiece. How much will 27 mackintoshes cost?

26. When wheat is 84¢ per bushel, how much will 384 bushels bring?

27. Find the price of 349 bushels of corn at 56¢ a bushel.

28. Cheese costs 13¢ per pound. Find the price of 54 pounds.

29. Find the cost of 325 pounds of sugar at 6¢ per pound.

30. An acre of land is worth $60.75. Find the value of 100 acres.

EXERCISE 10

On the map of the United States published by the General Land Office, Department of the Interior, 1 inch represents 37 miles. The distances in inches on this map between the cities named are given below:

1. New Orleans to Chicago 22.75.
2. Savannah to Indianapolis 16.6.
3. Mobile to Toledo 21.89.
4. Richmond to St. Louis 19.2.
5. Washington to San Antonio 37.88.
6. Boston to Jackson 34.6.
7. Atlanta to Des Moines 20.4.

8. Newport to St. Louis 27.9.

9. New York to Lincoln 32.2.

10. Chicago to San Francisco 50.8.

11. St. Louis to Portland, Oregon, 47.1.

12. Memphis to Seattle 51.2.

Find the distances in miles between the above-named cities.

Find the number of inhabitants in each of the following-named states:

	State	Areas in Sq. Mi.	Number of Inhabitants per Sq. Mi.
13.	Georgia	58,980	44.4
14.	Iowa	55,475	40.2
15.	Illinois	56,000	100.1
16.	Louisiana	45,420	37.4
17.	Michigan	57,430	49.2
18.	New Jersey	7,525	347.3
19.	Ohio	40,760	116.9
20.	Pennsylvania	44,985	168.9

DIVISION

Division is the process of determining how many times one number is contained in another number; as, 8 is contained in 24 three times.

The first number, 8, is the divisor; the second, 24, is the dividend; the result, 3, is the quotient.

Division may also be defined as the process of finding any part of a number, as one eighth of 24 is 3.

In this case, the given number, 24, is the dividend; the number of parts, 8, is the divisor; the part sought, 3, is the quotient.

This form of division is sometimes called **partition.**

If the division is not exact, there is a part left over, undivided, called the **remainder.**

The division of 24 by 4 may be written in three ways: $24 \div 4$, read, 24 divided by 4; $\frac{24}{4}$, read, 24 fourths; $\frac{1}{4}$ of 24, read, one fourth of 24. In each case, 24 is the **dividend**, 4 is the **divisor**, and the result is the **quotient.**

Whenever the divisor is contained in the dividend an exact number of times, that is, without a remainder,

$$\text{Dividend} = \text{divisor} \times \text{quotient}.$$

In case there is a remainder,

$$\text{Dividend} = (\text{divisor} \times \text{quotient}) + \text{remainder}.$$

In case there is a remainder, it is written as the numerator of a fraction of which the divisor is the denominator and is placed after the integral part of the quotient. Thus: $15 \div 4 = 3\frac{3}{4}$.

In division, if dividend and divisor are both concrete, they must represent things of the same kind. In such a division the quotient is abstract and shows the number of times that the dividend contains the divisor. Thus: 12 quarts $\div$ 4 quarts $= 3$. If the dividend is concrete and the divisor is abstract, the quotient is a concrete number representing the same things as the dividend. In this case the quotient shows one of the parts produced by separating the dividend into the number of parts indicated by the divisor. Thus, 12 quarts $\div 4 = 3$ quarts. If both dividend and divisor are abstract, the quotient is necessarily abstract. Thus: $12 \div 4 = 3$. The divisor cannot be concrete when the dividend is abstract. Thus we cannot speak of dividing the abstract number 15, by 3 apples.

In performing the work of division, the divisor is usually placed at the left of the dividend and separated from it by

a line. Then the quotient is placed above the dividend and separated from it by a line, the decimal point, if any, in the quotient; being placed just above the decimal point in the dividend. The first figure of the quotient is placed directly above the last figure of the dividend used for the first division. Instead of writing the remainder as a fraction with the divisor for its denominator, the division may be continued beyond the decimal point to any desired number of decimal places; three being the usual number.

Example 1. Divide 322021 by 1253.

```
         257
1253)322021
     2506
      7142
      6265
       8771
       8771
```

EXPLANATION. Beginning at the left-hand end of the dividend, consider as many figures as form a number larger than the divisor. This number is 3220. Then to find how many times 1253 is contained in 3220 we find how many times 12 is contained in 32. This gives 2, which we write in the quotient directly above the right-hand figure of 3220. Multiplying 1253 by 2 gives 2506, which subtracted from 3220 leaves a remainder of 714. Then we bring down the next figure from the dividend and write it with the remainder to form the next partial dividend, 7142. 12 is contained 5 times in 71. So we write 5 for the next figure in the quotient. Multiplying 1253 by 5 gives 6265, which subtracted from 7142 leaves a remainder 877. Then we bring down the next figure from the dividend. 12 in 87 goes 7 times. 1253 multiplied by 7 gives 8771. In case the divisor is not exactly contained in the dividend, the work may be continued by placing a decimal point in the quotient directly above the decimal point in the dividend and writing zeros to fill out decimal places of the dividend.

Example 2. Divide 90,961,266 by 3,026,000.

```
            30.059+
3,026,000)90961.266
          9078
           18126
           15130
            29966
            27234
             2732
```

Observe that the work can be shortened by cutting off zeros at the right of the divisor and pointing off as many places in the dividend as zeros cut off in the divisor. A + sign is placed after the quotient to show that the division is not exact.

Also we notice that it is convenient to use the first figure or the first two figures of the divisor as a **trial divisor.** Thus, 3 in 9 goes 3 times. Write 3 in the quotient and perform the multiplication and subtraction. 3026 is not contained in 181. Write 0 in the quotient and bring down the next figure of the dividend. 3026 is not contained in 1812. Write 0 in the quotient and bring down the next figure of the dividend. 3 in 18 goes six times. Write 6 in the quotient and perform the multiplication. 6 times 4026 is 18,156 which is larger than 18,126 and shows that the figure in the quotient is too large. Put 5 in the quotient in place of the 6. Perform the multiplication and subtraction. 3 in 29 goes 9 times. Write 9 in quotient. As the quotient is not to be continued, the multiplication and subtraction of this last step need not be performed except as a test of the correctness of the figure placed in the quotient.

EXERCISE 11

Divide and prove your answers correct:

1. 77,354 ÷ 16.	4. 92,738 ÷ 21.	7. 69,593 ÷ 23.
2. 79,358 ÷ 18.	5. 100,000 ÷ 24.	8. 85,376 ÷ 32.
3. 97,854 ÷ 20.	6. 73,948 ÷ 25.	9. 99,392 ÷ 36.

10. 59,738 ÷ 35.
11. 49,399 ÷ 40.
12. 99,988 ÷ 42.
13. 68,698 ÷ 48.
14. 123,456 ÷ 49.
15. 876,543 ÷ 50.
16. 789,295 ÷ 54.
17. 828,374 ÷ 56.
18. 528,739 ÷ 64.
19. 629,394 ÷ 72.
20. 273,579 ÷ 81.
21. 179,246 ÷ 84.
22. 739,264 ÷ 90.
23. 543,293 ÷ 56.
24. 665,670 ÷ 81.

EXERCISE 12

1. When sugar sells for 6 cents per pound, how many pounds can be bought for 84 cents?

2. If a boy walks at the rate of 3 miles per hour, how long will it take him to walk 87 miles?

3. In a peck there are 8 quarts. How many pecks are there in 3000 quarts?

4. If a bicyclist rides 9 miles an hour, how many hours will it take him to go from St. Louis to Indianapolis, a distance of 265 miles? After riding 19 hours, how far from Indianapolis will he be?

5. When coal costs $9 a ton, how many tons can be bought for $3456?

6. A teacher receives a salary at the rate of $4 a day for every day he teaches. His yearly salary is $716. How many days in the school year?

7. A brick mason receives $4 a day for every day he works. How many days must he work to earn $900?

8. How many feet are there in 2500 inches?

9. How many weeks are there in 364 days?

10. If a dozen penknives cost $9, how many dozen penknives can be bought for $126?

11. Plows cost $12 apiece. How many can be bought for $192?

12. A box of oranges is worth $3. How many such boxes can be bought for $111?

13. When a barrel of pork sells for $12, how many barrels must be sold to realize $5004?

14. A section foreman rides on a velocipede at the rate of 11 miles an hour. How long will it take him to go from Cincinnati to Cleveland, a distance of 264 miles?

15. Divide $1000 among 8 persons, giving to each the same sum of money.

16. A flock of sheep sells for $966. How many sheep in the flock, if each sheep sells for $6?

17. A man has $795 in 5-dollar gold pieces. How many coins has he?

18. Hogs sell for $8 apiece. At this price how many can be purchased for $360?

19. A box of soap is listed at $4. How many such boxes can be purchased for $980?

20. How many barrels of flour can be bought for $1002, when flour sells for $6 a barrel?

21. Oyster crackers cost 5 cents a pound. How many pounds can be bought for 95 cents?

22. By buying horses at $75 each and selling them at $84 each, a jobber makes a profit of $324. How many horses does he sell?

EXERCISE 13

1. How many bags of Rio coffee can be bought for $882, if one bag costs $21?

2. Currants sell for $14 a barrel. At this price how many barrels can be bought for $546?

3. Granulated sugar is worth $16 a barrel. How many barrels must be sold to realize $1264?

4. There are 36 inches in one yard. How many yards in 100,000 inches?

5. There are 32 quarts in one bushel. How many bushels in 7712 quarts?

6. How many days in 3000 hours?

7. A degree on a meridian of the earth's surface is 69 miles long. Two places on the same meridian are 2484 miles apart. How many degrees apart are they?

8. How many square yards in 3276 square feet? (There are 9 square feet in one square yard.)

9. A gallon contains 231 cubic inches. How many gallons in a barrel containing 8316 cubic inches?

10. Oolong tea costs $15 a chest. How many chests can be purchased for $495?

11. A barrel of sugar weighs 325 pounds. How many barrels in 105,625 pounds?

12. How long will it take a train, rate 30 miles an hour, to go from New York to San Francisco, a distance of 3270 miles, if 5 hours are allowed for stops?

13. There are 10 square chains in an acre. How many acres in 10,000 square chains?

14. Rhode Island contains in round numbers 800,000 acres. Find its area in square miles. (640 acres = 1 square mile.)

EXERCISE 14

1. If land is worth $68 an acre, how many acres can be bought for $4624?

2. A tract of land is sold for $4,795.50 at the rate of $69.50 an acre. How many acres in the tract?

3. When horses sell for $85.40 apiece, how many can be bought for $36,465.80?

4. If the price of wheat is 76¢ per bushel, how many bushels can be bought for $4,043.20?

5. When cans of asparagus sell for 35¢ each, how many can be bought for $85.75?

6. If a pair of patent leather shoes sells for $3.85, how many pairs must be sold to bring $1,482.25?

7. A clothier buys a stock of men's trousers for $392.04. If the average cost is $1.98 per pair, how many pairs of trousers does he buy?

8. If a keg of pickles cost $1.70, how many kegs can be purchased for $28.90?

9. Chipped beef is bought at 17¢ a pound. At this rate, how many pounds can be bought for $361.25?

10. A 12-pound sack of flour retails at 45¢. How many sacks can be bought for $322.65?

11. When a can of sardines retails for 27¢, how many cans will $218.70 buy?

12. A farmer gets for his apples $816.35 at the rate of $1.45 per barrel. How many barrels does he sell?

13. Corn is worth 45¢ per bushel. How many bushels can be bought for $7,876.35?

14. Oats are worth 36¢ per bushel. How many bushels can be bought for $142.56?

15. A share of railway stock is quoted at $78.50. How many shares must be sold to realize $3,061.50?

EXERCISE 15

The following table gives the area and the population of some of the principal countries in 1908. Find the population per square mile of each.

	Country	Area in Sq. Mi.	Population
1.	Austria-Hungary	261,200	50,499,000
2.	Belgium	11,370	7,386,000
3.	Denmark	15,360	2,574,000
4.	France	207,050	39,300,000
5.	German Empire	208,800	62,982,000
6.	Italy	110,600	33,604,000
7.	Japan	147,700	47,975,000
8.	Netherlands	12,560	5,592,000
9.	Russia	8,374,000	149,034,000
10.	Spain	194,800	19,713,000
11.	United Kingdom	121,370	44,547,000

BILLS OF GOODS

The most common use of decimal fractions is in computations which deal with United States money.

Bills for goods are the most general form of such computation.

A **bill** is a written statement showing the indebtedness of one person to another for goods bought or services received. The names of the persons concerned and the date of the bill appear in the heading. The separate items specify the date and nature of each transaction, giving unit price and total price for the articles bought or services rendered. When a bill is paid, it is marked **paid** and signed by the person receiving the payment. It is then **a receipted bill**; because it serves as a receipt for the money.

The symbol "@" is used to mean, "at the rate of"; thus, 3 lb. tea. @ 50¢, means three pounds of tea at the rate of fifty cents a pound. It is commonly read **at**.

The models on the next page illustrate two very common forms for bills.

SPECIMEN BILLS

GALVESTON, TEXAS, Jan. 31, 1907.

MR. A. B. C.

In account with K. M. & CO.,

DEALERS IN

FURNITURE, CARPETS, RUGS, &C.

Jan.	*2*	*3 Chairs @ $2.25*	*$6*	*75*		
	10	*1 Library Table*	*25*	*00*		
	15	*3 Rugs @ $6.75*	*20*	*25*		
	20	*40 yd. Matting @ 45¢*	*18*	*00*		
	24	*2 Wardrobes @ $17.50*	*35*	*00*	*$105*	*00*
		Paid				
		Feb. 1, 1903.				
		K. M. & CO.				
		Per M.				

DALLAS, TEXAS, Feb. 5, 1907.

MR. P. Q. R.

Bought of M. R. S.,

RETAIL GROCERS.

Jan.	*5*	*3 lb. Tea @ 50¢*	*$1*	*50*		
	"	*28 lb. Sugar @ 5¼¢*	*1*	*47*		
	"	*3 pks. Potatoes @ 40¢*	*1*	*20*		
	"	*7 lb. Bacon @ 15¢*	*1*	*05*		
	10	*8 lb. Butter @ 35¢*	*2*	*80*		
	12	*3 cans Salmon @ 17¢*		*51*		
	13	*6 lb. Sausage @ 12¢*		*72*	*$9*	*25*
		Paid				
		Feb. 8.				
		M. R. S.				
		Per X.				

EXERCISE 16

Make out the following bills and receipt them:

1. Mr. John Rye bought of William Merchant,

12 yd. Calico	@	9¢
15 yd. Sheeting	@	7¢
11 yd. Flannel	@	35¢
2 Hats	@	$3.75
18 yd. Carpet	@	75¢
3 Smyrna Rugs	@	$10.50

2. Mr. J. Hill bought of F. Warner & Co.,

5 Stoves	@	$6.50
3 doz. Knives	@	$4.80
2 Saws	@	$1.50
5 Iron Beds	@	$15.75
6 Wrenches	@	$1.25

3. H. Van Oppen bought of Hegel & Co.,

2 bu. Potatoes	@	$1.50
5 lb. Tea	@	75¢
2 boxes Herring	@	$1.95
25 lb. Ham	@	15¢
45 lb. Sugar	@	$4\frac{1}{3}$¢

4. Mr. James Kay bought of Simpson, Perdue & Co.,

50 lb. Sugar	@	$4\frac{1}{6}$¢
15 cans Tomatoes	@	13¢
27 cans Corn	@	16¢
10 packages Breakfast Food	@	$12\frac{1}{2}$¢
8 cans Salmon	@	18¢
5 gal. Maple Sirup	@	$1.30
25 lb. Butter	@	$37\frac{1}{2}$¢
6 lb. Y. H. Tea	@	75¢

FACTORS AND MULTIPLES

A number that is exactly divisible by only itself and 1 is called a **prime number.** Thus, 1, 2, 3, 5, 7, 11, 17, 19, 23, 29, etc., are prime numbers.

A number exactly divisible by another number, besides itself and one, is called a **composite number.**

Thus, 6, 8, 9, 10, 12, 14, 15, 16, 18, 20, etc., are composite numbers.

An **even number** is one that is exactly divisible by 2.

Thus, 4, 6, 8, 10, 12, etc., are even numbers.

An **odd number** is one that is not exactly divisible by 2.

Thus, 3, 5, 7, 9, 11, etc., are odd numbers.

One number is said to be a **measure** of another number when it is contained an exact number of times in that other number.

Instead of the word "measure," **factor** or **divisor** may be used. Example: 5 is a measure, factor, or divisor of 10, 15, 20, 25, etc.

If a number is contained an exact number of times in each of two or more numbers, it is called a **common measure or divisor** of those numbers.

Thus, 2, 3, and 6 are common measures of 12, 18, and 30.

The **Greatest Common Divisor** of two or more numbers is the largest number that will divide each of those numbers exactly. It is usually indicated by the letters, G. C. D.

Thus, 6 is the G. C. D. of 12, 18, 30.

Two or more numbers are **prime to each other** when they have no common measure but 1.

Thus, 8 and 9 are prime to each other; so also are 15 and 28 prime to each other. These numbers, while prime to each other, are not themselves prime numbers.

TESTS OF DIVISIBILITY

The following facts are often helpful. They are given here for purposes of reference and not to be memorized.

A number is exactly divisible by 2 if its units' figure is divisible by 2; that is, if it is an even number.

A number is exactly divisible by 3 if the sum of its digits is exactly divisible by 3. Thus, in 912, the sum of the digits is 12, which we know contains 3. This shows that 912 contains 3.

A number is exactly divisible by 4 if the number formed by its two right-hand digits is exactly divisible by 4. Thus, in 932 the 32 contains 4, and so we know that 932 contains 4.

A number is exactly divisible by 5 if its units' figure is 0 or 5.

A number is exactly divisible by 6 if it is an even number and the sum of its digits is exactly divisible by 3.

A number is exactly divisible by 8 if the number formed by its three right-hand digits is exactly divisible by 8. Thus, in 1336, by trial we find 336 contains 8, and so we know that 1336 contains 8.

A number is exactly divisible by 9 if the sum of its digits is divisible by 9.

A number is exactly divisible by 11 if the difference between the sum of its digits in the even places and the sum of its digits in the odd places is 0 or a multiple of 11. Thus, 94,853 is exactly divisible by 11; since $3 + 8 + 9 = 20$, and $5 + 4 = 9$, and $20 - 9 = 11$, which is divisible by 11.

A number is exactly divisible by 25 if the number formed by its two right-hand digits is exactly divisible by 25. Thus, in 12,675, we know that 75 contains 25, and so we know that 12,675 contains 25.

A number is exactly divisible by 125 if the number formed by its three right-hand digits is divisible by 125.

NOTATION

2^2 is a short way of writing 2×2.
2^3 is a short way of writing $2 \times 2 \times 2$.
2^4 is a short way of writing $2 \times 2 \times 2 \times 2$.
2^5 is a short way of writing $2 \times 2 \times 2 \times 2 \times 2$.
2^6 is a short way of writing $2 \times 2 \times 2 \times 2 \times 2 \times 2$.

The result of taking a number any number of times as factor is called a **power of the number**. Thus, $7^4 = 7 \times 7 \times 7 \times 7 = 2401$.

2401 is the 4th power of 7.

The 4 written to the right of 7 and slightly above it is called the **index** or **exponent** of the power.

2^2 is read **2 square**; 2^3 is read **2 cube**, or the **cube of 2**; 2^4 is read **2 fourth power**, or the **fourth power of 2**.

FINDING PRIME FACTORS

Example 1.

$$\begin{array}{r} 7\overline{)1001} \\ 11\overline{)143} \\ 13 \end{array}$$

Resolve 1001 into its prime factors. We begin with the lowest prime numbers and try them as factors. The units' digit of 1001 is not divisible by 2. This indicates to us that 2 is not a factor of 1001. The sum of its digits is not divisible by 3. This indicates that 3 is not a factor of 1001. Also we see that 5 is not its factor. So we try 7. By actual division we find 7 to be a factor, because it is contained an exact number of times, 143. The next prime number larger than 7 is 11. 11 is contained in 143 (see page 34), and the quotient, 13, is a prime number. Hence, the prime factors of 1001 are 7, 11, 13; and $1001 = 7 \times 11 \times 13$.

Example 2. Resolve 5040 into its prime factors, and express 5040 as the product of prime numbers.

2	5040
2	2520
2	1260
2	630
5	315
3	63
3	21

Divide by 2 as many times as possible. Since 315 ends in 5, 5 is a factor of 315. Divide next by 3 as often as possible.

The prime factors of 5040 are 2, 2, 2, 2, 3, 3, 5, 7.

$$5040 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 7$$
$$= 2^4 \times 3^2 \times 5 \times 7.$$

EXERCISE 17

Resolve into prime factors and express each number as the product of its prime factors:

1. 8, 12, 16, 18, 20, 24, 27, 28, 30, 32, 36, 39, 40, 42.

2. 45, 48, 49, 50, 56, 60, 65, 69, 72, 75, 77, 80, 84, 88, 92.

3. 98, 99, 111, 117, 119, 120, 124, 128, 132, 133, 135, 140, 144.

4. 240, 720, 343, 512, 216, 729, 736, 608, 544.

5. 1331, 11011, 1309, 858, 1274, 891, 3575.

6. Write all the measures of each of the following numbers: 36, 360, 200, 567, 576, 448.

7. Write all the common measures of: (*a*) 36, 24; (*b*) 18, 27; (*c*) 48, 72; (*d*) 21, 63; (*e*) 32, 96; (*f*) 18, 72.

When several numbers are to be taken as a whole and added, subtracted, multiplied, or divided like a single number, they may be inclosed in a sign, or symbol, known as a parenthesis. A number written immediately before a parenthesis with no sign between is to be multiplied by the quantity within the parenthesis. Thus, $3(8+5)$ means 3 times the sum of 8 and 5. Multiplication or

division signs are like parentheses in their effect upon the quantities between which they are written. Thus: $3 \times 14 + 4 \times 11$ means the same as $(3 \times 14) + (4 \times 11)$, which means that 3 times 14 is to be added to 4×11.

GREATEST COMMON DIVISOR

The **Greatest Common Divisor** is the largest factor that is common to two or more numbers. Thus, 12, 8, and 16 have as common factors both 2 and 4, but 4 is their G. C. D.

Example 1. Find the G. C. D. of 48, 120, 168.

Expressing these numbers as products of their prime factors,

$$48 = 2 \times 2 \times 2 \times 2 \times 3 = 2^4 \times 3.$$
$$120 = 2 \times 2 \times 2 \times 3 \times 5 = 2^3 \times 3 \times 5.$$
$$168 = 2 \times 2 \times 2 \times 3 \times 7 = 2^3 \times 3 \times 7.$$

The common factors are 2, 2, 2, 3. Therefore, the G. C. D. is the product of these, or $2^3 \times 3 = 24$.

To find the G. C. D. of two or more numbers, express each of the numbers as the product of its prime factors; then take the product of the prime factors common to all the numbers, each factor being taken the least number of times it occurs in any of the numbers.

EXERCISE 18

Find the G. C. D. of:

1. 16, 24.
2. 24, 32.
3. 18, 27.
4. 24, 36.
5. 45, 60.
6. 75, 90.
7. 54, 72.
8. 108, 180.
9. 84, 96.
10. 120, 156.
11. 91, 105.
12. 26, 117.
13. 57, 76.
14. 115, 161.
15. 144, 264.
16. 140, 252.
17. 20, 30, 40.
18. 30, 75, 105.

19.	36, 60, 84.	**24.**	63, 84, 105.	**29.**	144, 240, 336.
20.	39, 65, 91.	**25.**	64, 96, 224.	**30.**	162, 270, 378.
21.	60, 84, 132.	**26.**	72, 108, 180.	**31.**	168, 224, 392.
22.	54, 90, 108.	**27.**	88, 132, 220.	**32.**	252, 420, 588.
23.	64, 80, 96.	**28.**	126, 189, 252.	**33.**	264, 360, 600.

LEAST COMMON MULTIPLE

The result obtained by multiplying a number by an integer is called a multiple of the number.

Thus the multiples of 8 are 8, 16, 24, 32, 40, 48, etc.

The **Least Common Multiple of two or more numbers** is the least number that is a multiple of each of the numbers. In other words, the Least Common Multiple of two or more numbers is the least number that is exactly divisible by each of the numbers.

Least Common Multiple is denoted by L. C. M.

Example 1. What is the L. C. M. of 8 and 12?

Writing multiples of 8 we have many, including:

	8	16	24	32	40	48	64	72	80
and of 12,	12	24	36	48	60	72	84		

Notice that 24 is a common multiple of 8 and 12. So also are 48 and 72. 24 is the smallest or least common multiple (L. C. M.).

Example 2. What is the L. C. M. of 12 and 18?

Make a table of multiples of 12 and 18. Then 36, or 2 times 18, is the L. C. M. of 12 and 18.

Example 3. A man buys two kinds of sugar, one kind in 4-pound bags, and the other in 5-pound bags. What is the smallest number of pounds of each kind he can buy so as to have the same amount of each?

Here the answer is the L. C. M. of 4 and 5. The L. C. M. of 4 and 5 is 20. Hence, he buys 20 pounds of each kind.

Since the L. C. M. of two or more numbers is exactly divisible by each of the numbers, it follows that the L. C. M. contains all the factors of each of the given numbers.

This fact suggests a method of finding the L. C. M. of two or more numbers.

Example: Find the L. C. M. of 48, 60, 72.

$48 = 2^4 \times 3$. The prime factors are 2 and 3.

$60 = 2^2 \times 3 \times 5$. The prime factors are 2 and 3 and 5.

$72 = 2^3 \times 3^2$. The prime factors are 2 and 3.

Any multiple of 48 must contain 2 four times as a factor.

Any multiple of 60 must contain 5 as a factor.

Any multiple of 72 must contain 3 twice as a factor.

Hence, the number $2^4 \times 3^2 \times 5 = 720$ contains all the factors of the three numbers 48, 60, 72. Therefore, the L. C. M. of 48, 60, 72, is 720.

To find the L. C. M. of two or more numbers, resolve each of the numbers into its prime factors, then find the product of all the prime factors of the given numbers, taking each factor the greatest number of times it occurs in any of the numbers.

Another Method

Example: Find the L. C. M. of 48, 60, 72, 90.

$$\begin{array}{r|rrrr}
2 & 48 & 60 & 72 & 90 \\ \hline
2 & 24 & 30 & 36 & 45 \\ \hline
2 & 12 & \cancel{15} & 18 & 45 \\ \hline
3 & 6 & & \cancel{9} & 45 \\ \hline
 & 2 & & & 15
\end{array}
\qquad \text{L.C.M} = 15 \times 2 \times 3 \times 2 \times 2 \times 2 = 720.$$

NOTE. — As 15 in the third horizontal row is a factor of 45, it may be canceled out; also 9 in the following row.

Step 1. Arrange the numbers in a horizontal row.

Step 2. Divide by a prime factor common to two or more of the numbers. Set down the quotients and the undivided numbers.

Step 3. Treat the second horizontal row in the same manner, and so on until a horizontal row is obtained that consists only of numbers prime to one another of which no two are alike if larger than 1.

The continued product of the numbers in the last row and of the divisors will be the L. C. M.

EXPLANATION. This method is really the same as the other, in principle; each divisor used representing one of the factors which must appear in the L. C. M., while the quotients must include the other factors of each number. Thus: in the given example, the first division shows that the L. C. M. of 48, 60, 72, and 90 must contain the factor 2, and the L. C. M. of 24, 30, 36, and 45. But the next division shows that the L. C. M. of 24, 30, 36, and 45 must contain the factor 2, and the L. C. M. of 12, 15, 18, and 45. Similarly, the L. C. M. of 12, 18, and 45 must contain the factor 2, and the L. C. M. of 6, 9, and 45. But the L. C. M. of 6 and 45 must contain the factor 3, and the L. C. M. of 2 and 15. The L. C. M. of 2 and 15 is 2×15 or 30. Therefore the L. C. M. of 48, 60, and 72 is $2 \times 2 \times 2 \times 3 \times 2 \times 15$ or 720.

EXERCISE 19

Find the L. C. M. of:

1. 16, 20.	5. 21, 49.	9. 34, 51.
2. 21, 14.	6. 28, 70.	10. 48, 72.
3. 18, 60.	7. 42, 56.	11. 96, 120.
4. 18, 45.	8. 36, 53.	12. 28, 30.

13. 32, 80.
14. 26, 39.
15. 48, 84.
16. 60, 96.
17. 84, 108.
18. 55, 77.
19. 54, 90.
20. 72, 108.
21. 75, 125.
22. 36, 54, 72.
23. 36, 90, 60.
24. 48, 64, 36.
25. 12, 15, 18.
26. 14, 21, 35.
27. 30, 35, 21.
28. 28, 42, 70.
29. 32, 35, 150.
30. 30, 45, 48.
31. 15, 20, 25.
32. 12, 18, 20.
33. 45, 63, 70.
34. 14, 35, 40.
35. 12, 16, 18.
36. 14, 24, 40.
37. 15, 24, 25.
38. 77, 143, 22.
39. 18, 20, 45.
40. 30, 70, 105.
41. 30, 40, 48.
42. 21, 28, 35.
43. 12, 18, 27.
44. 12, 15, 16, 18.
45. 36, 40, 45.

FRACTIONS

Review carefully the definitions and explanations of fractions, given on pages 5 and 6.

What is a fraction? How is a fraction written? What does the numerator express? What does the denominator express? What is a proper fraction? An improper fraction? A mixed fraction? (See pages 5 and 6.)

A fraction may also be considered as an indicated division. The numerator is the dividend and the denominator is the divisor.

Is $\frac{2}{7}$ of 1 week equal to $\frac{1}{7}$ of 2 weeks?
Is $\frac{3}{7}$ of 1 week equal to $\frac{1}{7}$ of 3 weeks?
Is $\frac{4}{7}$ of 1 week equal to $\frac{1}{7}$ of 4 weeks?
Is $\frac{2}{5}$ of \$1 equal to $\frac{1}{5}$ of \$2?
Is $\frac{3}{5}$ of \$1 equal to $\frac{1}{5}$ of \$3?
Is $\frac{4}{5}$ of \$1 equal to $\frac{1}{5}$ of \$4?
Is $\frac{3}{4}$ of 1 foot equal to $\frac{1}{4}$ of 3 feet?
Is $\frac{7}{8}$ of 1 gallon equal to $\frac{1}{8}$ of 7 gallons?
Is $\frac{2}{3}$ of 1 yard equal to $\frac{1}{3}$ of 2 yards?

REDUCTION OF FRACTIONS

Is $1\frac{3}{4}$ feet $= \frac{7}{4}$ of a foot? Is $1\frac{3}{4} = \frac{7}{4}$?

Is $\frac{9}{2}$ of an hour $= 4\frac{1}{2}$ hours? Is $\frac{9}{2} = 4\frac{1}{2}$?

Is $\frac{1}{2}$ of an apple $= \frac{2}{4}$ of an apple? Is $\frac{1}{2} = \frac{2}{4}$?

Changing the form of a fraction without changing its value is called **reduction of fractions.**

An improper fraction is reduced to a mixed number by performing the division of numerator by denominator and expressing the result as a whole or mixed number.

Thus, $\frac{17}{4}$ reduces to $4\frac{1}{4}$.

EXERCISE 20

Reduce to integers or mixed numbers:

1. $\frac{11}{3}$.	**10.** $\frac{60}{8}$.	**19.** $\frac{27}{7}$.	**28.** $\frac{88}{13}$.
2. $\frac{13}{4}$.	**11.** $\frac{59}{9}$.	**20.** $\frac{39}{10}$.	**29.** $\frac{79}{14}$.
3. $\frac{27}{4}$.	**12.** $\frac{19}{12}$.	**21.** $\frac{97}{13}$.	**30.** $\frac{80}{7}$.
4. $\frac{24}{4}$.	**13.** $\frac{17}{11}$.	**22.** $\frac{78}{14}$.	**31.** $\frac{77}{14}$.
5. $\frac{31}{5}$.	**14.** $\frac{44}{5}$.	**23.** $\frac{79}{12}$.	**32.** $\frac{95}{14}$.
6. $\frac{32}{7}$.	**15.** $\frac{100}{9}$.	**24.** $\frac{72}{18}$.	**33.** $\frac{118}{18}$.
7. $\frac{38}{9}$.	**16.** $\frac{100}{11}$.	**25.** $\frac{48}{16}$.	**34.** $\frac{127}{6}$.
8. $\frac{40}{6}$.	**17.** $\frac{120}{12}$.	**26.** $\frac{94}{11}$.	**35.** $\frac{140}{8}$.
9. $\frac{50}{7}$.	**18.** $\frac{15}{14}$.	**27.** $\frac{89}{9}$.	**36.** $\frac{120}{11}$.

Reduce to an improper fraction $4\frac{5}{6}$.

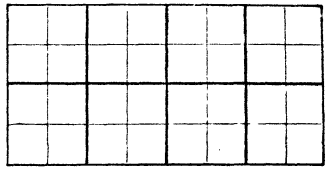
$$1 = 6 \text{ sixths.}$$
$$4 = 24 \text{ sixths.}$$
$$4\tfrac{5}{6} = 29 \text{ sixths} = \tfrac{29}{6}.$$

EXERCISE 21

Reduce to improper fractions:

1. $3\frac{1}{2}$.	**3.** $5\frac{3}{7}$.	**5.** $9\frac{2}{7}$.	**7.** $8\frac{7}{8}$.	**9.** $9\frac{5}{12}$.
2. $3\frac{2}{3}$.	**4.** $9\frac{3}{8}$.	**6.** $10\frac{3}{11}$.	**8.** $5\frac{5}{6}$.	**10.** $10\frac{10}{11}$.

11. $11\frac{1}{10}$. **13.** $19\frac{2}{7}$. **15.** $27\frac{5}{11}$. **17.** $39\frac{4}{5}$. **19.** $20\frac{10}{11}$.
12. $10\frac{1}{4}$. **14.** $18\frac{4}{9}$. **16.** $38\frac{5}{12}$. **18.** $19\frac{2}{3}$. **20.** $16\frac{15}{16}$.



How many small squares in the rectangle?

How many small squares in $\frac{1}{4}$ rectangle? in $\frac{2}{8}$? in $\frac{8}{32}$?

How many small squares in $\frac{3}{4}$ rectangle? in $\frac{6}{8}$? in $\frac{24}{32}$?

How many small squares in $\frac{16}{32}$ rectangle? in $\frac{8}{16}$? in $\frac{1}{2}$?

Take a ruler on which inches are divided into sixteenths. Compare $\frac{1}{2}$ in., $\frac{4}{8}$ in., $\frac{8}{16}$ in. Compare $\frac{3}{4}$ in., $\frac{6}{8}$ in., $\frac{12}{16}$ in.

From these instances we notice that $\frac{1}{4}$, $\frac{2}{8}$, and $\frac{8}{32}$ are equal; that $\frac{1}{2}$ is equal to $\frac{8}{16}$, or $\frac{16}{32}$, or $\frac{4}{8}$; that $\frac{3}{4}$ is equal to $\frac{6}{8}$ or $\frac{12}{16}$ or $\frac{24}{32}$.

Draw lines $\frac{7}{8}$, $1\frac{3}{4}$, $\frac{11}{16}$ inches long.

Draw a line; divide it into six equal parts, and show thereby that $\frac{1}{3}=\frac{2}{6}$, $\frac{1}{2}=\frac{3}{6}$, $\frac{2}{3}=\frac{4}{6}$.

Show by squared paper that $\frac{5}{8}=\frac{20}{32}$, $\frac{4}{7}=\frac{8}{14}$, $\frac{8}{9}=\frac{24}{27}$.

Notice that multiplying both numerator and denominator of $\frac{1}{2}$ by 16 gives the fraction $\frac{16}{32}$ which is equal to $\frac{1}{2}$; dividing both numerator and denominator of $\frac{12}{16}$ by 4 gives $\frac{3}{4}$, which is equal to $\frac{12}{16}$.

How can you change $\frac{1}{2}$ to $\frac{4}{8}$? $\frac{3}{4}$ to $\frac{6}{8}$? $\frac{24}{32}$ to $\frac{6}{8}$? $\frac{8}{16}$ to $\frac{1}{2}$?

These illustrate the general rule: **If both terms of a fraction are multiplied by the same number, or both divided by the same number, the value of the fraction remains unchanged.**

Example. Reduce $\frac{3}{7}$ to fourteenths. In order to produce fourteenths, the denominator is to be multiplied by 2. Therefore, the numerator is to be multiplied by 2.

Thus: $\dfrac{3\times 2}{7\times 2}=\dfrac{6}{14}$.

EXERCISE 22

Reduce:

1. $\frac{2}{3}$ to 9ths; to 15ths; to 24ths; to 30ths; to 36ths.
2. $\frac{3}{4}$ to 8ths; to 16ths; to 24ths; to 32ds; to 40ths.
3. $\frac{3}{5}$ to 10ths; to 20ths; to 25ths; to 35ths; to 40ths.
4. $\frac{4}{7}$ to 14ths; to 21sts; to 28ths; to 35ths; to 49ths.
5. $\frac{5}{8}$ to 16ths; to 24ths: to 40ths; to 48ths; to 64ths.
6. $\frac{3}{8}$ to 32ds; to 48ths; to 56ths; to 72ds; to 80ths.
7. $\frac{5}{9}$ to 18ths; to 27ths; to 45ths; to 63ds; to 72ds.
8. $\frac{9}{10}$ to 20ths; to 50ths; to 70ths; to 80ths.

Reduce $\frac{21}{28}$ by dividing its terms by 7.

$$\frac{21}{28} = \frac{3}{4}.$$

A fraction is said to be **in its simplest form** when its terms are integers prime to each other.

A fraction, in its simplest form, is also said to be **in its lowest terms.**

Example. Reduce to lowest terms $\frac{96}{120}$.

$\frac{96}{120} = \frac{48}{60} = \frac{12}{15} = \frac{4}{5}$. Dividing the terms of the fraction by 2, the result is $\frac{48}{60}$. Dividing the terms of this fraction by 4, the result is $\frac{12}{15}$. Dividing the terms of $\frac{12}{15}$ by 3, the result is $\frac{4}{5}$.

More directly, we may divide 96 and 120 each by 24, which will give the same result, $\frac{4}{5}$.

EXERCISE 23

Reduce to lowest terms:

1. $\frac{6}{12}, \frac{8}{18}, \frac{16}{24}, \frac{18}{27}, \frac{9}{24}, \frac{16}{28}, \frac{32}{40}, \frac{25}{30}, \frac{32}{48}$.
2. $\frac{18}{45}, \frac{24}{60}, \frac{30}{75}, \frac{44}{55}, \frac{35}{84}, \frac{30}{48}, \frac{40}{48}, \frac{84}{96}$.
3. $\frac{54}{81}, \frac{36}{54}, \frac{48}{64}, \frac{72}{96}, \frac{54}{72}, \frac{24}{64}, \frac{36}{96}, \frac{54}{144}$.
4. $\frac{36}{162}, \frac{48}{108}, \frac{60}{135}, \frac{49}{84}, \frac{63}{108}, \frac{84}{144}, \frac{126}{216}, \frac{140}{240}$.
5. $\frac{144}{216}, \frac{100}{150}, \frac{108}{162}, \frac{81}{108}, \frac{36}{64}, \frac{63}{81}, \frac{84}{108}, \frac{80}{192}$.

ADDITION

Example 1. Add $\frac{1}{2}$, $\frac{1}{3}$.

Reduce the fractions to fractions having a common denominator, and then add the numerators; thus,—

$$\frac{1}{2}=\frac{3}{6}.$$
$$\frac{1}{3}=\frac{2}{6}.$$

Hence, $$\frac{1}{2}+\frac{1}{3}=\frac{3}{6}+\frac{2}{6}=\frac{5}{6}.$$

Example 2. Add $\frac{3}{4}$, $\frac{5}{6}$.

Here the lowest denomination common to both fractions is 12ths. Notice, 12 is L. C. M. of 4, 6.

$$\frac{3}{4}=\frac{9}{12}$$
$$\frac{5}{6}=\frac{10}{12}.$$

Hence, $$\frac{3}{4}+\frac{5}{6}=\frac{9}{12}+\frac{10}{12}=\frac{19}{12}=1\frac{7}{12}$$

Example 3. Add $4\frac{1}{2}$, $2\frac{5}{6}$, $1\frac{5}{12}$.

$$\begin{array}{ll} 4\frac{1}{2} & = 4\frac{6}{12} \\ 2\frac{5}{6} & = 2\frac{10}{12} \\ 1\frac{5}{12} & = 1\frac{5}{12} \\ \hline 8\frac{3}{4} & \end{array}$$

First add the fractions $\frac{1}{2}$, $\frac{5}{6}$, $\frac{5}{12}$. To do this, find the L. C. M. of the denominators. The L. C. M. of 2, 6, 12, is 12.

$\frac{1}{2}=\frac{6}{12}$, $\frac{5}{6}=\frac{10}{12}$, $\frac{5}{12}=\frac{5}{12}$. Adding $\frac{6}{12}$, $\frac{10}{12}$, $\frac{5}{12}$, the sum is $\frac{21}{12}=\frac{7}{4}=1\frac{3}{4}$.

Write $\frac{3}{4}$ in the sum and carry 1. Add next the integers 4, 2, 1, and the 1 carried. The sum is $8\frac{3}{4}$.

To add fractions, first reduce them to equivalent fractions having a common denominator. Then add the numerators, and underneath the sum write the common denominator.

If the resulting fraction is not in its simplest form, reduce it to its simplest form.

Thus: $\frac{3}{8}+\frac{4}{5}+\frac{13}{40}=\frac{15}{40}+\frac{32}{40}+\frac{13}{40}=\frac{60}{40}=1\frac{1}{2}$.

To add mixed numbers, first add the fractions, and to this sum add the sum of the integers.

EXERCISE 24

Find the sum of:

1. $\frac{1}{2}, \frac{1}{4}, \frac{1}{3}$.
2. $\frac{1}{2}, \frac{3}{4}, \frac{2}{3}$.
3. $\frac{1}{2}, \frac{1}{6}, \frac{3}{4}$.
4. $\frac{1}{2}, \frac{5}{8}, \frac{3}{4}$.
5. $\frac{1}{3}, \frac{5}{6}, \frac{7}{12}$.
6. $\frac{2}{3}, \frac{1}{6}, \frac{5}{12}$.
7. $\frac{1}{6}, \frac{5}{12}, \frac{1}{4}$.
8. $\frac{7}{15}, \frac{2}{5}, \frac{1}{5}$.
9. $\frac{3}{8}, \frac{3}{4}, \frac{1}{2}$.
10. $\frac{2}{5}, \frac{1}{2}, \frac{9}{10}$.
11. $\frac{7}{10}, \frac{1}{2}, \frac{3}{5}$.
12. $\frac{3}{10}, \frac{2}{5}, \frac{1}{2}$.
13. $\frac{2}{3}, \frac{5}{9}$.
14. $\frac{7}{24}, \frac{7}{12}, \frac{5}{6}$.
15. $\frac{1}{2}, \frac{1}{4}, \frac{1}{3}, \frac{11}{16}$.
16. $\frac{1}{2}, \frac{3}{4}, \frac{7}{8}, \frac{9}{16}$.
17. $\frac{7}{12}, \frac{1}{3}, \frac{1}{6}$.
18. $\frac{11}{18}, \frac{2}{3}, \frac{7}{9}$.
19. $\frac{3}{4}, \frac{7}{12}, \frac{17}{24}$.
20. $\frac{5}{6}, \frac{5}{12}, \frac{13}{24}$.
21. $\frac{3}{4}, \frac{7}{10}, \frac{17}{20}$.
22. $\frac{1}{4}, \frac{4}{5}, \frac{9}{10}$.
23. $\frac{1}{4}, \frac{2}{5}, \frac{7}{10}, \frac{11}{20}$.
24. $\frac{3}{5}, \frac{3}{10}, \frac{19}{20}$.
25. $\frac{1}{2}, \frac{1}{6}, \frac{1}{12}, \frac{5}{24}$.
26. $\frac{7}{8}, \frac{2}{3}, \frac{3}{4}$.
27. $\frac{5}{8}, \frac{1}{3}, \frac{7}{12}$.
28. $\frac{3}{8}, \frac{2}{3}, \frac{5}{12}$.
29. $\frac{3}{4}, \frac{7}{12}, \frac{5}{6}$.
30. $\frac{2}{3}, \frac{1}{6}, \frac{5}{12}, \frac{13}{24}$.
31. $1\frac{1}{2}, 2\frac{1}{3}, 3\frac{3}{4}$.
32. $2\frac{1}{6}, 5\frac{1}{2}, 7\frac{7}{12}$.
33. $3\frac{1}{4}, 2\frac{1}{2}, 2\frac{7}{8}$.
34. $5\frac{2}{3}, 7\frac{5}{6}, 11\frac{11}{12}$.
35. $9\frac{1}{6}, 7\frac{2}{3}, 10\frac{1}{12}$.
36. $10\frac{5}{12}, 9\frac{3}{4}, 2\frac{2}{3}$.
37. $9\frac{1}{2}, 1\frac{2}{5}, 6\frac{7}{10}$.
38. $2\frac{5}{12}, 3\frac{1}{6}, 7\frac{5}{24}$.
39. $9\frac{1}{3}, 7\frac{3}{8}, 2\frac{7}{12}$.
40. $4\frac{1}{4}, 5\frac{1}{6}, 7\frac{11}{12}$.
41. $8\frac{2}{5}, 3\frac{3}{4}, 12\frac{9}{10}$.
42. $6\frac{7}{8}, 4\frac{13}{32}, 3\frac{5}{16}$.
43. A boy has \$$\frac{1}{2}$ and \$$\frac{1}{5}$. What part of **\$1** has he?
44. How much is $\frac{1}{2} + \frac{1}{3} + \frac{1}{4}$ of an hour?
45. Which is the largest and which is the smallest of the three fractions, $\frac{2}{3}, \frac{3}{4}, \frac{5}{6}$?

SUBTRACTION

Example 1. From $\frac{4}{5}$ take $\frac{2}{3}$.

$$\frac{4}{5} = \frac{12}{15}.$$
$$\frac{2}{3} = \frac{10}{15}.$$
$$\frac{12}{15} - \frac{10}{15} = \frac{2}{15}.$$

To subtract fractions, reduce them to equivalent fractions having a common denominator. Then find the dif-

ference between the numerators, write it over the common denominator, and reduce the result to lowest terms.

Example 2. From $17\frac{3}{10}$ take $12\frac{3}{4}$.

$$\begin{array}{rl} 17\frac{3}{10} = & 17\frac{6}{20} \\ 12\frac{3}{4} = & 12\frac{15}{20} \\ \hline & 4\frac{11}{20} \end{array}$$

Reduce the fractions to 20ths. $\frac{15}{20}$ cannot be taken from $\frac{6}{20}$; take it from $1\frac{6}{20}$; that is, from $\frac{26}{20}$; and carry 1. 1 and 12 are 13; 13 and 4 are 17. The remainder is $4\frac{11}{20}$.

To subtract mixed numbers, reduce the fractional parts to equivalent fractions having a common denominator, and subtract the fractions; then subtract the integers. In case the fraction of the subtrahend is larger than the fraction of the minuend, subtract it from one plus the fraction in the minuend and carry one.

EXERCISE 25

Find the value of;

1. $\frac{3}{4} - \frac{1}{2}$.
2. $\frac{2}{3} - \frac{1}{2}$.
3. $\frac{2}{3} - \frac{1}{6}$.
4. $\frac{5}{6} - \frac{2}{3}$.
5. $\frac{9}{10} - \frac{1}{2}$.
6. $\frac{7}{8} - \frac{3}{4}$.
7. $\frac{9}{10} - \frac{2}{5}$.
8. $\frac{11}{12} - \frac{3}{4}$.
9. $\frac{7}{12} - \frac{1}{3}$.
10. $\frac{11}{12} - \frac{5}{6}$.
11. $\frac{19}{24} - \frac{2}{3}$.
12. $\frac{4}{5} - \frac{3}{10}$.
13. $3 - 1\frac{1}{2}$.
14. $4 - 1\frac{3}{4}$.
15. $5 - 3\frac{3}{8}$.
16. $6 - 4\frac{7}{10}$.
17. $11 - \frac{4}{5}$.
18. $13 - \frac{7}{8}$.
19. $14 - 5\frac{1}{6}$.
20. $15 - 3\frac{7}{12}$.
21. $28 - 21\frac{9}{10}$.
22. $17\frac{3}{4} - 10\frac{1}{2}$.
23. $18\frac{2}{3} - 11\frac{1}{4}$.
24. $5\frac{4}{5} - 2\frac{3}{10}$.
25. $29\frac{7}{8} - 24\frac{3}{4}$.
26. $33\frac{2}{3} - 17\frac{1}{6}$.
27. $9\frac{5}{12} - 3\frac{1}{4}$.
28. $32\frac{17}{24} - 30\frac{5}{12}$.
29. $31\frac{13}{24} - 20\frac{1}{12}$
30. $40\frac{2}{3} - 30\frac{11}{24}$.
31. $4\frac{1}{2} - 1\frac{2}{3}$.
32. $7\frac{2}{3} - 3\frac{3}{4}$.
33. $9\frac{1}{2} - 5\frac{7}{12}$.
34. $10\frac{1}{3} - 9\frac{3}{4}$.
35. $16\frac{1}{4} - 9\frac{2}{3}$.
36. $4\frac{2}{5} - 1\frac{7}{10}$.
37. $9\frac{3}{5} - 2\frac{9}{10}$.
38. $8\frac{1}{2} - 3\frac{9}{10}$.
39. $9\frac{3}{4} - 4\frac{11}{12}$.
40. $18\frac{1}{3} - 9\frac{3}{5}$.
41. $28\frac{1}{3} - 8\frac{4}{9}$.
42. $17\frac{1}{4} - 9\frac{7}{20}$.

43. What number must be added to $1\frac{1}{2}$ to make $7\frac{1}{8}$?

44. A man buys a suit of clothes for $\$12\frac{3}{4}$ and gives 3 five-dollar bills in payment. How much change should he receive?

45. From a piece of cloth containing $17\frac{2}{3}$ yards $14\frac{3}{4}$ yards are sold. How many yards are left?

46. A boy buys two books costing $\$\frac{3}{4}$ and $\$\frac{3}{5}$. How much change should he get out of a $\$2\frac{1}{2}$ gold piece?

CANCELLATION

The method of reducing a fraction to lowest terms by dividing out all factors common to numerator and denominator is very useful in reducing fractions whose terms are continued products of several factors. Thus, in the fraction $\frac{9 \times 8 \times 3}{18 \times 27 \times 16}$, the terms are indicated products instead of each being one number. If each term were one number, we should reduce the fraction by dividing out the factors common to numerator and denominator. Therefore, we may do this with the given fraction; thus, 9 in 9, goes once; 9 in 18, goes twice: 8 in 8, once; 8 in 16, twice: 3 in 3, once; 3 in 27, nine times. The numerator reduces to $1 \times 1 \times 1$, or 1. The denominator reduces to $2 \times 9 \times 2$, or 36. The fraction becomes $\frac{1}{36}$.

The usual form for the work is:

$$\frac{\overset{1}{\cancel{9}} \times \overset{1}{\cancel{8}} \times \overset{1}{\cancel{3}}}{\underset{2}{\cancel{18}} \times \underset{9}{\cancel{27}} \times \underset{2}{\cancel{16}}} = \frac{1}{36}; \text{ also } \frac{\overset{3}{\cancel{75}} \times \overset{\cancel{9}}{\cancel{63}} \times \cancel{3}}{\underset{7}{\cancel{175}} \times \underset{\underset{10}{\cancel{30}}}{\cancel{270}}} = \frac{3}{10}.$$

This process of reducing a fraction by dividing out common factors is called **cancellation**.

Cancellation is very useful for shortening the work of division and should be used in all cases where it is possible.

EXERCISE 26

Find by cancellation the quotient:

1. $\frac{4 \times 18 \times 3 \times 24}{9 \times 4 \times 144}$.

2. $\frac{18 \times 90 \times 105}{14 \times 25 \times 3}$.

3. $\frac{27 \times 64 \times 8}{108 \times 32}$.

4. $\frac{343 \times 125}{35 \times 35}$.

5. $\frac{16 \times 16 \times 8 \times 81}{64 \times 32 \times 3}$.

6. $\frac{225 \times 216}{75 \times 9 \times 12}$.

7. $\frac{108 \times 27 \times 121}{22 \times 33 \times 18}$.

8. $\frac{22 \times 88 \times 15}{132 \times 4}$.

9. $\frac{1760 \times 99}{4 \times 88 \times 165}$.

10. $\frac{5280 \times 14}{176 \times 84}$.

11. $\frac{1728 \times 34}{27 \times 136 \times 8}$.

12. $\frac{640 \times 5200}{125 \times 512 \times 13}$.

13. $\frac{2380 \times 104}{119 \times 8 \times 13}$.

14. $\frac{111 \times 39 \times 12}{74 \times 27 \times 13}$.

15. A farmer exchanged 320 acres of land worth \$50 an acre for 25 city lots. Find the average price of a lot.

$$(\text{price of a lot}) \times 25 = \$50 \times 320.$$

Hence, price of lot $= \frac{\$50 \times 320}{25} = \$640$.

16. How many cows at \$40 a head cost as much as 15 horses at \$64 a head?

17. How many dozen eggs at 35¢ a dozen must be sold to pay for 7 barrels of apples at \$2.10 a barrel?

18. A laborer receives \$3.20 a day. How many days must he work to pay for 6 tons of coal at \$8 per ton?

19. A bicyclist rides at the rate of 9 miles an hour. How long will it take him to travel as far as a train goes in 6 hours at the rate of 33 miles an hour?

20. How many cattle at \$42 a head must be sold to pay for 11,200 bushels of wheat at 75¢ a bushel?

MULTIPLICATION OF FRACTIONS

Multiplication of a fraction by an integer.

Example 1. Multiply $\frac{3}{4}$ by 18.

$\frac{3}{4} \times 18 = 3$ fourths $\times\ 18 = 54$ fourths $= \frac{54}{4} = 13\frac{2}{4} = 13\frac{1}{2}$.

Example 2. Multiply $3\frac{3}{10}$ by 14.

$3\frac{3}{10} \times 14 = (3 \times 14) + (\frac{3}{10} \times 14) = 42 + 4\frac{1}{5} = 46\frac{1}{5}$.

To multiply a mixed number by an integer, first multiply the fractional part of the mixed number by the multiplier, next multiply the integral part by the multiplier; add the two results for the final product.

EXERCISE 27

Find the value of:

1. $\frac{2}{3} \times 18$.	**9.** $\frac{3}{14} \times 24$.	**17.** $1\frac{2}{3} \times 10$.	**25.** $7\frac{4}{11} \times 33$.
2. $\frac{3}{4} \times 14$.	**10.** $\frac{5}{16} \times 40$.	**18.** $1\frac{7}{8} \times 12$.	**26.** $8\frac{5}{11} \times 19$.
3. $\frac{4}{5} \times 7$.	**11.** $\frac{9}{16} \times 34$.	**19.** $3\frac{3}{8} \times 10$.	**27.** $9\frac{5}{12} \times 80$.
4. $\frac{9}{10} \times 18$.	**12.** $\frac{11}{12} \times 42$.	**20.** $5\frac{7}{12} \times 54$.	**28.** $6\frac{11}{12} \times 102$.
5. $\frac{7}{10} \times 16$.	**13.** $\frac{13}{16} \times 44$.	**21.** $6\frac{7}{8} \times 36$.	**29.** $7\frac{7}{16} \times 60$.
6. $\frac{3}{8} \times 12$.	**14.** $\frac{19}{20} \times 50$.	**22.** $9\frac{1}{2} \times 16$.	**30.** $11\frac{1}{10} \times 15$.
7. $\frac{5}{8} \times 19$.	**15.** $\frac{17}{20} \times 45$.	**23.** $7\frac{5}{8} \times 44$.	**31.** $12\frac{3}{10} \times 18$.
8. $\frac{7}{8} \times 28$.	**16.** $1\frac{3}{4} \times 9$.	**24.** $10\frac{5}{16} \times 24$.	**32.** $10\frac{2}{9} \times 13$.

33. Find the cost of a dozen cans of baking powder at $37\frac{1}{2}$ cents a can.

34. *A pail of mackerel cost $2\frac{1}{8}$ dollars. Find the cost of 20 pails.*

35. Find the cost of a barrel of sugar weighing 325 pounds, if the cost per pound is $4\frac{11}{16}$¢.

36. When starch sells for $3\frac{1}{2}$¢ a pound, find the price of 15 pounds of starch.

37. When wheat is $79\frac{1}{8}$ cents a bushel, how much will 164 bushels bring?

38. Find the cost of a 75-pound chest of Hyson tea at $42\frac{1}{2}$¢ per pound.

39. If a dozen cakes of yeast cost $42\frac{3}{4}$¢, find the cost of 9 dozen cakes of yeast.

40. Pepper sells for $14\frac{3}{4}$¢ a pound. Find the cost of 2 bags, each containing 120 pounds.

41. A pound package of chocolate costs $31\frac{1}{4}$¢. Find the cost of 25 such packages.

42. A square rod equals $30\frac{1}{4}$ square yards. Reduce 160 square rods to square yards.

43. A link of a surveyor's chain is $7\frac{23}{25}$ inches. If the chain contains 100 links, how many inches long is the chain?

44. When silk sells at $\$\frac{17}{20}$ a yard, what is the cost of 14 yards of silk?

45. A degree on a meridian of the earth's surface is about $69\frac{1}{10}$ miles. How many miles in 15 degrees? in 40 degrees?

46. A person owes debts amounting to \$9800. He has money with which to pay only $\$\frac{3}{4}$ for every dollar that he owes. How much money does he pay?

47. A mass of copper and lead weighs 2240 pounds; $\frac{4}{5}$ of the mass is copper. How much copper and how much lead in the mass?

48. A man invests \$2300. At the end of a year his gain is $\frac{7}{20}$ of his investment. Find his gain and the value of the investment at the end of the year.

Multiplication of a fraction by a fraction

Find $\frac{1}{3}$ of 6; $\frac{1}{3}$ of 6 apples; $\frac{1}{3}$ of 6 sevenths; $\frac{1}{3}$ of $\frac{6}{7}$.
Find $\frac{1}{4}$ of 12; $\frac{1}{4}$ of 12 apples; $\frac{1}{4}$ of 12 nineteenths; $\frac{1}{4}$ of $\frac{12}{19}$.
Find $\frac{2}{3}$ of 6; $\frac{2}{3}$ of 6 apples; $\frac{2}{3}$ of 6 sevenths; $\frac{2}{3}$ of $\frac{6}{7}$.
Find $\frac{3}{4}$ of 12; $\frac{3}{4}$ of 12 apples; $\frac{3}{4}$ of 12 nineteenths; $\frac{3}{4}$ of $\frac{12}{19}$.

Example 1. Find $\frac{4}{5} \times \frac{2}{3}$.

$\frac{4}{5} \times \frac{2}{3}$, which is the same as $\frac{4}{5}$ of $\frac{2}{3} = \frac{1}{5}$ of $4 \times \frac{2}{3} = \frac{1}{5}$ of $\frac{8}{3}$

$$= \frac{8}{5 \times 3} = \frac{8}{15}.$$

More briefly, $\frac{4}{5} \times \frac{2}{3} = \frac{4 \times 2}{5 \times 3} = \frac{8}{15}$.

Example 2. Multiply $3\frac{3}{8}$ by $2\frac{7}{9}$.

$$3\tfrac{3}{8} \times 2\tfrac{7}{9} = \frac{27}{8} \times \frac{25}{9} = \frac{\overset{3}{\cancel{27}} \times 25}{8 \times \cancel{9}} = \frac{75}{8} = 9\tfrac{3}{8}.$$

To multiply a fraction by a fraction, multiply the numerators together for the new numerator and the denominators together for the new denominator.

To multiply mixed numbers, reduce them to improper fractions and multiply.

EXERCISE 28

Multiply:

1. $\frac{1}{2}$, $\frac{2}{3}$.	**4.** $\frac{7}{8}$, $\frac{9}{10}$.	**7.** $\frac{16}{19}$, $\frac{57}{74}$.
2. $\frac{1}{2}$, $\frac{3}{4}$.	**5.** $\frac{3}{8}$, $\frac{17}{25}$.	**8.** $\frac{17}{31}$, $\frac{93}{119}$.
3. $\frac{4}{5}$, $\frac{10}{11}$.	**6.** $\frac{7}{12}$, $\frac{19}{28}$.	**9.** $\frac{14}{25}$, $\frac{75}{112}$.

10. $\frac{12}{27}$, $\frac{45}{64}$.
11. $\frac{15}{29}$, $\frac{87}{105}$.
12. $\frac{16}{35}$, $\frac{105}{112}$.
13. $\frac{3}{7}$, $\frac{11}{16}$, $\frac{8}{33}$.
14. $\frac{4}{9}$, $\frac{15}{18}$, $\frac{27}{40}$.
15. $\frac{4}{13}$, $\frac{26}{45}$, $\frac{9}{16}$.
16. $\frac{9}{20}$, $\frac{70}{81}$, $\frac{27}{28}$.
17. $\frac{11}{20}$, $\frac{90}{121}$, $\frac{33}{45}$.
18. $\frac{16}{55}$, $\frac{11}{48}$, $\frac{15}{17}$.
19. $\frac{1}{4}$, $\frac{7}{9}$, $\frac{72}{101}$.
20. $\frac{3}{5}$, $\frac{3}{9}$, $\frac{135}{243}$.
21. $1\frac{2}{3}$, $1\frac{2}{3}$.
22. $1\frac{1}{2}$, $1\frac{1}{2}$, $1\frac{1}{2}$.
23. $2\frac{1}{7}$, $\frac{7}{15}$, $\frac{3}{4}$.
24. $3\frac{3}{4}$, $\frac{4}{15}$, $\frac{4}{5}$.
25. $2\frac{2}{3}$, $2\frac{2}{3}$, $\frac{5}{64}$.
26. $7\frac{1}{4}$, $1\frac{3}{4}$, $\frac{16}{29}$.
27. $2\frac{5}{8}$, $3\frac{3}{5}$, $\frac{20}{21}$.
28. $4\frac{1}{6}$, $2\frac{3}{5}$, $\frac{64}{65}$.
29. $7\frac{1}{2}$, $7\frac{1}{2}$, $\frac{4}{15}$.
30. $3\frac{1}{3}$, $3\frac{1}{3}$, $\frac{7}{25}$.
31. $5\frac{1}{2}$, $5\frac{1}{2}$, $\frac{8}{11}$.
32. $7\frac{3}{4}$, $3\frac{1}{2}$, $\frac{61}{62}$.
33. $1\frac{4}{5}$, $\frac{5}{18}$, $\frac{2}{3}$.
34. $7\frac{2}{3}$, $1\frac{7}{8}$, $\frac{45}{46}$.
35. $4\frac{1}{4}$, $4\frac{1}{4}$, $3\frac{5}{9}$.
36. $1\frac{1}{5}$, $1\frac{1}{6}$, $\frac{5}{7}$.
37. $20\frac{1}{2}$, $20\frac{1}{2}$.
38. $4\frac{1}{2}$, $5\frac{1}{2}$, $1\frac{3}{11}$.
39. $6\frac{3}{4}$, $1\frac{7}{9}$, $\frac{7}{12}$.
40. $2\frac{2}{5}$, $1\frac{5}{12}$, $\frac{10}{17}$.
41. $2\frac{5}{7}$, $8\frac{3}{5}$, $11\frac{9}{13}$.
42. $6\frac{7}{8}$, $4\frac{2}{9}$, $13\frac{11}{15}$.

43. Find the value of $63\frac{1}{2}$ acres of land at \$$55\frac{1}{2}$ an acre.

44. Find the cost of $12\frac{1}{2}$ pounds of meat at $15\frac{1}{2}$ cents per pound.

45. Find the price of $4\frac{1}{2}$ bushels of wheat at $81\frac{1}{4}$ cents per bushel.

46. A speculator buys 10,000 bushels of wheat at $79\frac{5}{8}$ cents per bushel and sells it at $81\frac{1}{4}$ cents per bushel. Find his profit.

DIVISION OF FRACTIONS

REQUIRED. To divide $\frac{12}{23}$ by 6.

$\frac{12}{23}$ may be written 12 twenty-thirds.

12 twenty-thirds divided by 6 gives 2 twenty-thirds. Therefore, $\frac{12}{23} \div 6 = \frac{2}{23}$. But this is the same as if we wrote $\frac{1}{6} \times \frac{12}{23}$ and multiplied, using cancellation.

To divide a fraction by a whole number, multiply the fraction by a fraction having 1 for the numerator and the whole number for the denominator.

REQUIRED. To divide $\frac{8}{15}$ by $\frac{2}{3}$. $\frac{8}{15}$ may be written 8 fifteenths; and $\frac{2}{3}$ may be written 2 thirds. The divi-

dend is fifteenths, the divisor is thirds; and the division is impossible until we can make them alike in kind. We can do this by changing to a common denominator. The common denominator is 15; and the example becomes, — divide 8 fifteenths by 10 fifteenths. 8 fifteenths $\div$ 10 fifteenths gives $\frac{8}{10}$, which reduces to $\frac{4}{5}$.

Therefore, **to divide one fraction by another, change to a common denominator and divide the numerator of the dividend by the numerator of the divisor.**

Another Method

If a product and one factor are given, the other factor can be found by dividing the product by the given factor.

We found that $\frac{8}{15}$ is the product of $\frac{2}{3}$ by another fraction, the 2 being multiplied by the other numerator to give 8, and the 3 being multiplied by the other denominator to give 15. So if we divide 8 by 2 the quotient will be the numerator sought; and if we divide 15 by 3, the quotient will be the denominator sought. $8 \div 2 = 4$. $15 \div 3 = 5$. Therefore, $\frac{8}{15} \div \frac{2}{3} = \frac{4}{5}$.

If we write $\frac{8}{15} \times \frac{3}{2}$, and cancel, we see that the 8 is divided by the 2, and the 15 is divided by the 3, giving $\frac{4}{5}$ as before.

Observe that $\frac{8}{15} \div \frac{2}{3}$ gives the same result as $\frac{8}{15} \times \frac{3}{2}$.

Hence the rule for division of fractions: **Invert the divisor and multiply.**

This is a shorter method than to find a common denominator, divide the numerators and reduce the quotient to lowest terms as above, but the result is the same.

The quantity obtained by inverting a fraction is called its **reciprocal.** Thus, the reciprocal of $\frac{2}{3}$ is $\frac{3}{2}$, of $\frac{5}{3}$ is $\frac{3}{5}$, of $\frac{1}{7}$ is $\frac{7}{1}$, or 7.

The quantity obtained by dividing 1 by a number is

called the **reciprocal** of the number. It is the same as the fraction inverted. Thus, $\frac{1}{8}$ is the reciprocal of 8.

The same is true if the number is a fraction. Thus, $\frac{1}{\frac{2}{3}} = 1 \div \frac{2}{3} = 1 \times \frac{3}{2} = \frac{3}{2}$, or the fraction inverted.

Therefore: **To divide by a fraction, multiply by its reciprocal.**

This rule should be memorized, as it includes the other rules for division of fractions.

For division of mixed numbers, change the mixed numbers to simple fractions and then divide according to the rule for dividing fractions.

EXERCISE 29

Divide:

1. $\frac{18}{25}$ by 27.
2. 1 by $\frac{9}{82}$.
3. $\frac{2}{5}$ by $\frac{4}{18}$.
4. $\frac{20}{27}$ by 16.
5. $3\frac{3}{4}$ by 10.
6. $\frac{3}{10}$ by $\frac{6}{11}$.
7. $\frac{9}{14}$ by 12.
8. $2\frac{1}{8}$ by 3.
9. $\frac{7}{12}$ by $\frac{14}{15}$.
10. $\frac{12}{13}$ by 16.
11. $3\frac{5}{9}$ by 8.
12. $\frac{8}{9}$ by $\frac{4}{5}$.
13. $\frac{10}{13}$ by 15.
14. $4\frac{5}{11}$ by 14.
15. $\frac{9}{14}$ by $\frac{7}{12}$.
16. $\frac{10}{11}$ by 25.
17. $5\frac{5}{13}$ by 21.
18. $\frac{8}{15}$ by $\frac{4}{25}$.
19. $\frac{14}{15}$ by 21.
20. $9\frac{7}{12}$ by 46.
21. $\frac{9}{28}$ by $\frac{18}{35}$.
22. $\frac{18}{21}$ by 36.
23. $9\frac{9}{10}$ by 77.
24. $22\frac{10}{13}$ by 8.
25. 12 by $\frac{2}{3}$.
26. $9\frac{3}{13}$ by 45.
27. $4\frac{4}{5}$ by 9.
28. 14 by $\frac{2}{7}$.
29. $16\frac{4}{5}$ by 63.
30. $4\frac{2}{3}$ by $1\frac{3}{4}$.
31. 16 by $\frac{4}{7}$.
32. $18\frac{1}{5}$ by 26.
33. $3\frac{3}{4}$ by $2\frac{5}{7}$.
34. 18 by $\frac{6}{7}$.
35. $17\frac{1}{7}$ by 75.
36. $4\frac{3}{8}$ by $9\frac{2}{7}$.
37. 21 by $\frac{3}{4}$.
38. $10\frac{2}{7}$ by 48.
39. $7\frac{1}{5}$ by $3\frac{1}{2}$.
40. 25 by $\frac{5}{6}$.
41. $9\frac{2}{7}$ by 52.
42. $4\frac{1}{17}$ by $17\frac{1}{4}$
43. 26 by $\frac{4}{5}$.
44. $9\frac{1}{11}$ by 75.
45. $3\frac{1}{3}$ by $\frac{7}{9}$.
46. 26 by $\frac{13}{14}$.
47. $8\frac{1}{3}$ by 15.
48. 1 by $4\frac{3}{4}$.

49. 33 by $\frac{6}{11}$.
50. $7\frac{9}{13}$ by 80.
51. 1 by $7\frac{1}{7}$.
52. 40 by $\frac{5}{12}$.
53. $5\frac{15}{17}$ by 50.
54. 1 by $9\frac{2}{3}$.
55. 60 by $\frac{8}{9}$.
56. $8\frac{5}{8}$ by 46.
57. $10\frac{2}{3}$ by $2\frac{2}{3}$.
58. 11 by $\frac{2}{9}$.
59. $\frac{5}{14}$ by $\frac{3}{7}$.
60. $14\frac{2}{3}$ by $1\frac{11}{12}$.
61. 1 by $\frac{1}{2}$.
62. $\frac{1}{3}$ by $\frac{5}{12}$.
63. $\frac{19}{36}$ by $1\frac{7}{12}$.
64. 1 by $\frac{3}{4}$.
65. $\frac{9}{10}$ by $\frac{7}{12}$.
66. $\frac{14}{15}$ by $1\frac{3}{11}$.

Example **1.** Find the price of 2000 pounds of wheat at 84¢ a bushel. There are 60 pounds in a bushel.

To solve this question there are two steps to take.

Step 1. Find the number of bushels by dividing the number of pounds by the number of pounds in one bushel.

Step 2. Multiply the price of one bushel by the number of bushels.

SOLUTION. Number of bushels $= \frac{2000}{60}$.

$$\text{Price of the wheat} = 84¢ \times \frac{2000}{60} = \frac{\overset{14}{\cancel{84}} \times 200\cancel{0}}{\underset{\cancel{6}}{\cancel{6}\cancel{0}}}$$

$$= 2800¢ = \$28.$$

Example **2.** Three fifths of a man's money is \$2437. How much money has he?

$$\tfrac{3}{5} \text{ of his money} = \$2437.$$

$$\tfrac{1}{5} \text{ of his money} = \frac{\$2437}{3} \text{ or } \tfrac{1}{3} \text{ of } \$2437.$$

$$\tfrac{5}{5} \text{ of his money} = \frac{\$2437}{3} \times 5 \text{ or } \tfrac{5}{3} \text{ of } \$2437.$$

$$= \frac{\$12185}{3} = \$4061.66\tfrac{2}{3}.$$

Hence, his money = \$4061.67 (to nearest cent).

This method of solving a problem is known as the **analytical method.** It is called also the **unit method,** because the value of the unit of the quantity under consideration is first sought and from this the value of any number of units is then obtained.

Note that the answer is obtained by multiplying $2437 by $\frac{5}{3}$, the reciprocal of $\frac{3}{5}$. In this problem there are given the product of two factors and one of the factors. The other factor is sought. The problem is therefore one of division.

EXERCISE 30

1. Find the price of 78 acres of land if 25 acres are worth $1,375.

2. When 18 pounds of sugar sell for $1, find the cost of 45 pounds.

3. When 7 bushels of wheat sell for $5.95, how much can a person get for 255 bushels?

4. If 5 bushels of barley sell for $2, how much will 343 bushels sell for?

5. If 6 barrels of flour are sold for $45, at this rate how much will 84 barrels sell for?

6. Seven barrels of pork sell for $80.50. Find the cost of 50 barrels of pork.

7. Nine barrels of salt cost $11.70. Find the cost of 19 barrels of salt.

8. Eleven bushels of oats are sold for $4.51. Find the value of 168 bushels.

9. Six barrels of lard bring $115. How much will 46 barrels bring?

10. When 7 yards of sheeting cost 50¢, find how much must be paid for 98 yards.

11. Six yards of cambric sell for 75¢. How much must be given for 34 yards of cambric?

12. Four yards of flannel cost $1.15. How much will 29 yards of flannel cost?

13. Eight yards of gingham cost 60¢. How much will 103 yards cost?

14. Nine yards of cotton fabric cost 75¢. How much will 69 yards cost?

15. Six yards of cotton cheviot cost $1. How much will 81 yards cost?

16. Five eighths of a man's money is $75. How much money has he?

17. Three fourths of the length of a pole is 81 feet. Find the length of the pole.

18. The sum of the eighth and the twelth of a number is 15. What is the number?

19. A dealer sold $\frac{1}{3}$ of his coal and had 170 tons left. How many tons had he at first?

20. The sum of the fourth part and the sixth part of a number is 25. What is the number?

COMPLEX FRACTIONS

A **complex fraction** is a fraction at least one of whose terms contains a fraction or fractions.

Thus, $\frac{2\frac{1}{2}}{7}$, $\frac{3}{4\frac{1}{5}}$, $\frac{1+\frac{1}{2}-\frac{1}{3}}{\frac{2}{5}+\frac{2}{3}}$, are complex fractions.

Such expressions merely indicate that the quantity in the numerator is to be divided by the quantity in the denominator. The numerator and denominator should each be simplified and then the numerator should be divided

by the denominator by the usual methods for the division of fractions.

Example 1. Simplify $\dfrac{2\frac{2}{3}}{1\frac{5}{6}}$.

$$2\tfrac{2}{3} \div 1\tfrac{5}{6} = \tfrac{8}{3} \div \tfrac{11}{6} = \tfrac{8}{3} \times \tfrac{6}{11} = \tfrac{16}{11} = 1\tfrac{5}{11}.$$

Example 2. Simplify $\dfrac{3\frac{6}{25} - 2\frac{1}{4}}{1\frac{4}{5} - 1\frac{1}{2}}$.

The numerator, $3\frac{6}{25} - 2\frac{1}{4}$, reduces to $\frac{99}{100}$.
The denominator, $1\frac{4}{5} - 1\frac{1}{2}$, reduces to $\frac{3}{10}$.

$$\tfrac{99}{100} \div \tfrac{3}{10} = \tfrac{99}{100} \times \tfrac{10}{3} = 3.3.$$

EXERCISE 31

Simplify:

1. $\dfrac{4\frac{1}{2}}{39}$. 4. $\dfrac{3\frac{3}{8}}{18}$. 7. $\dfrac{11\frac{7}{8}}{6\frac{10}{11}}$. 10. $\dfrac{19\frac{1}{4}}{46\frac{1}{5}}$. 13. $\dfrac{2\frac{14}{25} - \frac{4}{9}}{2\frac{4}{15}}$.

2. $\dfrac{3\frac{2}{3}}{22}$. 5. $\dfrac{5\frac{3}{8}}{9\frac{5}{9}}$. 8. $\dfrac{13\frac{5}{7}}{9\frac{3}{5}}$. 11. $\dfrac{25\frac{1}{4}}{40\frac{2}{5}}$. 14. $\dfrac{1\frac{24}{25} - \frac{1}{9}}{2\frac{2}{15}}$.

3. $\dfrac{4\frac{4}{5}}{12}$. 6. $\dfrac{7\frac{2}{3}}{9\frac{1}{5}}$. 9. $\dfrac{14\frac{2}{3}}{26\frac{2}{5}}$. 12. $\dfrac{1\frac{7}{9} - \frac{49}{64}}{2\frac{5}{24}}$. 15. $\dfrac{\frac{4}{7} - \frac{1}{3} + \frac{3}{4}}{\frac{8}{35} - \frac{2}{15} + \frac{6}{20}}$.

16. $\dfrac{\frac{6}{7} + 1\frac{1}{2} - 1\frac{7}{8}}{1\frac{4}{7} - 1\frac{1}{4}}$. 17. $\dfrac{\frac{5}{8} + \frac{3}{4} \text{ of } 1\frac{1}{5}}{\frac{18}{25} + \frac{1}{2}}$. 18. $\dfrac{1\frac{4}{5} - \frac{2}{3} \text{ of } 1\frac{1}{4} \text{ of } 1\frac{1}{5}}{1\frac{1}{20} - \frac{2}{9} \text{ of } 2\frac{3}{8}}$.

19. $\dfrac{\frac{27}{64} - \frac{8}{27}}{\frac{5}{12}} \div 12\frac{1}{18}$. 20. $\dfrac{\frac{3}{4} \text{ of } \frac{5}{6} \div 1\frac{2}{3}}{2\frac{1}{4} \times \frac{5}{18} \text{ of } \frac{3}{5}} \times 1\frac{1}{4} \times 1\frac{1}{5}$.

DECIMALS

Fractions whose denominators are 10, 100, 1000, or some other power of 10, are usually written as decimals.

Before taking the following work on decimals, review thoroughly the explanation and exercises in decimals on pages 7 and 8.

ADDITION OF DECIMALS

Find the sum of 3.4, 2.38, 5.005, 6.2374, 11.1.

3.4
2.38
5.005
6.2374
11.1
28.1224

Write the numbers in a column so that the decimal point of each number is directly below the decimal point of the next number above. This brings all figures of the same denomination into the same vertical column.

Then add as in the case of integers, being careful to keep each figure in the vertical column to which it belongs.

Write the decimal point of the sum in the same vertical line with the decimal points of the numbers added.

EXERCISE 32

Add:

1. 2.2, .025, 37.3, 5.284, 6.294, 538.1, 77.77.
2. 3.5, 7.12, .339, 47.35, 39.28, .123, 54.275.
3. 9.28, 11.18, .999, 39.28, 7.451, 94.354, 98.76.
4. 12.49, 1.492, 38.75, 53.41, 98.69, 845.5, 892.9.
5. .009, 5.976, 40.99, 6.385, 9.278, 8.239, 64.271.
6. .098, 9.853, 19.47, 17.392, 28.394, 8.01, 77.47.
7. .285, 11.95, 29.99, 94.931, 1.732, 64.6, 78.75.
8. 11.4, 17.5, 99.37, 15.273, 9.394, 71.3, 92.95.
9. 1.21, 12.1, .121, 8.295, 7.777, 68.7, 78.28.
10. 15.9, 9.158, 91.58, 9.158, 2.293, 84.5, .139.
11. 98.5, 11.667, 66.66, 8.394, 9.928, 76.8, 9.359.
12. 77.8, 88.88, 99.99, 6.325, 7.384, 94.9, 1.798.
13. 1.412, 1.732, 3.142, 62.85, 19.76, 856.2.
14. 631.5, 729.8, 65.47, 18.19, 343.7, 685.9.

SUBTRACTION OF DECIMALS

Find the difference between 4001 and 1.7003.

$$\begin{array}{r} 4001.0000 \\ 1.7003 \\ \hline 3999.2997 \end{array}$$

Arrange the numbers so that the decimal points stand in the same vertical column. Ciphers may be inserted at the right of either number to give it as many decimal places as the other number. Perform the subtraction as if the numbers were integers. Place the decimal point in the remainder in the same vertical column as the decimal points of the minuend and subtrahend.

EXERCISE 33

Find the remainder and verify your answer in each case:

1. 7.73 − 6.78.
2. 9.29 − 3.47.
3. 6.34 − 1.95.
4. 9.82 − 7.78.
5. 7.45 − 3.59.
6. 10.71 − 7.79.
7. 8.94 − 3.95.
8. 5.012 − 2.9.
9. 10.943 − 7.97.
10. 8.325 − 4.378.
11. 8.924 − 5.938.
12. 7.312 − 2.7.
13. 9.419 − 5.57.
14. 10.1 − 7.325.
15. 9.24 − 5.3481.
16. 8.73 − 4.4444.
17. 12.32 − 5.6741.
18. 19.33 − 6.2734.
19. 9.271 − 4.3847.
20. 3.213 − .9875.
21. 4.321 − .73201.
22. 5.204 − 1.3256.
23. 8.731 − 5.4557.
24. 9.21 − 7.2349.
25. 7.29 − 3.4551.
26. 6.001 − 5.112.
27. From seven hundred four thousandths take two hundred five ten-thousandths.
28. From five hundred ten thousandths take five hundred ten-thousandths.

29. From two thousand take two thousandths.

30. How much does one thousandth exceed one hundred-thousandth?

31. Find the difference between a hundred and a hundredth.

32. From 39 tenths take 39 thousandths.

33. From 100 hundredths take 100 ten-thousandths.

34. How much must be added to one and five tenths to make ten?

MULTIPLICATION AND DIVISION BY POWERS OF TEN

Consider the two numbers,

(*a*) 320.12,
(*b*) 3201.2.

Both are expressed by the same figures written in the same order. The number (*b*) can be obtained from the number (*a*) by moving each figure one place to the left. But moving a digit one place to the left makes its value ten times as great, and, hence, moving all the digits of a number one place to the left makes the number they represent ten times as great.

The number (*b*) can also be obtained from (*a*) by moving the decimal point in (*a*) one place to the right. Also (*a*) can be obtained from (*b*) by moving the decimal point in (*b*) one place to the left.

To multiply a number by 10, move the decimal point in the number one place to the right.

To divide a number by 10, move the decimal point in the number one place to the left.

Consider the numbers,

(*a*) 320.12,
(*b*) 32,012.

The number (*b*) is obtained from (*a*) by moving each digit in (*a*) two places to the left. This multiplies each digit by 100.

(*b*) may also be obtained from (*a*) by moving the decimal point in (*a*) two places to the right; also (*a*) from (*b*) by moving the decimal point two places to the left.

To multiply a number by 100, move the decimal point in the number two places to the right.

To divide by 100, move the decimal point in the dividend two places to the left.

Consider the numbers,

(*a*) 320.12,
(*b*) 320,120.

(*b*) is here obtained from (*a*) by moving each digit in (*a*) three places to the left. (*b*) can also be obtained from (*a*) by moving the decimal point in (*a*) three places to the right.

To multiply a number by any power of 10, move the decimal point as many places to the right in the number as there are zeros in the multiplier.

To divide by any power of 10, move the decimal point as many places to the left in the number as there are zeros in the divisor.

Example 1. Multiply 86.4 by 10,000. Moving the decimal point four places to the right, the number becomes 864,000.

Example 2. Divide 12.3 by 100,000. Moving the decimal point five places to the left, the number becomes .000123.

EXERCISE 34

Multiply by 10:

1. 120, 14.2, .1431, .00012, 1.7320, .01234.

Multiply by 100:

2. 173, 172.8, 19.23, .001237, 8654, 17.1.

Multiply by 1000:

3. 1156, 32.5, 7.123, .93891, .01275, .00011.

Multiply by 10,000:

4. 345, 34.25, 5.1739, 6.001, .01793, .12.

5. Divide each of the following numbers by 10; by 100; by 1000; by 10,000; by 100,000:

32,734	9,285.773
3,745.3	325.298
928.49	127
72,173.5	325
12.792	17
99,999.9	18.326
1,201	1,000
3,450	7,100

MULTIPLICATION OF DECIMALS

Example 1. Multiply 3.23 by 25.

$$3.23 = 323 \text{ hundredths.}$$

$$323 \text{ hundreths} \times 25 = 8075 \text{ hundredths} = 80.75.$$

Example 2. Multiply 3.23 by .25.

$$.25 = \tfrac{25}{100} = \tfrac{1}{100} \text{ of } .25.$$

Therefore, $3.23 \times .25 = \tfrac{1}{100}$ of 3.23×25.

$$323 \times 25 = 80.75.$$

$$\tfrac{1}{100} \text{ of } 80.75 = .8075.$$

The multiplication may be performed as follows:

$$\begin{array}{r} 3.23 \\ .25 \\ \hline 1615 \\ 636 \\ \hline .8075 \end{array}$$

Multiply as if both numbers were integers. Then, in the product, place the decimal point so that the number of figures at the right of the decimal point equals the number of figures at the right of the decimal point in the multiplicand, plus the number of figures at the right of the decimal point in the multiplier.

Example 3. Multiply .32 by .018.

$$\begin{array}{r} .018 \\ .32 \\ \hline 36 \\ 54 \\ \hline .00576 \end{array}$$

Point off five places.

EXPLANATION

$$\tfrac{32}{100} \times \tfrac{18}{1000} = \tfrac{576}{100000} = .00576.$$

EXERCISE 35

1. Find .04 of $108; .05 of $274; .06 of $720; .07 of $144.

2. Find .09 of $34.50; .3 of $75.30; .08 of $75.80; .07 of $84.70.

3. Find .4 of $29.75; .5 of $69.48; .6 of $68.32; .1 of $328.50.

4. Find .125 of $80.80; .75 of $54; .6 of $300.50; .25 of $98.84.

5. Find .625 of $688; .875 of $792.80; .375 of $900.80.

6. Find .375 of 84 acres; .0625 of 64 acres; .3125 of 96 acres.

7. Find .1 of .1; .3 of .4; .3 of .3; .01 of .2; .01 of 1.2.

8. Multiply 27.9 by 18.
9. Multiply 1327 by 1.6.
10. Multiply 3927 by .46.
11. Multiply 120.01 by 3.6.
12. Multiply 25 by .017.
13. Multiply 37.5 by .07.
14. Multiply 11.9 by 2.4.
15. Multiply 182.54 by 1.49.
16. Multiply .286 by 1.96..
17. Multiply 92.24 by 2.7.
18. $.148 \times 1.15 = ?$
19. $.82 \times .51 = ?$
20. $1.875 \times .32 = ?$
21. $1.78 \times 1.89 = ?$
22. $18.24 \times .95 = ?$
23. $1.18 \times .1695 = ?$
24. $.97 \times .97 = ?$
25. $.68 \times .68 = ?$
26. $.373 \times .373 = ?$
27. $.901 \times .901 = ?$
28. $.803 \times .803 = ?$
29. $.693. \times .693 = ?$
30. $.1 \times .1 \times .1 = ?$
31. $.3 \times .3 \times .3 = ?$
32. $.4 \times .4 \times .4 = ?$
33. $.7 \times .7 \times .7 = ?$
34. $1.04 \times 1.04 \times 1.04 = ?$
35. $1.06 \times 1.06 \times 1.06 = ?$
36. $1.08 \times 1.08 \times 1.08 = ?$
37. $.25 \times .25 \times .25 = ?$

38. Of the asphalt found in West Virginia .7645 is carbon, .10783 is hydrogen, .1346 is oxygen, and the remainder is ash. How much of each substance in 254 tons of asphalt? Check your answers.

39. Of the asphalt found in Oregon .7217 is carbon, .078 is hydrogen, .1461 is oxygen, and the remainder is ash. Find the amount of each in 385 tons of asphalt. Check your answer.

DIVISION OF DECIMALS

Numbers in the decimal system of notation may be regarded in many ways. Thus, 32.25 may be regarded as (*a*) 32 and 25 hundredths; (*b*) 3225 hundredths; (*c*) 32,250 thousandths; (*d*) 322,500 ten-thousandths; (*e*) 322.5 tenths; (*f*) 3.225 tens.

Example 1. Divide 1.293 by 8.

$$\begin{array}{r} 8\overline{)1.293000} \\ .161625 \end{array}$$

8 into 12 tenths gives 1 tenth, with a remainder 4 tenths. 4 tenths = 40 hundredths; 40 hundredths and 9 hundredths = 49 hundredths. 8 into 49 hundredths gives 6 hundredths, with a remainder 1 hundredth. Change 1 hundredth to thousandths, and proceed as before.

Example 2. Divide .01234 by 4.

$$\begin{array}{r} 4\overline{).012340} \\ .003085 \end{array}$$

EXERCISE 36

Divide:

1. 73.21 by 8.	9. 8.218 by 7.	17. 5.472 by 6.
2. 3.45 by 4.	10. 3.942 by 6.	18. 8.2548 by 9.
3. 19.362 by 6.	11. 6.475 by 7.	19. .34794 by 9.
4. 1.791 by 9.	12. 9.143 by 8.	20. .67356 by 9.
5. 4.564 by 5.	13. .1234 by 5.	21. .999999 by 7.
6. 3.927 by 8.	14. .73206 by 6.	22. 7.3745 by 7.
7. .015 by 5.	15. 1.1466 by 7.	23. 6.2676 by 6.
8. 8.846 by 6.	16. 6.2751 by 8.	24. 1.7346 by 7.

Find the difference between .07858 and .078; also find the difference between .07858 and .079.

$$\begin{array}{rr} .07858 & .07900 \\ .078 & .07858 \\ \hline .00058 & .00042 \end{array}$$

Hence .07858 is nearer to .079 than it is to .078. If, therefore, one is asked to give the value of .07858 correct to three figures, write for answer .079.

Express .73948 correct to three figures. *Ans.* .739.

Express .25764 correct to three figures. *Ans.* .258.

Whenever asked to give a decimal correct to any number of figures, discard the remaining figures if the first

one of them is less than 5; if it is 5 or more than 5, increase the last figure retained by 1.

The sign $^+$ or $^-$ may be written after such a number to show whether it is smaller or larger than the correct value. Thus, the .739 might be written $.739^+$ to show that figures have been dropped; and the .258 might be written $.258^-$ to show that it is larger than the correct value.

When the divisor is a decimal, the following method is helpful in determining the position of the decimal point in the quotient.

Example 1. Divide .0732 by .8.

If dividend and divisor are both multiplied by the same number, the quotient is not changed. So we multiply the divisor by 10 to make it an integer, and also multiply the dividend by 10. The multiplication is performed by moving the decimal point one place to the right in each.

$$\begin{array}{r} 8\overline{).7320} \\ .0915 \end{array}$$

EXPLANATION

$$.0732 \div .8 = \frac{732}{10000} \div \frac{8}{10} = \frac{723}{10000} \times \frac{10}{8}$$
$$= .732 \div 8.$$

Example 2. Divide 12 by .125.

Move the decimal point in the divisor to the right so that the divisor becomes an integer. Move the decimal point of the dividend the same number of places to the right.

$$\begin{array}{r} 96 \\ 125\overline{)12000} \\ \underline{1125} \\ 750 \\ 750 \end{array}$$

$$12 \div .125 = 12 \div \frac{125}{1000} = 12 \times \frac{1000}{125}$$
$$= 12000 \div 125.$$

Example 3. Divide 3.274 by 6.25.

$$\begin{array}{r@{}l} & .523 \\ 625) & \overline{327.400} \\ & \underline{3125} \\ & 1490 \\ & \underline{1250} \\ & 2400 \\ & \underline{1875} \\ & 525 \end{array}$$

EXPLANATION

$$3.274 \div 6.25 = \tfrac{3274}{1000} \div \tfrac{625}{100} = \tfrac{3274}{1000} \times \tfrac{100}{625}$$
$$= 327.4 \div 625.$$

Hence the rule: **Whenever the divisor is a decimal, make it an integer by moving the decimal point to the right. Move the decimal point in the dividend the same number of places to the right, supplying ciphers if necessary. Divide as with integers and place the decimal point in the quotient in the same vertical line with the new decimal point of the dividend.**

EXERCISE 37

Perform the following divisions:

1. $22.3 \div .223$.
2. $3.74 \div .374$.
3. $173.2 \div 1.732$.
4. $7.3 \div .073$.
5. $1.25 \div .0125$.
6. $9.28 \div .00928$.
7. $11.34 \div .01134$.
8. $7.04 \div .0704$.
9. $100 \div .01$.
10. $1000 \div .001$.
11. $.012 \div .12$.
12. $1.24 \div 124$.
13. $9.53 \div 9530$.
14. $7.1 \div 7100$.
15. $6.5 \div 65{,}000$.
16. $11.79 \div 11{,}790$.
17. $.001 \div 10$.
18. $.005 \div 100$.
19. 9.265×926.5.
20. $12.335 \div 1232.5$.
21. $1.534 \div 153.4$.
22. $1001 \div .1001$.
23. $54 \div .054$.
24. $792 \div .0792$.

25. $113 \div .0113$.

26. $79.28 \div .7928$.

27. $6.45 \div 6450$.

28. $99.29 \div 99,290$.

29. $7.35 \div 73,500$.

30. $9.24 \div 92,400$.

31. $8.123 \div .008123$.

32. $.04567 \div 45.670$.

EXERCISE 38

Divide:

1. 2.34 by .8.
2. .012 by .5.
3. 3.475 by .4.
4. 1.2348 by .6.
5. .1798 by .5.
6. 3.144 by 1.2.
7. 5.96 by 1.6.
8. 3.2903 by 1.3.
9. .27 by .2.
10. 5.376 by 1.6.
11. 9.4851 by 1.5.
12. 3.2 by 6.4.
13. 20 by .5.
14. 10 by .16.
15. 40 by .32.
16. 56 by 1.12.
17. 84 by 5.6.
18. 392 by 7.84.
19. 100 by .625.
20. 100 by .008.
21. 400 by .05.
22. 144 by .288.
23. 15.4 by .616.
24. .096 by .192.
25. 1 by .001.
26. 5. by .004.
27. .1 by .0001.
28. .04 by .0008.
29. .32 by .00128.
30. .45 by .0018.
31. .078 by .00312.
32. .067 by .0268.
33. .01 by .8.
34. .002 by 1.6.
35. .018 by 45.
36. .54 by 81.
37. .243 by 1.944.
38. .216 by 1.44.
39. 5.12 by .16.
40. 7.29 by 270.
41. 34.7231 by .713.
42. 31.8791 by 3.97.
43. .267584 by 2.96.
44. .348336 by .492.

45. .190256 by .188.
46. 59.4204 by 5860.
47. 55.9911 by 108.3.
48. .575484 by 54.6.
49. .461071 by 122.3.
50. 4.50775 by 123.5.

EXERCISE 39

The mileage and valuation by counties in Texas of the St. Louis and San Francisco Railway as given by the Texas Railroad Commission for the year 1906 are as follows:

	County	Mileage	Valuation
1.	Collin	19.51	$346,538.13
2.	Dallas	2.7	53,300.16
3.	Denton	9.99	188,311.64
4.	Grayson	27.44	843,427.59
5.	Hardeman	8.68	183,997.77
6.	Tarrant	4.56	191,208.29
7.	Wilbarger	12.77	192,843.01

Find the valuation per mile in each of the above counties.

8. On July 16, 1907, a contract for paving Broadway, Denver, Colorado, was awarded on the following itemized specifications and prices:

3050 ft. 6″ × 18″ stone curb	@ $ 1.05*
2750 yd. brick gutter	@ $ 2.25
22,900 yd. street asphalt pavement	@ $ 2.25
704 ft. oak header	@ $ .50
945 ft. 27″ pipe sewer	@ $ 2.40
580 ft. 24″ pipe sewer	@ $ 2.00

*6″ × 18″ means 6 inches by 18 inches.

580 ft. 21″ pipe sewer	@ $ 1.75
580 ft. 15″ pipe sewer	@ $ 1.10
398 ft. 12″ pipe sewer	@ $ .86
516 ft. 10″ pipe sewer	@ $ .75
12 manholes	@ $45.00
17 catch basins	@ $65.00
10 M ft. lumber	@ $30.00

Find the total cost. If the portion to be paved is 3050 ft. long, what would be the cost for paving one mile at the same average rate?

REDUCTION OF COMMON FRACTIONS TO DECIMALS AND REDUCTION OF DECIMALS TO COMMON FRACTIONS

Example 1. Reduce $\frac{7}{8}$ to a decimal.

$$8\overline{)7.000}$$
$$.875$$

Example 2. Reduce $\frac{7}{11}$ to a decimal.

$$11\overline{)7.00000}$$
$$.63636^{+}$$

A fraction is changed to a decimal by performing the indicated division of numerator by denominator.

A fraction in its lowest terms having for denominator a number whose prime factors are 2's or 5's or both can always be exactly expressed as a decimal.

A fraction in its lowest terms having for denominator a number containing prime factors other than 2's aud 5's cannot be exactly expressed as a decimal. In such cases the value may be found to several decimal places and the remainder dropped. Three decimal places usually are sufficient; thus $\frac{1}{3}$ reduces to $.333^{+}$ and $\frac{3}{7}$ reduces to $.428^{+}$.

EXERCISE 40

Reduce to decimals:

1. $\frac{3}{8}$, $\frac{5}{8}$, $\frac{7}{16}$, $\frac{9}{16}$, $\frac{11}{16}$, $\frac{13}{16}$, $\frac{15}{16}$, $\frac{3}{16}$.
2. $\frac{4}{15}$, $\frac{7}{15}$, $\frac{11}{15}$, $\frac{13}{15}$, $\frac{14}{15}$, $\frac{7}{12}$, $\frac{11}{12}$.
3. $\frac{3}{10}$, $\frac{79}{100}$, $\frac{87}{1000}$, $\frac{183}{10000}$, $\frac{2779}{10000}$.
4. $\frac{3}{80}$, $\frac{11}{80}$, $\frac{29}{80}$, $\frac{19}{20}$, $\frac{37}{40}$, $\frac{31}{60}$, $\frac{53}{60}$.
5. $\frac{1}{32}$, $\frac{3}{32}$, $\frac{5}{32}$, $\frac{9}{32}$, $\frac{13}{32}$, $\frac{19}{32}$, $\frac{31}{32}$.
6. $\frac{2}{7}$, $\frac{5}{7}$, $\frac{1}{13}$, $\frac{4}{13}$, $\frac{3}{14}$, $\frac{9}{14}$, $\frac{11}{14}$, $\frac{13}{14}$.
7. $\frac{5}{9}$, $\frac{19}{99}$, $\frac{147}{999}$, $\frac{49}{900}$, $\frac{274}{9990}$, $\frac{569}{9900}$.

A decimal is reduced to a fraction by expressing in figures the denominator which is indicated by the position of the decimal point, and then reducing the resulting fraction to lowest terms.

Example. Reduce .0625 to a common fraction. .0625 is read 625 ten-thousandths; $\frac{625}{10000}$ is read in the same way.

$$.0625 = \frac{625}{10000} = \frac{125}{2000} = \frac{25}{400} = \frac{5}{80} = \frac{1}{16}.$$

EXERCISE 41

Reduce to common fractions:

1. .3, .8, .25, .125, .1875.
2. .07, .0125, .00875, .0625, .0075.
3. .009, .0225, .1125, .0275.
4. .072, .0104, .035, .0112, 0375.
5. .144, .0504, .0768, .162, .0112.
6. .288, .0176, .0325, .0175, .425.
7. .2875, .3375, .5125, .7375.

EXERCISE 42

1. A man walks 3 miles an hour. At this rate, how long will it take him to walk 12 miles?

2. A train goes 25 miles an hour. How long will it take it to go 300 miles at this rate?

3. A bicyclist travels at the rate of 9 miles an hour. How long will it take him to go 60 miles?

4. How would you find the time required for going any given distance, if you knew the distance traveled in a unit of time?

5. A man walks 3.5 miles an hour. At this rate, how long would it take him to go 49 miles?

6. The distance from London to Glasgow is 401.5 miles. An express train goes this distance in 8 hours. Find its rate per hour.

7. From London to Edinburgh is 393.5 miles. The daily mail train takes 7.75 hours to go this distance. Find its rate per hour.

8. The Empire State Express goes from New York City to Buffalo, a distance of 440 miles, in 8.25 hours. Find its rate per hour.

9. The mail train from Paris to Bayonne goes 486.25 miles in 8.983 hours. Find its rate per hour.

10. The distance from New York City to Cleveland is 568 miles. A train goes this distance in 19.5 hours. Find its average speed.

11. A steamer goes from New York City to Bremen, a distance of 4235 miles, in 7.75 days. Find its rate per day. Also its rate per hour.

12. The earth moves in its orbit at the rate of 1110

miles a minute. How many times faster does the earth move than a train which goes 54 miles an hour?

13. A city lot is worth $1800. If this sum is .75 of the value of the house on it, what is the value of the house?

14. If .7 of a sum of money is $196, what is the sum of money?

15. Cast iron is 7.2 times as heavy as water. How many cubic feet of cast iron weigh as much as 6120 cubic feet of water?

16. Coal is 1.3 times as heavy as water. How many cubic feet of coal weigh as much as 546 cubic feet of water?

17. There are 231 cubic inches in a gallon. How many gallons in 1 cubic foot? (1 cu. ft. = 1728 cu. in.)

18. If 2000 pounds of coal cost $8.75, find the price of 8750 pounds of this kind of coal.

19. If 3.5 yards of cloth cost $12.25, find the price of 7.5 yards of this cloth.

20. If 1.6 yards of velvet cost $2.88, find the price of 9.75 yards of velvet.

COMPUTATION ON THE BASIS OF 100, OF 1000 AND 2000

When a price is stated at so much per hundred it is often convenient to state the quantity of goods as so many hundreds and fractions of a hundred. This can easily be done by moving the decimal point two places to the left. To show the number of thousands the decimal point is moved three places to the left.

Example 1. Find the cost of transporting 5 bales of cotton weighing respectively 510 lb., 515 lb., 508 lb., 496 lb., 487 lb., at 46¢ per 100 lb.

$$510 + 515 + 508 + 496 + 487 = 2516.$$

2516 may be written 25.16 hundred.

At 46¢ per hundred the cost of 25.16 hundred is:

$$25.16 \times 46¢ = \$11.5736.$$ *Ans.* $11.57.

Example 2. Find the cost of shipping 7 head of cattle, average weight 1089 lb., at 97¢ per 100 lb.

The weight of 7 head is: $7 \times 1089 = 7623$ lb., or 76.23 hundredweight.

At 97¢ per hundred the cost of shipping 76.23 hundred is: $76.23 \times 97¢ = \$73.9431$. *Ans.* $73.94.

Example 3. Find the value of a carload of coal weighing 43,275 lb. at $4.80 per ton of 2000 lb.

For convenience, point off three places in 43,275 and divide $4.80 by 2, which really gives the number of thousands and the price per thousand; thus:

$$\frac{43275}{2000} \times \$4.80 = \frac{43.275 \times 4.80}{2} = 43.275 \times \$2.40 = \$103.86.$$

Example 4. How much will it cost a man a year to insure his life for $9000, if the annual premium is $32.80 per $1000.

9000 may be written 9 thousand.

At $32.80 per 1000, the premium on 9 thousand is:

$$9 \times \$3280 = \$295.20.$$

EXERCISE 43

The following rates in cents per 100 lb. are taken from the annual Report of the Railroad Commission of the state of Texas for the year 1911.

Find the cost of shipping:

1. 5 bales cotton, average weight 503 lb., @ 45¢.
2. 12 bales cotton, average weight 496 lb., @ 48¢.

3. 15 bales cotton, average weight 490 lb., @ 8¢.

4. 130 bbl. flour, 200 lb. to the barrel, @ 16¢.

5. 124 bbl. flour, 200 lb. to the barrel, @ 17¢.

6. 1 carload grain, weighing 27,500 lb., @ 14¢.

7. 256 sacks flour, 98 lb. to the sack, @ 12¢

8. 32,800 lb. grain @ $7\frac{1}{2}$¢.

9. 1 carload cotton seed products, weighing 23,800 lb., @ 12¢.

10. 1 carload cotton seed hulls, weighing 28,600 lb., @ $14\frac{1}{2}$¢.

11. 1 carload cotton seed meal, weighing 42,000 lb., @ 16¢.

12. 1 carload cotton seed oil, weighing 43,600 lb., @ 5¢.

13. 1 carload brick, weighing 45,000 lb., @ $5\frac{1}{2}$¢.

14. 1 carload fire brick, weighing 27,000 lb., @ $14\frac{1}{2}$¢.

15. 1 carload common brick, weighing 47,000 lb., @ $5\frac{1}{2}$¢.

16. 1 carload mules, weighing 29,000 lb., @ 23¢.

17. 1 carload cattle, weighing 25,000 lb., @ 14¢.

18. 1 carload sheep, weighing 15,500 lb., @ 15¢.

19. 1 carload crude petroleum, weighing 42,000 lb., @ 9¢.

20. 1 carload asphaltum, weighing 27,000 lb., @ 15¢.

21. 1 carload melons, weighing 20,500 lb., @ 19¢.

22. 5880 lb. molasses @ $7\frac{1}{2}$¢.

23. 19,200 lb. sugar @ 48¢.

24. The freight rate on coal in cents per ton of 2000 lb. from Eagle Pass to the points was in 1906:

Weimer 138 ¢	Flatonia 127 ¢	Columbus 140 ¢
Beaumont 217 ¢	Gonzales 121 ¢	Schulenburg 134 ¢

Find the cost of shipping 1 carload of coal, weighing 39,000 lb., from Eagle Pass to each of these points.

25. Find the cost of shipping 105,000 lb. gravel from Austin to San Antonio at 60 ¢ per ton of 2000 lb.

26. Find the cost of shipping 116,000 lb. crushed rock from Clay Quarry to Houston at $67\frac{1}{2}$ ¢ per ton of 2000 lb.

27. Find the cost of shipping 130,000 lb. crushed rock from Jacksboro to Fort Worth at 50 ¢ per ton of 2000 lb.

28. Find the cost of shipping a carload of sand, weighing 50,000 lb., from Sand Pit to San Antonio at 40 ¢ per ton of 2000 lb.

29. Find the premium on a $5500 life insurance policy at $21.50 per $1000.

30. Find the premium on a life insurance policy for $4500 at $25.30 per $1000.

31. What is the premium on a life insurance policy of $6500 at $19.92 per $1000?

32. Find the premium on a life insurance policy for $10,500 at $29.80 per $1000.

33. Find the premium on a life insurance policy for $8500 at $51.20 per $1000.

34. A man insured his life for $9450. Find the annual premium at $62.40 per $1000.

35. If the man (Ex. 34) lived 19 years after taking out his policy, did the Insurance Company gain or lose?

PERCENTAGE

Fractions having 100 for denominator are used so extensively that they receive special attention and give us some new expressions.

Thus: $\frac{6}{100}$ is read six hundredths; but may also be read as six per cent, and it may be written as .06 or 6%.

Per cent means one in each hundred, or by the hundred.

The symbol, %, means **per cent**, and is placed directly after the number with which it is used. It signifies that the number is the numerator of a fraction whose denominator is 100.

Any **per cent** is readily written as a decimal by pointing off two places. Thus: 7% = .07, 25% = .25, 115% = 1.15.

7% means 7 for each 100; $\frac{7}{100}$; or .07.

25% means 25 for each 100; $\frac{25}{100}$; or .25.

115% means 115 for each 100; $\frac{115}{100}$; or 1.15.

By writing the denominator and then reducing the fraction it is easy for us to see that:

100% of a number = the number.

50% of a number = $\frac{1}{2}$ the number.

$33\frac{1}{3}$% of a number = $\frac{1}{3}$ the number.

25% of a number = $\frac{1}{4}$ the number.

$16\frac{2}{3}$% of a number = $\frac{1}{6}$ the number.

Memorize the above values.

In solving problems we usually write the per cent as a decimal and use it according to the methods that we have learned for decimals.

Example 1. In a city school system there are 5250 children in attendance. If 84% are promoted, how many are promoted?

84% of 5250 is the same as $.84 \times 5250 = 4410$.

Example 2. Find $58\frac{1}{3}\%$ of 3880.

$58\frac{1}{3}\%$ of 3880 means $.58\frac{1}{3} \times 3880 = 2263\frac{1}{3}$.

EXERCISE 44

Find:

1. 9 % of \$84.
2. 8 % of \$425.
3. 6 % of \$800.
4. 5 % of \$2000.
5. 8 % of \$3250.
6. 7 % of \$4500.
7. 10 % of \$2250.
8. 11 % of \$4000.
9. 12 % of \$7250.
10. 4 % of \$3600.
11. 5 % of \$983.
12. 8 % of \$750.
13. 6 % of \$850.
14. 7 % of \$1250.
15. 25 % of \$4840.
16. 30 % of \$3290.
17. 40 % of \$4500.
18. 50 % of \$3250.
19. 75 % of \$4000.
20. 70 % of \$3500.
21. 80 % of \$2450.
22. 100 % of \$7800.
23. 16 % of \$3200.
24. 18 % of \$9200.
25. 125 % of \$4000.
26. 225 % of \$5400.

27. A man whose salary is \$1750 a year saves 15 % of it. How much does he save?

28. In a city school system there are 8250 children; 54 % of this number are girls. How many girls in these schools? How many boys?

29. A farm of 175 acres has 24 % woodland. How many acres of woodland in the farm?

30. A house costs \$4740. The lot on which it is built cost 32 % of the value of the house. Find the cost of the lot.

31. In a certain year the number of rainy days was 20 % of the number of days in the year. How many rainy days were there? How many fair days?

32. A lawyer charged 6% for collecting a debt of \$3720. Find his fee. How much did he remit to his client?

EXERCISE 45

Find:

1. $33\frac{1}{3}\%$ of \$9600.

2. $66\frac{2}{3}\%$ of \$3240.

3. 25% of \$4920.

4. 20% of \$4500.

5. $16\frac{2}{3}\%$ of \$636.

6. $83\frac{1}{3}\%$ of \$792.

7. $12\frac{1}{2}\%$ of \$3280.

8. $37\frac{1}{2}\%$ of \$4640.

9. $62\frac{1}{2}\%$ of \$5720.

10. $8\frac{1}{3}\%$ of \$5640.

11. $41\frac{1}{3}\%$ of \$9120.

12. $58\frac{1}{3}\%$ of \$7560.

13. $6\frac{2}{3}\%$ of \$4515.

14. $13\frac{1}{3}\%$ of \$4845.

15. $26\frac{2}{3}\%$ of \$3900.

16. $46\frac{2}{3}\%$ of \$2400.

17. $5\frac{5}{9}\%$ of \$3600.

18. $116\frac{2}{3}\%$ of \$672.

19. A man sells his house for \$1800. If he paid for it $83\frac{1}{3}\%$ of the price at which it was sold, what did the house cost?

20. A shoe dealer sold \$720 worth of shoes. The shoes cost him $66\frac{2}{3}\%$ of the selling price. Find the cost price of the shoes.

21. In an apple orchard of 840 trees $58\frac{1}{3}\%$ bore fruit. How many trees were fruit-bearing?

22. A ranchman lost during a blizzard $16\frac{2}{3}\%$ of his sheep. If the number in his flock was originally 960, how many did he lose, and how many were left?

23. If the area of a country is 1230 square miles, and 75% of it arable land, how many square miles are arable land?

24. Piles used in the construction of a railroad bridge are 42 ft. long, and $83\frac{1}{3}\%$ of their length is beneath the water. Find the length in the water.

25. The railroad mileage of the United States in the year 1910 was 238,609.28. Of this the railroad mileage of Florida was $1\frac{4}{5}$ %. Find the railroad mileage of Florida in 1910.

INTEREST

Interest is money paid for the use of money.

The sum loaned is the **principal.**

Interest is always reckoned as a **rate per cent** of the principal. The rate per cent is for one year unless otherwise stated.

To illustrate: The interest on $100 for 1 yr. at 6 % is $6. For 2 yrs. at 6 % it is $12. For 1 yr. at 5 % it is $5. For $\frac{1}{2}$ yr. at 5 % it is $2.50.

The interest on $200 for 1 yr. at 6 % is $12. What is the interest on $200 for 2 yrs. at 6 %? for 1 yr. at 5 %? for $\frac{1}{2}$ yr. at 5 %? What is the interest on $300 for 1 yr. at 6 %? at 7 %? at 3 %?

Example. What is the interest on $250 for 2 years at 3 %?

The interest on $250 for 1 year at 3 % is:

.03 × $250 = $7.50. The interest on $250 for 2 years at 3 % is 2 × $7.50 = $15.00.

EXERCISE 46

Find the interest on:

1. $600 for 1 yr. at 4 %; for 1 yr. at 5 %; for 1 yr. at 6 %; for 1 yr. at 8 %.

2. $850 for 1 yr. at 7 %; for 1 yr. at 8 %; for 1 yr. at 9 %.

3. $950 for 1 yr. at 3 %; for 1 yr. at 4 %; for 1 yr. at 8 %.

4. \$982 for 2 yr. at 4 % ; for 3 yr. at 5 % ; for 1 yr. 6 mo. at 6 %.

5. \$738 for $\frac{1}{2}$ yr. at 5 % ; $\frac{1}{2}$ yr. at 6 %.

6. \$920 for $\frac{1}{4}$ yr. at 6 % ; $\frac{1}{4}$ yr. at 7 %.

7. \$1200 for 4 mo. at 5 % ; 4 mo. at 6 %.

8. \$1100 for 6 mo. at 7 % ; 6 mo. at 4 %.

9. \$1280 for 3 mo. at 8 % ; 3 mo. at 6 %.

PROPERTY INSURANCE

Property insurance is a provision for making good to the owner part of the value of property lost or destroyed.

The written contract, in which the insurance company agrees to pay for the goods lost or destroyed, is called the **insurance policy**.

The sum paid to the insurance company for this insurance is called the **premium.**

The premium is usually reckoned as a certain amount for each \$100 of the amount of the policy.

Example. What is the premium on an insurance policy of \$15,350 at \$1.35 per \$100?

$$\frac{15350}{100} \times \$1.35 = 153.5 \times \$1.35 = \$207.225, \text{ or}$$
$$15350 \times \$.0135 = \$207.225.$$

In the first solution, the number of 100's is multiplied by the rate on \$100. In the second solution, the number of dollars is multiplied by the rate on \$1.00.

EXERCISE 47

Find the premium for insuring dwellings against loss by fire at the rates specified per \$100:

1. \$2500 at \$1.30.
2. \$2000 at \$1.15.
3. \$4500 at \$1.50.
4. \$3000 at \$1.25.

5. \$2500 at \$1.90.
6. \$5500 at \$1.70.
7. \$6500 at \$1.50.
8. \$4000 at \$1.80.
9. \$5400 at \$1.70.
10. \$3300 at \$1.80.
11. \$7500 at \$1.60.
12. \$7250 at \$1.40.
13. \$10,500 at \$1.30.
14. \$19,250 at \$1.25.
15. \$16,450 at \$1.60.
16. \$7900 at \$1.35.
17. \$22,500 at \$1.10.
18. \$18,250 at \$1.20.
19. \$2400 at \$1.80.
20. \$9300 at \$1.50.
21. \$8500 at \$1.60.
22. \$9450 at \$1.50.
23. \$6500 at \$1.90.
24. \$5400 at \$1.60.
25. \$9500 at \$1.90.
26. \$12,000 at \$1.75.
27. \$18,000 at \$1.75.
28. \$200,000 at \$1.25.
29. \$15,500 at \$1.60.
30. \$16,200 at \$1.60.
31. \$1800 at \$1.90.
32. \$1750 at \$1.75.

33. A man insures his residence, valued at \$5000, at $\frac{3}{4}$ of its value at the rate of \$1.20 on the \$100. Find the premium paid.

34. A jobber insures a quantity of cotton, worth \$30.000. at $\frac{2}{3}$ of its value at the rate of 75¢ on the \$100. Find his premium.

CHAPTER II

COMPOUND QUANTITIES

It is frequently desirable to compare two objects in regard to size, weight, or some other quality. Sometimes the comparison can be made directly; thus, the length of a pencil can be compared with the length of a table by applying the pencil along the edge of the table in such a way as to find out how many times the length of the table contains the length of the pencil; or the size of a cup can be compared with the size of a pail by counting the number of cupfuls of water required to fill the pail.

This process of comparing two objects in respect to a common quality is called measuring.

When the objects cannot be thus directly compared, they can still be compared by first measuring each in terms of a third object that can be conveniently compared with each; thus, if the length of one table measures 6 pencil-lengths, and the length of another measures 3 pencil-lengths, the first table is twice as long as the second. In this case the pencil-length is the common measure of the lengths of the two tables.

Any length, volume, weight, etc., used to measure another is called a **unit of measure**; or simply a **measure.**

A unit measure established by government or by scientists is called a **standard measure.**

To assure measures that are uniform and accurate, the government of each country decides what measures shall be standard and defines them.

There is a simple legal standard unit for each class of measures; as, for length, area, weight, and so forth. There are also smaller units, or **denominations**, derived from the legal standard units.

For example, the yard is the standard unit of length; other units are foot, rod, etc., all being either multiples or fractions of a yard.

Often it is convenient to use more than one unit in measuring a giving distance, by using a large unit first as many times as possible, then applying smaller units to any remainder, instead of expressing the remainder as a fractional part of the large unit. In this way we may say that the length of a room is 4 yards, 1 foot, and 6 inches instead of saying $4\frac{1}{2}$ yards. Also, the weight of an object may be given in pounds and ounces; thus, 5 pounds, 4 ounces instead of $5\frac{1}{4}$ pounds. Such expressions made up of two or more units of the same kind are called **compound quantities.**

A number that designates a concrete quantity in units of measure is called a **denominate** number; thus, 5 feet is a denominate number.

A denominate number that uses two or more units to state a measurement is called a **compound quantity**, a **compound number**, or a **compound denominate quantity.**

Quantities composed of units of one denomination are generally called **simple quantities.**

The following tables give the measures in common use in this country. Other tables are given at the end of the book.

LINEAR OR LONG MEASURE

The yard is the legal unit of length. Originally it was the "length of the king's arm."

Now the length of the standard yard in the United

States is the distance marked on a certain metal bar preserved at Washington. Other units are derived from the yard as shown in the following table:

LINEAR OR LONG MEASURE

12 inches (in.)	= 1 foot (ft.)
3 feet	= 1 yard (yd.)
$5\frac{1}{2}$ yards	= 1 rod (rd.)
320 rods	= 1 mile (mi.)
1 mi. = 320 rd.	= 1760 yd. = 5280 ft.
6080 feet	= 1 knot, geographical or nautical mile
3 knots	= 1 marine league

How many inches in 1 ft.? in 2 ft.? in 1 yd.?

How many yards in 72 in.?

How many feet in 1 yd.? in 10 yd.? in 1 rd.? in 2 rd.? in 1 mile?

How many yards in 12 ft.? in 21 ft.?

How many yards in 1 rd.? in 2 rd.? in 1 mile? in $\frac{1}{2}$ mile? in $\frac{1}{4}$ mile?

How many rods in 11 yd.? in 66 ft.?

How many rods in 1 mile? in 10 mi.? in $\frac{1}{2}$ mi.?

SQUARE MEASURE

Most of the units used for measuring surfaces are derived from units of length.

They are all squares.

One square inch is an amount of surface equal to the surface of a square which measures one inch on a side. Similarly, the square foot, square yard, square rod, and square mile denote amounts of surface, each equal to the surface of a square whose side is the corresponding linear unit. The acre is a unit of area not corresponding to any linear units.

The table shows the relation between these units:

144 square inches (sq. in.)	= 1 square foot (sq. ft.)
9 square feet	= 1 square yard (sq. yd.)
$30\frac{1}{4}$ square yards	= 1 square rod (sq. rd.)
160 square rods	= 1 acre (A.)
640 acres	= 1 square mile (sq. mi.)
1 acre	= 4840 square yards
1 section	= 1 square mile
36 sections	= 1 township

The **square yard** is the legal **unit of surface.**

How many square inches in 1 sq. ft.?

How many square yards in 1 sq. rd.? in 27 sq. ft.?

How many acres in 1 sq. mi.? in 320 sq. rd.?

CUBIC OR SOLID MEASURE

The **volume** of a solid means the amount of space it occupies.

A cube is a solid bounded by six squares.

The unit of volume is the amount of space occupied by a cube whose edge is the corresponding linear unit.

1728 cubic inches (cu. in.)	= 1 cubic foot (cu. ft.)
27 cubic feet	= 1 cubic yard (cu. yd.)

The **cubic yard** is the legal **unit of volume.**

How many cubic inches in 1 cu. ft.?

How many cubic feet in 1 cu. yd.? in 2 cu. yd.?

How many cubic yards in 27 cu. ft.? in 81 cu. ft.? in 54 cu. ft.?

MEASURE OF CAPACITY

Capacity, or **contents**, may be measured by units of Liquid Measure, or Dry Measure, depending upon the nature of the material. These units can be reduced to cubic units.

Liquid Measure is used in the measurement of water, milk, molasses, oil, etc.

Dry Measure is used in the measurement of fruit, vegetables, grain, etc.

LIQUID MEASURE

4 gills (gi.)	= 1 pint (pt.)
2 pints	= 1 quart (qt.)
4 quarts	= 1 gallon (gal.)

A gallon contains 231 cu. in.

DRY MEASURE

2 pints (pt.)	= 1 quart (qt.)
8 quarts	= 1 peck (pk.)
4 pecks	= 1 bushel (bu.)

One bushel contains 2150.42 cu. in. It is the volume of a cylindrical vessel $18\frac{1}{2}$ in. in diameter and 8 in. deep.

How many quarts in 1 gal.? in 4 gal.? in 12 pt.?

How many gallons in 8 qt.? in 24 pt.?

How many quarts in 1 bu.? in 3 pk.?

How many pecks in 24 qt.? in 16 pt.?

How many bushels in 32 qt.? in 64 qt.?

AVOIRDUPOIS WEIGHT

Avoirdupois Weight is used in weighing all commercial quantities excepting the precious metals, jewels, and drugs when sold by retail druggists.

16 ounces (oz.)	= 1 pound (lb.)
100 pounds	= 1 hundredweight (cwt.)
20 hundredweight, or 2000 pounds	= 1 ton (T.)
2240 pounds	= 1 long ton

The **grain**, which is the basis of units of weight, is a small fixed weight originally the weight of a grain of wheat.

1 pound (Avoirdupois) = 7000 grains

The long ton is used in the United States custom-houses, and in weighing coal and mineral products at the mines.

How many ounces in 1 lb.? in 2 lb.? in $\frac{1}{2}$ lb.? in $\frac{1}{4}$ lb.? in 5 lb.?

How many pounds in 1 T.? in 5 T.? in 4 cwt.? in 1 long ton? in 48 ounces?

TROY WEIGHT

24 grains (gr.)	= 1 pennyweight (pwt.)
20 pennyweights	= 1 ounce (oz.)
12 ounces	= 1 pound (lb.)
1 pound Troy	= 5760 grains
1 grain (Troy)	= 1 grain (Avoirdupois)

Troy Weight is used in weighing precious metals and jewelry.

How many grains in 1 pwt.? in 1 lb. (Troy)? in 1 lb. (Avoir.)?

How many ounces in 1 lb. of gold?

Which is the heavier, a pound of gold or a pound of feathers? Why?

TIME MEASURE

60 seconds (sec.)	= 1 minute (min.)
60 minutes	= 1 hour (hr.)
24 hours	= 1 day (da.)
7 days	= 1 week (wk.)
365 days	= 1 common year (yr.)
366 days	= 1 leap year (yr.)
100 years	= 1 century

There are twelve calendar months in a year.

The following lines will enable one to remember the number of days in each month:

> "Thirty days hath September,
> April, June, and November,
> February twenty-eight alone,
> And all the others thirty-one;
> But leap year, coming once in four,
> Gives February one day more."

A century is named by the number of hundreds next larger than that used in expressing dates within that century. Thus, the 5th century is the hundred years from the year 401 and including the year 500; the 19th century is the years from the year 1801 and including the year 1900.

The length of a year is 365 days, 5 hours, 48 minutes, 46 seconds. The common year has 365 days. The difference in length between the common year and the actual, or solar year, gave rise to the introduction of **leap years.** Years are leap years when their numbers are exactly divisible by 4. The year 1907 is not a leap year, as the number 1907 is not exactly divisible by 4. The year 1828 was a leap year, as 1828 is exactly divisible by 4. But the year with which a century ends is a leap year only when the number of the year is exactly divisible by 400. Thus, the year 2000 is a leap year because 2000 is exactly divisible by 400; but the year 1900 was not a leap year because 1900 is not exactly divisible by 400.

CIRCULAR ARC MEASURE

A **plane** is a flat surface.

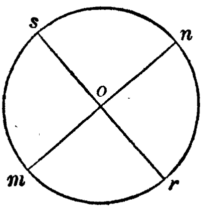
A figure in a plane is called a **plane figure.**

A **circle** is a plane figure bounded by a line called the

circumference, every point of which is equally distant from a point within the figure called the **center.**

A straight line from the center of a circle to the circumference is called a **radius.** *so* is a radius; name other **radii.**

A straight line drawn through the center and terminated by the circumference at two points is called a **diameter.** The lines *mn* and *sr* are diameters. Could you draw more diameters?



Any portion of a circumference is called an arc. *sn* is an **arc.**

An arc equal to one half of a circumference is called a **semicircumference.** *msn* is a semicircumference.

An arc equal to one fourth of a circumference is called a **quadrant.** *nr* is a quadrant.

Instead of measuring the circumference of a circle in feet it is sometimes convenient to use **arc units.** The whole circumference is divided into 360 equal parts called **degrees.** Each degree is divided into 60 equal parts called **minutes**, and each minute into 60 equal parts called **seconds.**

As one degree is $\frac{1}{360}$ of any circumference, a degree on one circle is not of the same size as a degree on a larger or smaller circle.

60 seconds (″)	=	1 minute (′)
60 minutes	=	1 degree (°)
360 degrees	=	1 circumference

How many seconds in 1 min.? in 2 min.?

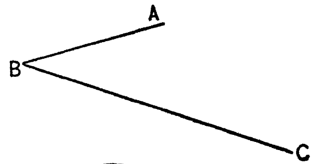
How many minutes in 1 degree? in 50°? in 120″? in 600″? in 30″?

How many degrees in 1 circumference? in $\frac{1}{2}$ circumference? in 1 quadrant? in 180′? in 300′?

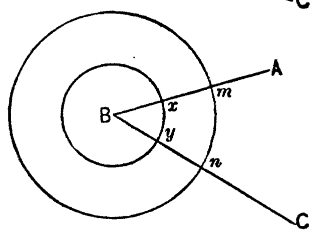
ANGULAR MEASURE

An **angle** is the opening between two straight lines which meet. Thus, the lines *AB* and *BC* form the angle *ABC*.

The point where the two lines meet is called the **vertex** of the angle. *B* is the vertex of angle *ABC*.



If an angle has its vertex at the center of a circle, the sides of the angle can be extended, if necessary, and will meet the circumference of the circle. The arc thus cut from the circle will contain a certain number of degrees. Thus, the angle *ABC* in Fig. 2 cuts from the circle the arc *mn*.



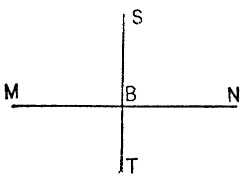
Now draw another circle of different size but with *B* for its center. The angle *ABC* will cut an arc from this circle, as arc *xy*. Suppose *mn* contains 30° of its circumference. Then arc *xy* contains 30° of its circumference. Also if any other circle should be drawn with *B* as its center, the arc cut out by the angle *ABC* would contain 30° on that circle. So the angle *ABC* is called an angle of 30° because its sides would cut out an arc of 30° on any circle having its center at *B*.

Thus a system for comparing angles is obtained.

The **unit of angular measure** is **1 degree.** It is the angle whose sides cut out an arc of one degree upon the circumference of any circle, the vertex of the angle being at the center. The size of an angle refers to the width of the opening between the two lines forming its sides.

60 seconds (″)	= 1 minute (′)
60 minutes	= 1 degree (°)
90 degrees	= 1 right angle
2 right angles	= 1 straight angle

A **right angle** is also defined as one of the angles formed by two lines which meet in such a way as to form equal angles. Thus, lines *MN* and *ST* meet at point *B* and form the four right angles, *MBS*, *SBN*, *NBT*, and *TBM*.



The sides of a right angle are **perpendicular** to each other.

A **straight angle** is a straight line considered as an angle formed by two lines proceeding in opposite directions from a single point. Thus, *MBN* and *SBT* are straight angles.

EXERCISE 48

1. What part of 1′ is 1″?

2. How many seconds in $\frac{1}{2}$ minute?

3. Reduce 1′ 30″ to seconds.

4. What part of a straight angle is a right angle?

5. What part of a right angle is an angle of 45°? 30°? 15°? 18°? 60°? 75°?

6. How many degrees in 1 straight angle?

7. What part of a straight angle is an angle of 15°? 24°? 30°? 45°? 60°? 80°? 100°? 105°? 120°? 135°? 150°?

NUMBERS

12 units	= 1 dozen (doz.)
12 dozen	= 1 gross
12 gross	= 1 great gross
20 units	= 1 score

PAPER MEASURE

24 sheets of paper	= 1 quire
20 quires	= 1 ream
2 reams	= 1 bundle
5 bundles	= 1 bale

How many pencils in $\frac{1}{2}$ gross? How many eggs in 10 doz.? How many men in 5 score? How many dozen in 2 gross? In 1 great gross?

In 200 units, how many dozen? How many score? How many gross?

How many quires in one ream? How many sheets in 1 ream? In 2 quires?

MISCELLANEOUS MEASURE

(The measures in this paragraph are inserted merely for reference.)

1 bushel of barley	= 48 lb.
1 bushel of wheat	= 60 lb.
1 bushel of oats	= 32 lb.
1 bushel of rye	= 56 lb.
1 bushel of potatoes (Irish)	= 60 lb.
1 bushel of potatoes (sweet)	= 55 lb.
1 bushel of buckwheat	= 48 lb.
1 bushel of beans	= 60 lb.
1 bushel of shelled corn	= 56 lb.
1 bushel of peas	= 60 lb.
1 bushel of clover seed	= 60 lb.
1 barrel of flour	= 196 lb.
1 barrel of pork or beef	= 200 lb.
1 cental of grain	= 100 lb.

REDUCTION DESCENDING

Changing an expression to a different expression having the same value is called **reduction**. Thus, the changes of $\frac{1}{2}$ to $\frac{6}{12}$; .5 to $\frac{5}{10}$; 2 ft. to 24 in., are examples of reduction.

The process of changing an expression to an equivalent in terms of a lower denomination is called **reduction descending**.

Example. Reduce 4 rd. 3 yd. 2 ft. to feet.

$$\begin{array}{r} 4 \\ 5\frac{1}{2} \\ \hline 2 \\ 20 \\ \hline 22 \\ 3 \\ \hline 25 \\ 3 \\ \hline 75 \\ 2 \\ \hline 77 \end{array}$$

As there are $5\frac{1}{2}$ yd. in 1 rd., in 4 rd. there are $4 \times 5\frac{1}{2} = 22$ yd. 22 yd. added to the 3 yd. given make 25 yd. As there are 3 ft. in 1 yd., in 25 yd. there are 25×3 ft. $= 75$ ft. 75 ft. added to the 2 ft. given make 77 ft.

EXERCISE 49

1. Reduce 4 yd. 2 ft. to inches.
2. Reduce 110 yd. 1 ft. to inches.
3. Reduce $5\frac{1}{2}$ mi. to yards.
4. Reduce 7 mi. 120 rd. to yards.
5. Reduce 10 mi. 110 rd. 4 yd. to yards.
6. Reduce $445\frac{3}{4}$ mi. to yards.

Example 1.

Reduce 4 sq. rd. 3 sq. ft. to sq. ft.

$$\begin{array}{r} 30\frac{1}{4} \\ 4 \\ \hline 1 \\ 120 \\ \hline 121 \\ 9 \\ \hline 1089 \\ 3 \\ \hline 1092 \end{array}$$

EXPLANATION. Since there are $30\frac{1}{4}$ sq. yd. in 1 sq. rd., in 4 sq. rd. there are $4 \times 30\frac{1}{4}$ sq. yd. $= 121$ sq. yd. Since there are 9 sq. ft. in 1 sq. yd., in 121 sq. yd. there are 121×9 sq. ft. $= 1089$ sq. ft. Adding the 3 sq. ft. gives 1092 sq. ft.

Example 2. Reduce 2 cubic yards to cubic inches.

$2 \times 27 \times 1728$ cu. in. $= 93312$ cu. in.

Example 3. Reduce 5 gal. 2 qt. 1 pt. 2 gi. to gills.

5 gal. 2 qt. 1 pt. 2 gi.

$$\begin{array}{r l}
4 & \\
\hline
20 & = \text{number of quarts in 5 gal.} \\
2 & \\
\hline
22 & = \text{number of quarts in 5 gal. 2 qt.} \\
2 & \\
\hline
44 & = \text{number of pints in 5 gal. 2 qt.} \\
1 & \\
\hline
45 & = \text{number of pints in 5 gal. 2 qt. 1 pt.} \\
4 & \\
\hline
180 & = \text{number of gills in 5 gal. 2 qt. 1 pt.} \\
2 & \\
\hline
182 & = \text{number of gills in 5 gal. 2 qt. 1 pt. 2 gi.}
\end{array}$$

EXERCISE 50

Reduce:

1. 1 sq. mi. to sq. rd.
2. $2\frac{1}{2}$ sq. mi. to A.
3. 12 A. to sq. ft.
4. 27 sq. rd. to sq. ft.
5. 3 mi. 50 rd. to ft.
6. 8 mi. 40 rd. to ft.
7. $2\frac{3}{4}$ mi. to yd.
8. 3.75 mi. to yd.
9. 2.125 mi. to ft.
10. 25 cu. yd. to cu. ft.
11. 38 cu. yd. 20 cu. ft. to cu. ft.
12. $17\frac{1}{2}$ cu. yd. to cu. ft.
13. 18.75 cu. yd. to cu. ft.
14. 20.25 cu. yd. to cu. in.
15. 2 gal. 2 qt. to qt.
16. 5 gal. 3 qt. to qt.
17. 3 gal. 1 pt. to pt.
18. 7 gal. 1 pt. to pt.
19. 19.25 gal. to pt.
20. 4 bu. to qt.
21. $3\frac{7}{8}$ bu. to qt.
22. 3.625 bu. to qt.
23. 7 pk. to qt.
24. 7.375 pk. to qt.
25. $18\frac{7}{8}$ bu. to pt.
26. 13 bu. 3 qt. to pt.

27. How many feet in $\frac{1}{2}$ mi.? in $\frac{1}{4}$ mi.? in $\frac{1}{11}$ mi.?

28. How many yards in $\frac{1}{4}$ mi.? in $\frac{1}{5}$ mi.? in $\frac{1}{3}$ mi.?

29. What fraction of a mile is 440 yd.? 176 yd.? 88 yd.?

30. How many square yards in $\frac{1}{4}$ of an A.? in $\frac{1}{2}$ A.?

31. What part of a township is 1 sq. mi.?

32. How many square rods in $\frac{7}{8}$ A.?

33. How many square rods in .7 A.? in .9 A.?

34. How many square feet in $\frac{3}{4}$ sq. rd.?

35. How many cubic inches in 1 pt., Dry Measure?

36. How many cubic inches in 1 pt., Liquid Measure?

37. How many quarts in $\frac{3}{4}$ pk.?

38. How many gallons required to fill 10 bu. measures?

Example. Reduce 7 T. 3 cwt. 12 lb. 10 oz. to ounces.

T.	CWT.	LB.	OZ.
7	3	12	10

```
 20
143 cwt.
 100
14312 lb.
   16
 85882
14312
229002 oz.
```

Reduce the 7 T. to hundredweights by multiplying by 20. Add 3 cwt. to the product and get 143 cwt. Multiply 143 by 100 and add 12 lb. to the product. This gives 14,312 lb. Multiply this by 16, adding 10 oz., when the first figure is multiplied by 6.

EXERCISE 51

1. Reduce 19 T. to pounds.

2. Reduce 14 T. 4 cwt. to pounds.

3. Reduce 17 T. 3 cwt. to pounds.

4. Reduce 25 T. 2 cwt. to ounces.

5. Reduce 3 T. 15 cwt. 2 lb. to pounds.

6. Reduce 4 T. 11 cwt. 58 lb. to pounds.

7. Reduce 8 T. 2 cwt. 73 lb. to pounds.

8. A dealer buys 50 long tons of coal and sells it by the short ton. How many short tons does he sell?

9. A dealer buys 100 long tons of coal at $6.75 per ton. He sells it by the short ton at $6.75 per ton. How much profit does he make?

10. Convert 784 short tons into long tons.

11. Convert 550 long tons into short tons.

12. Three horses together weigh 2 T. 4 cwt. 91 lb. Find in pounds the average weight of the horses.

REDUCTION ASCENDING

The process of changing an expression to an equivalent expression in terms of higher denominations is called **reduction ascending.**

In **reduction ascending** it is sometimes desirable to change to a definite denomination; as, changing 54 in. to yards gives $1\frac{1}{2}$ yd.; but, usually, it is more convenient to express in integers any quantity which would become a fraction if changed to a higher denomination; as changing 21 pints (Liquid) to higher denominations, gives 2 gals. 2 qts. 1 pt.

Example 1. Reduce 5000 ft. to higher denominations.

$5000 \div 3 = 1666$ yd. 2 ft.
$1666 \div 5\frac{1}{2} = 302$ rd. 5 yd.
Therefore, 5000 ft. = 302 rd. 5 yd. 2 ft.

EXPLANATION. Since there are 3 ft. in 1 yd., in 5000 ft. there are $5000 \div 3$, or 1666 yd., and 2 ft. remain. Since there are $5\frac{1}{2}$ yd. in 1 rd., in 1666 yd. there are $1666 \div 5\frac{1}{2}$, or 302 rd., and 10 yd. remain. So the 5000 ft. equals 302 rd. plus the remainders, 10 yd. and 2 ft.

As it takes 320 rd. for the next higher denomination, 302 rd. is not large enough for this reduction without fractions.

Example 2. Reduce 5000 sq. in. to higher denominations.

$5000 \div 144 = 34$ sq. ft. and 194 sq. in.
$34 \div 9 = 3$ sq. yd. and 7 sq. ft.
5000 sq. in. $= 3$ sq. yd. 7 sq. ft. 104 sq. in.

The pupil should complete the explanation.

Example 3. Reduce 1,000,201 oz. to higher denominations.

$$\begin{array}{r|l} 16 & 1000201 \\ \hline 100 & 62512 \text{ lb. } 9 \text{ oz.} \\ \hline 20 & 625 \text{ cwt. } 12 \text{ lb.} \\ \hline & 31 \text{ T. } 5 \text{ cwt.} \end{array}$$

Divide by 16 to get the number of pounds. Divide by 100 to get the number of hundredweights. Divide by 20 to get the number of tons. *Ans.* 31 T. 5 cwt. 12 lb. 9 oz.

Weights are generally expressed in tons or in pounds.

EXERCISE 52

Reduce to higher denominations:

1. 7800 oz.	3. 75,497 oz.	5. 7987 lb.
2. 9763 oz.	4. 1,000,000 oz.	6. 32,721 lb.

7. How many ordinary or short tons in 100 long tons?

8. Reduce 10,000 lb. to long tons.

Example 1. Reduce 85 pt. to higher denominations.

$$\begin{array}{r|l} 2 & 85 \text{ pt.} \\ \hline 4 & 42 \text{ qt. } 1 \text{ pt.} \\ \hline & 10 \text{ gal. } 2 \text{ qt. } 1 \text{ pt.} \end{array}$$

Example 2. The length of one degree of latitude at 40° north is 364,280 ft. Express this length in miles.

Divide 364,280 by 5280, the number of feet in one mile. This division gives $68\frac{992}{5280}$, which reduces to $68\frac{31}{165}$.

Therefore, 364,280 ft. $= 68\frac{31}{165}$ miles.

EXERCISE 53

Reduce to higher denominations:

1. 234 pt. (Liquid Measure).
2. 47,385 cu. in.
3. 3456 pt. (Liquid Measure).
4. 10,240 rd.
5. 2000 pt. (Dry Measure).
6. 393,000 cu. in.
7. 20,000,000 A.
8. 15,000 sq. in.

9. The diameter of the earth at the equator is 41,852,404 ft. Express this distance in miles and the decimal of a mile correct to two decimal figures.

10. The diameter of the earth at the pole is 41,709,790 ft. What is the polar diameter of the earth in miles correct to two decimal figures?

11. By how many miles does the equatorial diameter exceed the polar diameter?

12. Light takes 8 min. 18 sec. to come from the sun to the earth. The average distance of the sun from the earth is 92,790,000 mi. Find the velocity of light per second.

EXERCISE 54

Reduce to seconds:

1. $18° 20' 20''$.
2. A quadrant.
3. $12° 5' 10''$.
4. $7\frac{3}{4}°$.
5. $120.3°$.
6. $45° 30' 20''$.

Reduce to minutes:

7. $14\frac{1}{4}°$.
8. $75.75°$.
9. $254.125°$.
10. $4\frac{2}{3}°$.
11. $18\frac{5}{6}°$.
12. $13\frac{7}{8}°$.

13. Reduce a common year to minutes.

14. Find the number of minutes in the years 1903, 1904, 1905.

15. Find the number of minutes in February, 1904.

16. Find the number of minutes in the first three months of the year 1903; also in the first three months of the year 1904.

17. Find the number of seconds in a solar year, consisting of 365 da. 5 hr. 48 min. 46 sec.

18. The pulse of the average healthy person beats about 70 times a minute. At this rate, how many times will it beat in a leap year? How many times will it beat in the four successive years, beginning 1904?

19. Reduce 30 wk. 6 da. 12 hr. to minutes.

20. Reduce $1\frac{3}{5}$ common years to days.

21. Reduce $5\frac{5}{7}$ wk. to hours.

22. Reduce 20.4 yr. to hours, allowing five of them to be leap years.

23. How many days are there between Jan. 30, 1902, and Jan. 30, 1910?

24. Reduce $\frac{2}{15}$ of a circumference to degrees.

25. Reduce $\frac{29}{30}$ of a straight angle to degrees.

ADDITION OF COMPOUND QUANTITIES

In **adding compound quantities**, proceed as follows:

Step 1. Arrange the quantities so that numbers of the same denomination stand in the same vertical column, the highest denomination being written first, the next to the highest second, and so on.

Step 2. Add the numbers in each denomination.

Step 3. Reduce the sum to higher denominations where possible without fractions.

Example. Add: 7 ft. 5 in., 10 yd., 8 yd. 7 in., 9 yd. 2 ft. 4 in.

YD.	FT.	IN.
0	7	5
10	0	0
8	0	7
9	2	4
27	9	16
30	1	4

EXPLANATION. Arrange the work in columns for yards, feet, and inches. Then add and reduce to higher denominations as high as yards.

LINEAR MEASURE

EXERCISE 55

Add:

(1)

YD.	FT.	IN.
9	0	8
11	2	4
6	1	10
5	0	6

(2)

YD.	FT.	IN.
26	1	6
33	2	6
20	1	0
70	1	9

(3)

YD.	FT.	IN.
5	1	11
3	1	2
4	1	3
11	2	5

SQUARE MEASURE

(4)

A.	SQ. RD.
76	144
85	131
37	33
63	99

(5)

A.	SQ. RD.
33	79
173	27
254	28
45	53

(6)

A.	SQ. RD.
127	38
192	99
238	77
413	25

CAPACITY

7. Add : 2 gal. 3 qt. 1 pt., 3 gal. 2 qt. 1 pt., 5 gal. 2 qt. 1 pt., 4 gal. 2 qt. 1 pt.

8. Add : 7 gal. 2 qt. 1 pt., 9 gal. 3 qt., 4 gal. 1 qt. 1 pt., 6 gal. 3 qt. 1 pt., 9 gal. 1 pt., 7 gal. 1 pt.

9. Add : 3 bu. 3 pk. 5 qt., 4 bu. 2 pk. 4 qt., 9 bu. 2 pk. 7 qt., 9 bu. 7 qt., 8 bu. 2 pk. 3 qt., 6 bu. 3 pk. 2 qt.

10. Add : 4 bu. 7 qt., 3 bu. 4 pk. 6 qt., 7 bu. 2 pk. 6 qt., 8 bu. 3 qt., 9 bu. 2 pk. 3 qt., 4 bu. 3 pk. 2 qt.

11. Add : 17 gal. 1 pt., 14 gal. 2 qt. 1 pt., 2 gal. 2 qt. 1 pt., 15 gal. 1 pt., 13 gal. 1 qt. 1 pt., 14 gal. 3 qt. 1 pt.

12. Add : 14 bu. 2 pk. 7 qt., 29 bu. 3 pk. 5 qt., 23 bu. 2 pk. 6 qt., 39 bu. 6 qt., 28 bu. 3 pk., 17 bu. 2 pk. 5 qt.

13. Add : 38 bu. 3 pk. 2 qt., 16 bu. 2 pk. 1 qt., 28 bu. 3 pk. 7 qt., 3 bu. 7 qt., 5 bu. 3 pk., 24 bu. 2 pk. 2 qt.

14. Add : 15 bu. 5 qt., 12 bu. 3 pk., 17 bu. 7 qt., 18 bu. 6 qt., 29 bu. 2 pk. 3 qt., 71 bu. 3 pk. 2 qt., 18 bu. 3 pk.

AVOIRDUPOIS WEIGHT

15. Add : 20 T. 215 lb., 18 T. 425 lb., 17 T. 328 lb., 92 T. 411 lb.

16. Add : 384 lb. 12 oz., 125 lb. 15 oz., 82 lb. 14 oz., 73 lb. 11 oz.

17. Add : 425 lb. 10 oz., 17 lb. 14 oz., 30 lb. 12 oz., 72 lb. 9 oz.

18. Add : 15 T. 290 lb., 17 T. 184 lb., 12 T. 127 lb., 15 T. 9 lb., 18 T. 18 lb.

19. Add : 18 lb. 8 oz., 64 lb. 7 oz., 82 lb. 6 oz., 90 lb. 5 oz., 16 lb. 13 oz.

20. Add : 16 T. 175 lb., 71 T. 29 lb., 28 T. 245 lb., 97 T. 159 lb., 13 T. 1300 lb.

TIME

21. Add : 5 da. 4 hr. 15 min., 17 da. 17 hr. 17 min., 92 da. 14 hr. 14 min., 27 da. 23 hr. 12 min., 29 da. 16 hr. 14 min., 45 da. 15 hr. 18 min.

22. Add : 4 wk. 5 da. 7 hr., 9 wk. 6 da. 11 hr., 18 wk. 5 da. 12 hr., 23 wk. 11 hr., 28 wk. 4 da. 4 hr., 73 wk. 6 da. 19 hr., 82 wk. 5 da. 21 hr.

23. Add : 20 hr. 30 min. 18 sec., 17 hr. 45 min. 37 sec., 14 hr. 18 min. 18 sec., 14 hr. 12 min. 12 sec., 9 hr. 48 min. 48 sec., 8 hr. 39 min. 39 sec.

24. Add : 12 da. 17 hr. 44 min., 15 da. 18 hr. 18 min., 31 da. 19 hr. 19 min., 33 da. 21 hr. 27 min., 12 da. 12 hr. 36 min., 34 da. 20 hr. 23 min.

25. Add : 3 wk. 5 da. 23 hr., 8 wk. 6 da. 16 hr., 9 wk. 5 da. 18 hr., 4 wk. 4 da. 14 hr., 10 wk. 5 da. 13 hr.

26. Add : 14 hr. 14 min. 14 sec., 9 hr. 54 min. 38 sec., 11 hr. 12 min. 19 sec., 4 hr. 31 min. 27 sec., 5 hr. 45 min. 43 sec., 8 hr. 41 min. 42 sec.

VOLUME

27. Add : 4 cu. ft. 1421 cu. in., 9 cu. ft. 294 cu. in., 18 cu. ft. 998 cu. in., 7 cu. ft. 778 cu. in., 9 cu. ft. 499 cu. in., 15 cu. ft. 498 cu in.

28. Add : 27 cu. yd. 19 cu. ft., 84 cu. yd. 24 cu. ft., 87 cu. yd. 19 cu. ft., 16 cu. yd. 22 cu. ft., 55 cu. yd. 17 cu. ft., 34 cu. yd. 16 cu. ft.

29. Add : 37 cu. yd. 13 cu. ft., 38 cu. yd. 26 cu. ft., 49 cu. yd. 25 cu. ft., 62 cu. yd. 26 cu. ft., 77 cu. yd. 17 cu. ft., 94 cu. yd. 28 cu. ft.

30. Add : 15 cu. ft. 578 cu. in., 18 cu. ft. 902 cu. in., 18 cu. ft. 978 cu. in., 15 cu. ft. 293 cu. in.

SUBTRACTION OF COMPOUND QUANTITIES

NOTE. Use either method of subtraction, preferably the additive.

From 19 sq. yd. 5 sq. ft. 20 sq. in. take 14 sq. yd. 7 sq. ft. 45 sq. in.

SQ. YD.	SQ. FT.	SQ. IN.
19	5	20
14	7	45
4	6	119

Step 1. Write the quantities so that units of the same denomination are in the same column and subtract the subtrahend from the minuend.

Step 2. Find what concrete quantity added to 45 sq. in. will give 1 sq. ft. 20 sq. in., *i.e.* 164 sq. in. Write the remainder, 119 sq. in., in the column for square inches. Carry 1 sq. ft.

Step 3. Find what concrete quantity added to 8 sq. ft. will give 1 sq. yd. 5 sq. ft., *i.e.* 14 sq. ft. Write the remainder, 6 sq. ft., in the column for square feet. Carry 1 sq. yd.

EXERCISE 56

CIRCULAR ARC OR ANGULAR MEASURE

1. Subtract $5° 12' 13''$ from $84° 14' 30''$.
2. Subtract $19° 14' 14''$ from $27° 15' 10''$.
3. Subtract $38° 15' 45''$ from $90° 10' 10''$.
4. Subtract $54° 14' 54''$ from $172° 0' 19''$.
5. Subtract $84° 5' 15''$ from $90°$.
6. Subtract $113° 13' 54''$ from $180°$.
7. Subtract $94° 53' 50''$ from 180.
8. Subtract $87° 15'$ from $133° 12'$.
9. Subtract $119° 54' 17''$ from $180°$.
10. Subtract $15° 14' 17''$ from $94° 14' 7''$.
11. The tropic of Cancer is $23° 27' 6''$ north of the equator; the Arctic circle is $23° 27' 6''$ south of the north pole. Find the width of the north temperate zone.

CAPACITY

12. From 3 bu. 9 pk. 4 qt. take 1 bu. 2 pk. 5 qt.

13. From 12 gal. 3 qt. 1 pt. take 4 gal. 3 qt.

14. From 11 gal. take 4 gal. 3 qt. 1 pt.

15. From 17 gal. take 11 gal. 1 qt. 1 pt.

16. From 37 bu. 2 pk. 4 qt. take 17 bu. 3 pk. 7 qt.

17. From 29 bu. 1 pk. 2 qt. take 19 bu. 3 pk. 5 qt.

18. From 37 gal. take 17 gal. 1 qt. 1 pt.

19. From 134 gal. take 112 gal. 3 qt. 1 pt.

20. From 43 bu. take 3 pk. 4 qt. 5 pt.

21. From $1\frac{1}{2}$ gal. take $\frac{3}{4}$ gal, and express the result in quarts.

22. From $1\frac{3}{4}$ bu. take $\frac{7}{8}$ bu. and express the result in quarts.

23. From $5\frac{1}{2}$ bu. take $1\frac{3}{4}$ bu. and express the result in quarts.

TIME*

24. From 3 da. 4 hr. 11 min. take 1 da. 7 hr. 14 min.

25. From 11 da. 5 hr. 10 min. take 4 da. 11 hr. 19 min.

26. Almanacs give the time of sunrise in Florida, Louisiana, and Texas on March 5 at 6.22 A.M., and that of sunset as 6.02 P.M. Find the length of the day.

27. On April 1, 1903, the moon rose at 10.28 P.M. On April 4 following, it rose at 12.28 A.M. How many hours and minutes earlier did it rise on April 1 than on April 4?

*The abbreviations A.M. and P.M. stand for forenoon and afternoon respectively. The dot after the 6 is to separate hours from minutes. If there were seconds given, a dot would be used to separate the minutes from the seconds; thus, 5.13.12 P.M. means 13 minutes and 12 seconds after 5 o'clock in the afternoon.

Time between events happening in two different years.

Example. How many years, months, and days were there between Aug. 27, 1880, and Jan. 22, 1901?

YR.	MO.	DA.
1901	1	22
1880	8	27
20	4	25

Since January is the first month of the year and August is the eighth month of the year, we write 1 instead of January and 8 instead of August. In finding the difference, a month is taken as 30 days. The work is then performed as in the subtraction of compound quantities.

EXERCISE 57

1. The battle of New Orleans was fought on Jan. 8, 1815. Find the time from that date to the present day.

2. The first telegraph message was sent by Professor Morse on May 24, 1844. Find the time from that date to the present day.

3. The Spanish fleet under Cervera was destroyed near Santiago on July 3, 1898. Find the time from that date to Feb. 1, 1903.

4. The Mecklenburg Declaration of Independence was signed May 20, 1775. Find the time from this date to the surrender of Cornwallis, Oct. 19, 1781.

5. The following named men were born and died on the dates specified. Find how long each lived.

	BORN	DIED
John Milton	Dec. 9, 1608.	Nov. 8, 1674.
William Shakespeare .	April 23, 1564.	April 23, 1616.
Edmund Burke . . .	Jan. 12, 1730.	July 9, 1797.
Robert E. Lee . . .	Jan. 19, 1807.	Oct. 12, 1870.
U. S. Grant	April 27, 1822.	July 23, 1885.

	BORN	DIED
Oliver Goldsmith . .	Nov. 10, 1728.	April 4, 1774.
Benjamin Franklin .	Jan. 17, 1706.	April 17, 1790.
Alexander Hamilton .	Jan. 11, 1757.	July 12, 1804.
H. W. Longfellow .	Feb. 27, 1807.	March 24, 1882.
J. H. Newman . . .	Feb. 21, 1801.	Aug. 11, 1890.
W. E. Gladstone . .	Dec. 9, 1809.	May 19, 1898.

MULTIPLICATION OF COMPOUND QUANTITIES

Multiply 5 yd. 2 ft. 10 in. by 7.

YD.	FT.	IN.
5	2	10
		7
41	1	10

Multiply 10 in. by 7 and get 70 in. = 5 ft. 10 in. Write 10 in. and carry 5 ft. 7 times 2 ft. are 14 ft. 14 ft. and 5 ft. = 19 ft. = 6 yd. 1 ft. Write 1 ft. Carry 6 yd. 7 times 5 yd. are 35 yd. 35 yd. and 6 yd. = 41 yd.

EXERCISE 58

Multiply:

1. 4 yd. 2 ft. 3 in. by 9.

2. 6 yd. 1 ft. 9 in. by 11.

3. 9 yd. 2 ft. 11 in. by 8.

4. 3 bu. 2 pk. 7 qt. by 7.

5. 4 bu. 1 pk. 6 qt. by 12.

6. 9 gal. 3 qt. 1 pt. by 6.

7. 6 gal. 2 qt. 1 pt. by 12.

8. 3 bu. 3 pk. 7 qt. by 7.

9. 12 T. 400 lb. by 12.

10. 13 T. 387 lb. by 9.

11. 17 T. 254 lb. by 10.

12. $5° 29' 28''$ by 16.

13. $16° 38' 32''$ by 15.

14. 64 A. 150 sq. rd. by 12.

15. 15 A. 27 sq. rd. by 11.

16. $18° 9' 54''$ by 14.

17. Multiply $\frac{5}{8}$ T. by 9 and express the result in pounds.

18. Multiply $\frac{3}{4}$ mile by 19 and give the result in feet.

DIVISION OF COMPOUND QUANTITIES

Divide 97° 10′ 50″ by 8.

$$8\underline{)97° \quad 10' \quad 50''}$$
$$\quad 12° \quad 8' \quad 51\tfrac{1}{4}''$$

The eighth part of 97° is 12°, with a remainder of 1°. 1° 10′ = 70′. The eighth part of 70′ is 8′, with a remainder of 6′. 6′ 50″ = 410″. 8 into 410 goes $51\frac{1}{4}$ times.

EXERCISE 59

Divide:

1. 21 yd. 2 ft. 3 in. by 9.
2. 93° 15′ 15″ by 7.
3. 84° 14′ 14″ by 12.
4. 34 yd. 2 ft. 8 in. by 6.
5. 77 yd. 2 ft. 4 in. by 7.
6. 13 bu. 3 pk. 4 qt. by 6.

7. How many times is 231 cu. in. contained in 1 cu. ft. 582 cu. in.?

8. How many times is 7 yd. 1 ft. contained in 1 mi.?

9. A meter is a French unit of length; it equals 39.37 inches. How many meters equal 1 mile?

10. The planet Mercury revolves around the sun in 88 days. Find in degrees, minutes, and seconds its daily progress.

11. Civil engineers use a chain 100 feet long. How many of these chains in 5 miles?

REDUCTIONS INVOLVING FRACTIONS

The method is the same as for integers.

Example 1. Reduce .875 yd. to feet and inches.

.875 yd. = .875 × 3 ft. = 2.625 ft. = 2 ft. plus .625 ft.
.625 ft. = .625 × 12 in. = 7.5 in.

Hence, .875 yd. = 2 ft. 7.5 in.

Example **2.** Reduce $\frac{7}{12}$ bu. to lower denominations.

$$\frac{7}{12}\text{ bu.} = \frac{7}{12}\text{ of }4\text{ pk.} = \frac{7}{3}\text{ pk.} = 2\tfrac{1}{3}\text{ pk.}$$
$$\frac{1}{3}\text{ pk.} = \frac{1}{3}\text{ of }8\text{ qt.} = \frac{8}{3}\text{ qt.} = 2\tfrac{2}{3}\text{ qt.}$$
$$\frac{2}{3}\text{ qt.} = \frac{2}{3}\text{ of }2\text{ pt.} = \frac{4}{3}\text{ pt.} = 1\tfrac{1}{3}\text{ pt.}$$

Hence, $\frac{7}{12}$ bu. = 2 pk. 2 qt. $1\frac{1}{3}$ pt.

Example **3.** Express $\frac{7}{12}$ A. in square yards.

$$\frac{7}{12}\text{ of 1 A.} = \frac{7}{12}\text{ of 4840 sq. yd.} = \frac{7 \times 4840}{12}\text{ sq. yd.}$$
$$= \frac{7 \times 1210}{3}\text{ sq. yd.} = 2823\tfrac{1}{3}\text{ sq. yd.}$$

EXERCISE 60

Reduce:

1. $\frac{7}{8}$ T. to pounds.

2. $.15°$ to minutes.

3. $\frac{7}{9}$ da. to hours and minutes.

4. .2345 T. to pounds.

5. .95 da. to hours and minutes.

6. .375 bu. to quarts.

7. 18.4. mi. to feet.

8. $\frac{7}{11}$ mi. to yards.

9. .1875 mi. to rods.

10. $15\frac{3}{4}°$ to minutes.

11. $\frac{2}{3}$ of a common year to days and hours.

12. .3125 common years to days, hours, and minutes.

13. .45 bu. to a compound quantity.

14. $\frac{2}{3}$ gal. to a compound quantity.

15. .85 A. to square rods.

16. $\frac{39}{44}$ A. to square yards.

17. $\frac{13}{16}$ bu. to quarts.

18. $\frac{7}{16}$ gal. to pints.

EXPRESSING ONE QUANTITY AS A FRACTION OF ANOTHER QUANTITY

To express one quantity as a fraction of another quantity, reduce the quantities to the same denomination, and divide the first by the second.

Example 1. Express 27 rd. 4 yd. 2 ft. as a fraction of 1 mi.

$$27 \times 5\tfrac{1}{2} = 148\tfrac{1}{2} \text{ yd}$$
$$148\tfrac{1}{2} \text{ yd.} + 4 \text{ yd.} = 152\tfrac{1}{2} \text{ yd.}$$
$$152\tfrac{1}{2} \times 3 \text{ ft.} = 457\tfrac{1}{2} \text{ ft.}$$
$$457\tfrac{1}{2} \text{ ft.} + 2 \text{ ft.} = 459\tfrac{1}{2} \text{ ft.}$$

Therefore, 27 rd. 4 yd. 2 ft. $= 459\tfrac{1}{2}$ ft. A mile equals 5280 ft.

Therefore, $459\tfrac{1}{2}$ ft. equal $\dfrac{459\frac{1}{2}}{5280}$ of 1 mi., or $\dfrac{919}{10560}$ of 1 mi.

Example 2. Express 2 yd. 2 ft. 8 in. as a decimal of a mile.

Step 1. Expressing 2 yd. 2 ft. 8 in. as fraction of 1 mi. gives $\frac{13}{660}$ of 1 mi.

Step 2. Reducing $\frac{13}{660}$ to a decimal gives .0197.

Therefore, 2 yd. 2 ft. 8 in. = .0197 mi.

EXERCISE 61

Reduce:

1. 4400 ft. to the decimal of 1 mi.
2. 293 yd. 1 ft. to the decimal of 1 mi.
3. 117 yd. 1 ft. to the decimal of 1 mi.
4. 1 qt. 1 pt. to the decimal of a gal.
5. 1 pk. 6 qt. to the decimal of a bu.
6. 2 pk. 2 qt. to the decimal of a bu.
7. 3 pk. 1 qt. 1 pt. to the decimal of a bu.
8. $1\tfrac{1}{8}$ in. to the decimal of 1 ft.
9. 2.34 in. to the decimal of 1 ft.
10. $3° 15'$ to the fraction of a right angle.
11. 1 da. 18 hr. to the decimal of 1 wk.

MEASUREMENTS

If the length and width of a rectangle are expressed in the same linear unit, the number of corresponding square units in its area is equal to the product of the numbers in its length and width.

Review the explanation on page 14.

If it is desired to find area in acres, sections, or townships, the area must first be found in other units and then reduced. Why?

EXERCISE 62

1. Find the number of acres in the area of a rectangle whose dimensions are 360 ft. and 121 ft.

2. Find the area of a rectangle 1331 ft. by 720 ft.

3. Find, in acres, the area of a rectangular plot of ground 201 yd. by 10 rd. 5 yd.

4. A railway company acquires the right of way through a territory 154 mi. long, and fences in a strip 80 ft. wide. How many acres does it thus inclose, and how much does it pay for the land at $25 an acre?

5. Find the area of a park 396 ft. by 396 ft. Give your answer in acres.

6. How many acres in a rectangular farm 1.5 mi. long by $1\frac{1}{4}$ mi. wide? Find the value of the farm at $49 an acre.

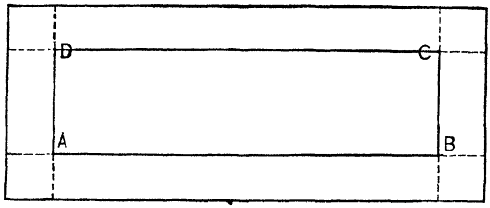
AREAS OF RECTANGULAR FIGURES

EXERCISE 63

1. A room is 40 ft. × 30 ft. × 18 ft. How many square yards in its walls and ceiling? (What are the various areas to be found and the dimensions of each?)

2. Find the area, in square yards, of the walls and ceiling of a room 24 ft. × 16 ft. × 12 ft.

3. *ABCD* is a rectangular plot of ground 400 ft. by 160 ft. On all four sides of it is a road 15 ft. wide. Find the area of the road.



4. Skirting a rectangular park 600 ft. long by 560 ft. wide is a road 24 ft. wide, on all sides of the park. Find the area of the road. Suppose a fence is built to inclose the road with the park. How many feet of fencing will be required?

5. A rectangular grass plot 252 feet by 180 feet has a walk around it. The width of the walk is 9 feet. How many flags, 9 inches square, will be required to flag the walk?

6. Find the area of each of the following rectangles, in square feet, correct to two decimal figures:

(*a*) 136 feet 8 inches by 115 feet 4 inches.
(*b*) 225 feet by 93 feet 10 inches.
(*c*) 78 feet 5 inches by 56 feet 6 inches.
(*d*) 25 feet 9 inches by 50 feet 2 inches.
(*e*) 104 feet 2 inches by 153 feet 11 inches.
(*f*) 203 feet by 53 feet 9 inches.
(*g*) 223 feet 10 inches by 78 feet.
(*h*) 618 feet 1 inch by 130 feet 7 inches.

HINT. Reduce the inches in each example to the fraction of 1 foot

7. Find the area of the following rectangles, giving the results in square yards, correct to two decimal figures:

(*a*) 84.5 feet by 76.75 feet.
(*b*) 90.67 feet by 84.33 feet.
(*c*) 96.34 feet by 85.28 feet.
(*d*) 177.33 feet by 82.54 feet.
(*e*) 129.55 feet by 79.63 feet.

8. A cornfield is $213\frac{1}{3}$ rods long and 96 rods wide. How many bushels of corn will it produce at 32 bushels to an acre? Find the value of the crop at $\$48\frac{3}{4}$ per bushel.

9. A city block is 110 yards long by 90 yards wide. How many acres in a park which extends 7 blocks one way and 5 blocks the other way?

10. A street is 1760 yards long and 20 yards wide. How many thousand bricks, 8 inches by 4 inches, will be needed to pave it?

11. How many tiles, 4 inches on a side, will be required to tile a hall 60 feet by 16 feet?

12. A room is 16 ft. × 12 ft. × 10 ft. How many square yards in the four walls of the room? How many square yards in the walls and ceiling?

13. If a hurricane exerts a pressure of 19.47 pounds per square foot, find in tons the total pressure exerted against the side of a building 50 ft. long and 45 ft. high.

Land surveyed by the United States Government surveyors is divided into townships and sections. This system is common in most of the region west of the Mississippi River, but is not used at all in some of the older states.

A **township** is a tract of land 6 mi. square, and it is divided into 36 **sections**, each 1 mi. square. The sections are numbered as shown in Fig. 1, on the next page; and

are subdivided into half sections, quarter sections, etc., named by the part of the section in which they are situated.

As a section contains 640 acres, the area of subdivisions can easily be computed.

A section is subdivided as indicated in Fig. 2. There are two divisions, E. $\frac{1}{2}$ and W. $\frac{1}{2}$. The E. $\frac{1}{2}$ is divided into two equal squares called N.E. $\frac{1}{4}$ and S.E. $\frac{1}{4}$. The W. $\frac{1}{2}$ is divided into two equal squares called N.W. $\frac{1}{4}$ and S.W. $\frac{1}{4}$. These may be again subdivided as shown in the figures below.

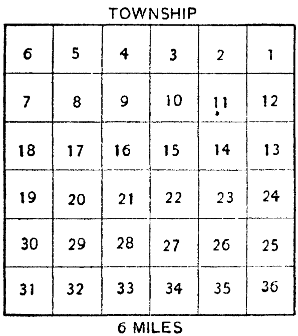


FIG. 1.

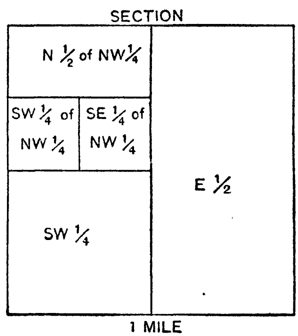


FIG. 2.

EXERCISE 64

1. Draw a figure and locate S.W. $\frac{1}{4}$ of S.E. $\frac{1}{4}$; N.W. $\frac{1}{4}$ of S.E. $\frac{1}{4}$; N.E. $\frac{1}{4}$ of S.W. $\frac{1}{4}$; S.E. $\frac{1}{4}$ of S.W. $\frac{1}{4}$.

2. Locate N.E. $\frac{1}{4}$ of the N.E. $\frac{1}{4}$; S.W. $\frac{1}{4}$ of N.E. $\frac{1}{4}$. How may acres in N.W. $\frac{1}{4}$ of S.W. $\frac{1}{4}$? How many acres in N.W. $\frac{1}{4}$? in S.E. $\frac{1}{4}$? in S. $\frac{1}{2}$ of S.W. $\frac{1}{4}$?

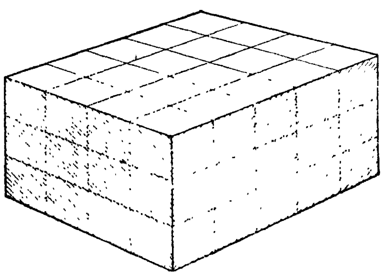
3. A man buys $\frac{1}{2}$ of the S. $\frac{1}{2}$ of N.W. $\frac{1}{4}$ and also $\frac{1}{2}$ of the N.W. $\frac{1}{4}$ of S.W. $\frac{1}{4}$ at the rate of \$20 an acre. Find the cost of his purchase.

VOLUMES OF RECTANGULAR SOLIDS

Suppose that the figure below represents a rectangular solid 5 in. × 4 in. × 3 in. If it were cut into inch cubes, there would be one of these cubes for every square inch of the bottom layer, and as many similar layers as there are inches in the height. The square inches in the bottom are $5 \times 4 = 20$ sq. in. So there are 20 of the inch cubes in each layer. As there are 3 layers, there are 3×20, or 60 of these cubes in the solid. As the volume of an inch cube is 1 cu. in., the volume of 60 inch cubes is 60 cu. in. But 60 is the number obtained by multiplying $5 \times 4 \times 3$.

The **volume of a rectangular solid** is obtained by multiplying together its three dimensions expressed in units of the same denomination.

EXERCISE 65



1. Find the volume of a box 12 ft. by 7 ft. by 6 ft.

2. Find the cubical contents of a room 18 ft. by 12 ft. and 9 ft. high.

3. Find the number of cubic feet in a room 32 ft. by 24 ft. by 12 ft. high.

4. How many cubic yards of earth are removed for the foundation of a house 75 ft. by 54 ft., if the earth is removed to the depth of 21 ft.?

5. A cistern, in the shape of a rectangular solid, is 22 ft. by 14 ft. by 6 ft. How many gallons of water can it contain?

6. A bin is 8 ft. by 3 ft. by 6 ft. How many bushels can it hold?

7. In order to build a concrete wall, earth is removed to the depth of 6 ft. If the wall is 210 ft. long and 12 ft. wide, how many cubic yards of earth must be removed?

8. How many cubic yards of gravel are required to fill, to the depth of 6 in., a street 1 mi. long and 36 ft. wide?

9. How many cubical boxes 2 ft. each way would a storeroom 18 ft. by 12 ft. by 10 ft. hold?

10. The Sault Ste. Marie Canal is 1.6 mi. long, 160 ft. wide, and 25 ft. deep. Express in cubic yards the volume of water required to fill it.

11. A block of marble is 4 ft. by 3 ft. by 24 ft. How many tons does it weigh, if a cubic foot of marble weighs 170 lb.?

12. How many pounds does a cedar beam weigh that is 14 in. by 10 in. by 40 ft., if a cubic foot of cedar wood weighs 38.1 lb.?

13. A cubic foot of clay weighs 75 lb. Find, in tons, the weight of a clay bank 10 ft. by 4 ft. by 80 ft.

14. A box 9 in. by 8 in. by 6 in. is filled with mercury. Find its weight in pounds, if a cubic foot of mercury weighs 13,570 oz.

15. How many 3-in. cubes are required to fill a cubical box each of whose edges is 1 yd.?

16. A pile of 4-ft. wood 8 ft. long and 4 ft. high contains a cord. How many cords of wood in a pile of 4-ft. wood 120 ft. long and 12 ft. high?

17. Find the weight of the water covering an acre to the depth of 4 in. 1 cu. ft. of water weighs 1000 oz.

A **quadrilateral** is a plane figure bounded by four straight lines. Figures 1 and 2 represent quadrilaterals.

Parallel lines are lines which lie in the same plane and do not meet if extended indefinitely.

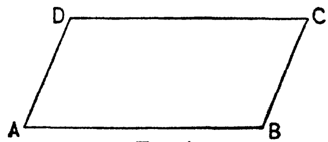


FIG. 1.

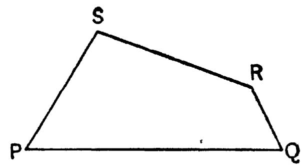


FIG. 2.

A quadrilateral having its opposite sides parallel is called a **parallelogram.** Figure 1 is a parallelogram.

The sides *AB*, *CD* are parallel. Also the sides *AD*, *BC* are parallel.

The side on which a figure is thought of as resting is called the **base.**

In the parallelogram *ABCD*, we may call *AB* the base, or **lower base.** The side opposite the base of a parallelogram is sometimes called the **upper base.** *DC* is the upper base of parallelogram *ABCD*.

The **altitude** of a parallelogram is the perpendicular distance between the bases. *EB* is the **altitude** of parallelogram *ABCD*.

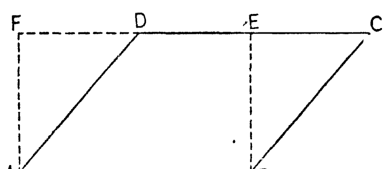


FIG. 3.

Draw a parallelogram *ABCD*, and, on the same base, *AB*, draw the rectangle *ABEF*. With scissors cut out the whole figure *ABCF*. Then fold on line *AD*, and notice that the parallelogram is left after the removal of triangle *AFD*. Fold this triangle back into place, and then fold on line *EB*. Notice that the removal of triangle *EBC* leaves the rectangle. Then cut on the line *AD* and on the line *EB*. Place the two triangles together carefully and notice that they are equal.

The parallelogram $ABCD = ABCF - ADF$.
The rectangle $ABEF = ABCF - BEC$.

Hence, the parallelogram $ABCD =$ the rectangle $ABEF$.

As the area of the rectangle $ABEF$ is equal to the product of its base AB by its altitude EB, and the area of the parallelogram $ABCD$ is equal to the area of the rectangle $ABEF$, then the area of the parallelogram $ABCD$ is equal to the product of its base AB by its altitude EB.

The area of a parallelogram is equal to the product of its base by its altitude.

Thus, the area of a parallelogram, whose base is 12 ft. and altitude 5 ft. equals $12 \times 5 = 60$ sq. ft.

EXERCISE 66

Find the areas of the following parallelograms:

1. Base 10 ft. 6 in., altitude 6 ft. 4 in.
2. Base 17 ft. 3 in., altitude 9 ft. 8 in.
3. Base 12 ft. 6 in., altitude 8 ft. 3 in.
4. Base 15 ft. 5 in., altitude 9 ft. 4 in.
5. Base 36 ft. 9 in., altitude 8 ft. 4 in.
6. Base 40 ft. 3 in., altitude 7 ft. 6 in.
7. Base 27 ft. 9 in., altitude 8 ft. 7 in.
8. Base 28 ft. 4 in., altitude 6 ft. 3 in.

A **triangle** is a plane figure bounded by three straight lines. ABC is a triangle.

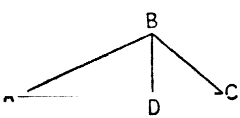


FIG. 1.

In the triangle ABC, we may call AC the **base** (Fig. 1).

The **altitude** of a triangle is the perpendicular distance from the base to the opposite vertex. BD is the **altitude** of triangle ABC.

In order to find the area of triangle ABC (Fig. 2), draw a line through C parallel to AB and a line through B parallel to AC, and draw the altitude BH. As $ACDB$ is a parallelogram, its area is equal to the product of AC by BH.

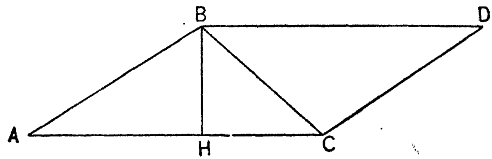


FIG. 2.

Cut on the line BC. Place the triangle BCD on the triangle ABC. It will be seen that the two triangles are equal. Therefore, the triangle ABC is equal to half of the parallelogram $ABCD$.

Therefore, the area of the triangle ABC is equal to half the product of AC by BH.

The area of a triangle is equal to half the product of its base by its altitude.

Thus, the area of a triangle, whose base is 12 ft. and altitude 5 ft., equals one half of 12 times 5 = 30 sq. ft.

EXERCISE 67

Find the areas of the following triangles:

1. Base 50 ft. 30 in., altitude 23 ft. 9 in.

2. Base 60 ft. 4 in., altitude 42 ft. 8 in.

3. Base 75 ft. 6 in., altitude 35 ft. 8 in.

4. Base 48 ft. 4 in., altitude 29 ft. 4 in.

5. Base 56 ft. 9 in., altitude 27 ft. 4 in.

6. Base 82 ft. 6 in., altitude 64 ft. 2 in.

7. Find the area of a right triangle whose base is 26 ft. and altitude 19 ft. (A right triangle is a triangle having a right angle.)

8. Find the area of a right triangle whose base is 96 ft. 6 in. and altitude 84 ft.

9. Find the area of a right triangle having the base 72 ft. 6 in. and altitude 63 ft. 9 in.

A quadrilateral having only two sides parallel is called a **trapezoid.** *ABCD* (Fig. 1) is a trapezoid, having *AB* parallel to *DC*. *AB* and *CD* are respectively the lower and upper bases of the trapezoid.

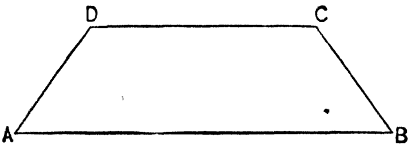


FIG. 1.

In order to find the area of trapezoid *ABCD* (Fig. 2), draw the diagonal *AC*. Two triangles, *ABC* and *ADC*, are thus formed. In triangle *ABC*, if *AB* is considered as the base, *CH* is the altitude, and the area equals half the product of *AB* by *CH*. In triangle *ADC*, if *DC* is considered as the base,

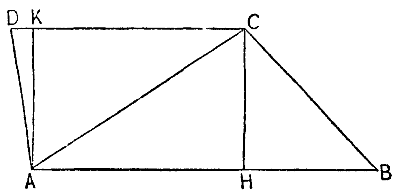


FIG. 2.

then *AK* is the altitude, and the area equals half the product of *DC* by *AK*. The sum of these two triangles equals the trapezoid. But *AB* and *CD* are the bases of the trapezoid. *CH* and *AK* are equal because they measure the distance between the parallel lines *DC* and *AB*. *CH* and *AK* each measure the altitude of the trapezoid.

Therefore, the area of the trapezoid is equal to the sum of the products of each base by half the altitude. This may also be stated: **The area of a trapezoid equals half the sum of its parallel sides, multiplied by the perpendicular distance between them.**

Thus, the area of a trapezoid, whose parallel sides are 6 ft. and 12 ft. and whose altitude is 5 ft., equals $\frac{1}{2} \times (6+12) \times 5$. $6+12=18$. $\frac{1}{2}$ of $18=9$. $9 \times 5 = 45$. Area $= 45$ sq. ft.

EXERCISE 68

1. Find the area of the upper surface of a board in the form of a trapezoid whose parallel sides are 12′ 6″ and 5′ 6″ and height 8′ 6″.

2. Find the area of a trapezoid whose parallel sides are 18′ 4″ by 12′ 8″ and 24′ apart.

3. Find the area of a field whose parallel sides are 20 rd. and 36 rd. and width 18 rd.

4. Find the area of a lot whose shape is a trapezoid having for parallel side 60 ft. and 40 ft. and length 84 ft.

BOARD MEASURE

Lumber is measured by the board foot. A **board foot** is a rectangular solid 1 ft. by 1 ft. by 1 in.

If a board is 1 in. thick, the number of board feet in it is measured by the number of square feet in its upper or lower surface. Thus, a board 8 ft. long, 8 in. wide, and 1 in. thick contains $8 \times \frac{8}{12}$ board feet.

The number of board feet in a regular piece of timber is found by multiplying the number of square feet in its upper or lower surface by the number of inches in its thickness.

Thus, a board 9 ft. by 14 in. and $2\frac{1}{2}$ in. thick contains $9 \times \frac{14}{12} \times 2\frac{1}{2}$ board feet.

The number of board feet in a board less than 1 in. in thickness is measured by the number of square feet in its upper or lower surface without regard to thickness. Thus, a board 10 ft. by 15 in. and $\frac{2}{3}$ in. thick contains $10 \times \frac{15}{12}$ board feet.

EXERCISE 69

1. How many board feet in a board 8 ft. by $1\frac{1}{2}$ ft. and 1 in. thick?

2. How many board feet in a board 9 ft. by 16 in. and $\frac{3}{4}$ in. thick? Find the cost of the board at $3\frac{1}{2}$¢ per board foot.

3. How many board feet in a plank 24 ft. by 15 in. and 2 in. thick? In a plank 20 ft. by 12 in. and 3 in. thick?

4. How many board feet in a railroad tie 24 ft. by 8 in. and 6 in. thick?

5. Find the number of board feet in each of the following pieces of lumber:

(*a*) 18 ft. by 16 in. and $1\frac{1}{2}$ in. thick.
(*b*) 12 ft. by 8 in. and 6 in. thick.
(*c*) 24 ft. by 9 in. and 3 in. thick.
(*d*) 16 ft. by 8 in. and 4 in. thick.
(*e*) 18 ft. by 12 in. and 3 in. thick.
(*f*) 21 ft. by 16 in. and 3 in. thick.
(*g*) 30 ft. by 14 in. and $2\frac{1}{2}$ in. thick.
(*h*) 28 ft. by 15 in. and $3\frac{1}{2}$ in. thick.

6. Find the cost of 480 boards, each $1\frac{1}{2}$ in. by 10 in. and 16 ft. long, @ $27.50 per M. ("Per M" means "by the 1000" board feet.)

7. Find the cost of 840 boards, each $1\frac{3}{4}$ in. by 10 in. and 12 ft. long, @ $25 per M.

8. Find the cost of 56 joists 2 in. by 8 in. and 18 ft. @ $27 per M.

9. Find the cost of:

94 pieces 2 in. by 4 in. by 24 ft. and
24 pieces 4 in. by 4 in. by 24 ft. @ $24 per M.

10. How many board feet in a cubical block of wood, each of whose dimensions is 2 ft.?

11. How many board feet in a beam 9 in. by 9 in. and 54 ft. long?

12. How many board feet in 16 railroad ties 8″ by 6″ and 8′ 6″ long?

13. How many cubic feet are equivalent to 120 board feet?

14. How many board feet in a beam 11 in. by 12 in. by 36 ft.?

15. A piece of lumber contains 2980 cu. in. Express this in board feet, understanding that the piece is more than 1 in. in thickness.

16. Find the number of board feet in a beam $10\frac{1}{2}$ in. by 6 in. and 27 ft. long.

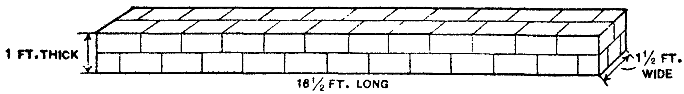
MASONRY AND BRICKLAYING

Stone work and **brick work** are often estimated by the **perch**, instead of by the cubic foot.

A perch of masonry is $24\frac{3}{4}$ cu. ft. In Great Britain, perch is another name for rod, and a perch of masonry originally meant a wall 1 rd. long, $1\frac{1}{2}$ ft. wide, and 1 ft. high. Of the $24\frac{3}{4}$ cu. ft. in a perch, 22 cu. ft. are allowed for the stone or brick, and $2\frac{3}{4}$ cu. ft. allowed for the mortar, *i.e.* eight ninths for stone or brick and one ninth for mortar.

In estimating the number of perches of masonry in the walls of a building, the outside dimensions of the walls are taken. This method of reckoning counts the corners twice. In estimating the amount of material, the computation should be exact, inside and outside dimensions being reckoned.

Twenty-two common bricks, *i.e.* bricks 8 in. by 4 in. by 2 in., and the mortar in which they are laid are sufficient to build 1 cu. ft. of wall.

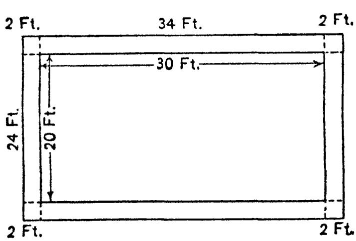


ONE PERCH

Example. A cellar is 30 ft. long, 20 ft. wide, and 7 ft. deep. How many cubic feet of masonry in its walls if they are 2 ft. thick? What is the actual number of cubic feet in the wall? If this wall were to be built of brick, how many bricks would be required, no allowance being made for openings?

If the wall is 2 ft. thick the total length of the cellar and of each of the long walls is 34 ft. and the total width is 24 ft. So the total length of the wall, outside measurment, is 116 ft. A wall 116 ft. long, 7 ft. high, and 2 ft. thick contains $116 \times 7 \times 2 = 1624$ cu. ft., the amount of masonry.

But this makes no account of the fact that the corners have been counted twice. So use the following method:



The side wall, 34 ft. by 7 ft. by 2 ft. contains $34 \times 7 \times 2 = 476$ cu. ft. The two side walls contain $2 \times 476 = 952$ cu. ft. The two end walls contain $2 \times 20 \times 7 \times 2 = 560$ cu. ft. The whole wall contains $952 + 560 = 1512$ cu. ft., exact measure. As 22 bricks are required for each cubic foot, the wall requires 1512×22 bricks $=$ 33,264 bricks.

EXERCISE 70

1. How many perches of masonry in a wall 84 ft. long, 16 ft. high, and $1\frac{1}{2}$ ft. thick? How many common bricks are required to build this wall?

2. A cellar is 36 ft. long, 18 ft. wide, and 8 ft. deep. How many cubic feet of masonry in its walls if they are 2 ft. thick?

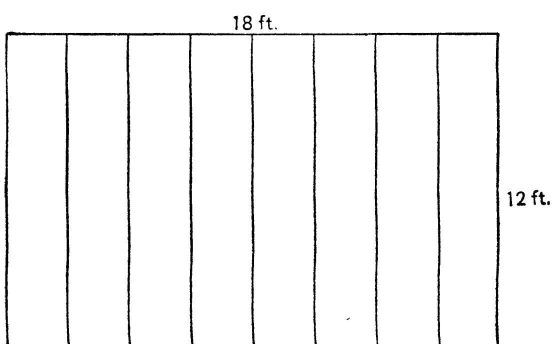
3. How many bricks are necessary to build a wall 25 ft. long, 12 ft. high, and 24 in. in thickness? What is the cost of the bricks at $9 per M? (M stands for 1000.)

4. How many cubic feet of mortar are required to build a wall 600 ft. long, 10 ft. high, and 18 in. in thickness? How many common bricks are required to build this wall?

5. At $9.50 per M, what is the cost of the bricks required to build a house 40 ft. by 36 ft. and 40 ft. high to the eaves, the highest point of the gable end being 52 ft. above the ground, and the walls $1\frac{1}{2}$ ft. thick? Allow 270 cu. ft. for openings.

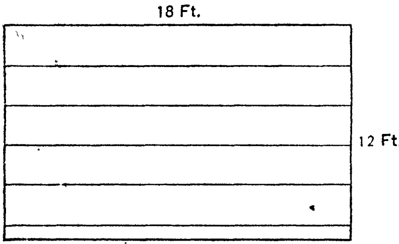
CARPETING

1. How many yards of carpet are needed for a room 18 ft. by 12 ft., if the carpet is 27 in. wide?



SOLUTION. (1) Let the carpet be placed crosswise. Number of strips = 18 ft. ÷ 27 in. = 18 ft. ÷ $2\frac{1}{4}$ ft. = 8.

∴ Number of yards = 12 × 8 ÷ 3 = 32. *Ans.* 32 yd.



(2) Let the carpet be placed lengthwise.

Number of strips = 12 ft. ÷ $2\frac{1}{4}$ ft. = $5\frac{1}{3}$.

Here 6 strips are needed. The fraction of a strip may be either cut off or turned under.

∴ Number of yards = 6 yd. × 6 = 36 yd. *Ans.* 36 yd.

To find the number of yards of carpet required to carpet a room :

1. Draw a diagram of the room.
2. Find the number of strips.
3. Multiply the number of strips by the number of yards in one strip.

EXERCISE 71

1. How many yards of carpet are required to cover a room 18 ft. by 16 ft. with carpet 30 in. wide, if the strips are laid crosswise?

2. How many yards of carpet $\frac{3}{4}$ of a yard wide are needed to carpet a room 24 ft. by 18 ft., if the carpet runs lengthwise?

3. How many yards of matting 36 in. wide are required to cover a room 16 ft. by 12 ft., the matting being laid crosswise?

4. How many yards of carpet 30 in. wide are needed to carpet a room 24 ft. long, 20 ft. wide, if the carpet is laid crosswise?

5. Find the cost of carpeting a room 16 ft. by 14 ft. with carpet 27 in. wide at 90¢ per yard, the strips being laid lengthwise.

EXERCISE 72

1. What fraction of a yard is one foot? What fraction of a yard is 2 feet?

2. What fraction of 1 foot is 1 inch? 3 inches? 4 inches? 5 inches? 7 inches? 8 inches? 9 inches? 10 inches?

3. What fraction of 1 yard is 1 inch? What fraction of a yard is 2 inches? 3 inches? 4 inches? 5 inches? 6 inches? 9 inches? 12 inches? 16 inches? 17 inches? 19 inches? 24 inches? 27 inches?

4. There are 8 quarts in 1 peck. What fraction of a peck is 1 quart? What fraction of a peck is 2 quarts? 3 quarts? 4 quarts? 5 quarts? 6 quarts?

5. What fraction of a square yard is 2 square feet? 3 square feet? 4 square feet? 5 square feet? 6 square feet? 7 square feet?

6. What fraction of 10 is 2? What fraction of 10 is 7?

7. What fraction of 11 is 4? What fraction of 13 is 9?

8. What fraction of 100 is 80?

9. Which of the four fundamental rules enables us to solve a problem of this character: What fraction of a number is some other number?

10. If 4 men can do a piece of work in 7 days, how long will it take 1 man to do the same work?

11. If a team of horses can plow a 40-acre lot in 16 days, how long will it take 4 teams, working together, to plow the same lot?

12. If a man can do a piece of work in 9 days, what fraction of the work can he do in 1 day? in 2 days? in 3 days? in 4 days? in 6 days?

MISCELLANEOUS EXERCISES

EXERCISE 73

1. How long will it take a person to earn \$44.40, if he earns \$1.85 a day?

2. If $\frac{5}{8}$ of a quantity of coal is $71\frac{1}{4}$ tons, how many tons of coal are there in all?

3. A train runs at the rate of $33\frac{1}{3}$ mi. per hour. How many hours will it take the train to run 350 mi.?

4. A man being asked his age replied, "Three elevenths of my age is $13\frac{1}{2}$ years." Find the man's age.

5. A's share is $\frac{3}{5}$ of the joint capital, or \$2540. What is the joint capital?

6. A dealer sells a piano for \$240, thereby gaining $\frac{1}{5}$ of the cost of the piano. What did the piano cost?

7. If a dealer sold a piano for \$240 and lost thereby $\frac{1}{5}$ of the cost price, what did the piano cost?

8. If the dividend is $18\frac{3}{4}$ and the quotient is $10\frac{1}{2}$, what is the divisor?

9. The sum of two numbers is 25, and one of the numbers is $1\frac{1}{2}$ times the other number. Find the numbers.

10. A farmer sells $\frac{2}{9}$ of his cattle to one jobber and $\frac{4}{9}$ to another jobber. If he keeps the remainder, what part does he keep? Suppose the number remaining is 27, how many head of cattle had the farmer originally?

11. A dealer sells $\frac{1}{2}$ of his coal, and then $\frac{1}{3}$ of it, and has $12\frac{1}{2}$ tons left. How many tons had he originally?

12. If the $\frac{1}{5}$ of a number and the $\frac{1}{6}$ of the same number together make $60\frac{1}{2}$, what is the number?

13. If .85 of A's money is \$289, how much has he?

14. By gaining .15 of his outlay a man's property amounts to \$7245. What was his outlay?

15. Five sevenths of a farm is sold for \$1385. What is the value of the farm?

16. How many times will a wheel 12 ft. $4\frac{1}{2}$ in. in circumference revolve in going $3\frac{3}{4}$ mi.?

17. How long will it take a person to go $10\frac{2}{5}$ mi. at the rate of $6\frac{1}{2}$ mi. per hour?

18. A train travels at the rate of $52\frac{1}{2}$ mi. in $1\frac{3}{4}$ hr. Find its rate per hour. How many hours and minutes will it take this train to travel 208 mi.?

19. From the ground to the first floor of a house is $\frac{4}{15}$ of the height of the house; if the first floor is 10 ft. above the ground, how high is the house?

20. What part of $4\frac{5}{6}$ is $\frac{1}{9}$ of $14\frac{1}{2}$?

21. If .625 of a gallon of maple sirup cost \$1.25, what will 1 gal. of maple sirup cost?

22. When 3 oranges sell for 5¢ how many oranges can I buy for 80¢?

23. If $8\frac{1}{4}$ yd. of calico cost 66¢, how many yards can I buy for \$3.30?

24. A merchant, by selling tea at 69¢ per pound, gains $\frac{3}{20}$ of the cost of the tea. Find the cost per pound of the tea.

25. By selling cloth at 78¢ per yard a clothier loses $\frac{1}{7}$ of the cost of the cloth. Find the cost price per yard of the cloth.

26. The rent of a house for 17 mo. is $225. Find the rent of this house for 12 mo.

27. Lead pencils cost $3\frac{1}{3}$¢ each. At this price, how many can be bought for $1.40?

28. A laborer gets 4¢ per cubit foot for digging a cellar. At this rate how much will he get if the cellar is 15 ft. by 12 ft. by 6 ft.?

29. If 3 qt. of oil cost $13\frac{1}{2}$¢, at this rate how much will 1 gal. 2 qt. cost?

30. If 1 lb. 8 oz. of cheese cost 42¢, find the cost of 4 lb.

31. How many tiles 6 in. square will be needed to pave a hearth 5 ft. by 2 ft.?

32. Express $\frac{1}{3} + \frac{1}{4} + \frac{1}{5}$ as a decimal.

33. Reduce 5.875 hr. to seconds.

34. Find the cost of a carload of coal weighing 35,880 lb. at $5.50 per ton.

35. A man having a salary of $1500 a year spends 25% of it for board, 10% for clothing, and 15% for other things. How much money does he spend?

36. Reduce 2876 in. to yards.

37. Find the value of 32 tubs of butter each weighing 56 lb. at $37\frac{1}{2}$¢ per pound.

38. The wages of a motorman on the Metropolitan West Side Elevated Railway, Chicago, are $30\frac{1}{2}$¢ per hour. How much does he earn in ten weeks, working 8 hr. a day?

39. How many feet in $87\frac{1}{2}$% of 1 mile?

40. How many square yards in $62\frac{1}{2}$% of 1 A.?

41. A farmer having 320 bushels of apples sells 75% of them. How many bushels does he sell?

42. How much will a man earn in 10 weeks at $1.50 per day, Sundays excepted?

43. A cubic foot of air weighs .08073 lb. Find the weight of the air in a room 15 by 16 ft. by 9 ft.

44. How many grains in $83\frac{1}{3}\%$ of 1 lb. Troy?

45. A railroad train runs 1 mi. in 1 min. 15 sec. Find its rate per hour.

46. Find the number of acres in a field 90 rd. by 40 rd.

47. Find the cost of 18,750 bd. ft. of lumber at $30 per M.

48. Find the commission on sales amounting to $4750 at 2%.

49. A man borrows money on April 1, and agrees to pay it in 90 days. On what date should he pay it?

50. How many square inches in the surface of an 18-in. cube?

51. Find in square rods 15% of 1 A.

52. Out of a class of 64 pupils $12\frac{1}{2}\%$ failed to be promoted. How many were promoted?

53. A lot is 42 ft. by 120 ft. Find the cost of fencing it at 85¢ per yard.

54. Find the cost of making a concrete sidewalk 63 by 12 ft. at $1.75 per square yard.

55. Express in minutes .075 of 1 day, 16 hours.

56. A railroad train runs 72 ft. in one second. At this rate how far will it run in one hour?

57. Find the cost of plastering the walls of a room 20 ft. long, 18 ft. wide, and 10 ft. high at 48¢ a square yard.

58. A watch loses 1 min. 5 sec. in 3 da. At this rate how much will it lose during the month of April?

59. How many pounds Avoirdupois in 420 lb. Troy?

60. A tank is 8 ft. by 6 ft. by 7 ft. How many gallons can it hold? (1. cu. ft. = $7\frac{1}{2}$ gal. nearly.)

61. Find the cost of 24 yd. of cloth at \$$1.87\frac{1}{2}$ per yd.

62. How many cords of wood in a pile 50 ft. long, 12 ft. wide, and 8 ft. high?

63. When coal sells for \$7.00 per ton, what is the cost of a sack of coal weighing 200 lb.?

64. A bin is 18 ft. long, 6 ft. wide, and 4 ft. deep. How many bushels will it contain? (1 bu. = $1\frac{1}{4}$ cu. ft.)

65. Change to decimals $\frac{17}{25}$, $\frac{15}{22}$, $\frac{3}{250}$, $\frac{6}{7}$.

66. Change to fractions in lowest terms .875, .00525, $.66\frac{2}{3}$.

67. Express in simplest form

$$7 \times 3.04 \times 10{,}000 - 125{,}000.7.$$

68. A man rents a house for \$550 a year. His rent the last year was 9% less. What was his rent last year?

69. A grocer having on hand 15 gal. 2 qt. 1 pt. of oil buys 20 gal. 2 qt. and sells 30 gal. 2 qt. 1 pt. How much has he left?

70. Find $8\frac{1}{3}$% of 1 cu. ft. Give result in cubic inches.

71. The following is the lowest bid in detail for improving Lick Run Pike, Cincinnati, Ohio, submitted July 19, 1907:

20,000 cu. yd. embankment @ \$$0.18\frac{1}{2}$
500 cu. yd. excavation @ \$1.00
1530 cu. yd. stone @ \$2.25
265 cu. yd. screening @ \$2.25
100 cu. yd. cement @ \$8.00
400 ft. 12″ pipe @ \$1.00
10 ft. 12″ slant @ \$.80

30 ft. 4″ box culvert concrete @ \$6.00
60 ft. 5″ box culvert concrete @ \$7.00
2350 ft. 7″ box culvert concrete @ \$16.00
650 ft. 8″ box culvert concrete @ \$18.00
18,400 sq. yd. rolling @ 3¢
6 manholes @ \$40.00

Find the amount of the bid.

REVIEW OF FRACTIONS

EXERCISE 74

Add :

1. $\frac{1}{6}$, $\frac{9}{60}$, $\frac{9}{40}$, $\frac{5}{16}$.
2. $2\frac{8}{9}$, $3\frac{19}{24}$, $4\frac{11}{15}$.
3. $\frac{1}{9}$, $\frac{1}{14}$, $\frac{3}{20}$, $\frac{7}{12}$.
4. $1\frac{7}{9}$, $\frac{11}{27}$, $\frac{7}{24}$, $1\frac{1}{12}$.
5. $\frac{1}{4}$, $\frac{7}{18}$, $\frac{13}{24}$, $\frac{7}{10}$.
6. $2\frac{13}{24}$, $2\frac{29}{36}$, $4\frac{5}{27}$.
7. $\frac{4}{15}$, $\frac{5}{16}$, $\frac{17}{120}$.
8. $\frac{7}{8}$, $3\frac{11}{12}$, $1\frac{11}{16}$, $6\frac{8}{9}$.
9. $\frac{3}{8}$, $\frac{3}{14}$, $\frac{1}{10}$.
10. $\frac{7}{10}$, $3\frac{2}{15}$, $\frac{17}{20}$, $5\frac{23}{45}$.
11. $1\frac{3}{4}$, $1\frac{5}{6}$, $1\frac{8}{9}$, $1\frac{11}{12}$, $1\frac{13}{27}$.
12. $2\frac{1}{12}$, $3\frac{13}{14}$, $2\frac{1}{3}$, $\frac{15}{16}$.

Find the difference between :

13. $4\frac{5}{14}$ and $1\frac{3}{4}$.
14. $1\frac{33}{100}$ and $\frac{78}{100}$.
15. $1\frac{27}{100}$ and $\frac{7}{10}$.
16. $\frac{1}{11}$ and $\frac{1}{12}$.
17. $\frac{1}{9}$ and $\frac{1}{12}$.
18. $\frac{1}{8}$ and $\frac{1}{18}$.
19. $\frac{9}{10}$ and $\frac{8}{9}$.
20. $\frac{5}{6}$ and $\frac{4}{5}$.
21. $\frac{11}{12}$ and $\frac{10}{11}$.
22. $\frac{17}{24}$ and $\frac{13}{20}$.
23. $\frac{17}{18}$ and $\frac{14}{15}$.
24. $7\frac{5}{7}$ and $3\frac{11}{14}$.
25. $8\frac{5}{9}$ and $7\frac{11}{12}$.
26. $9\frac{5}{12}$ and $6\frac{13}{18}$.
27. $5\frac{7}{16}$ and $2\frac{17}{24}$.
28. $6\frac{7}{18}$ and $5\frac{11}{24}$.
29. $8\frac{17}{20}$ and $6\frac{29}{30}$.
30. $12\frac{5}{22}$ and $8\frac{13}{33}$.

31. What number must be added to $3\frac{3}{4}$ to give $5\frac{2}{3}$?

32. A man's capital amounts to \$$1727\frac{1}{2}$. By how much must he increase it so that it may amount to \$3000?

33. What number must be taken from $30\frac{7}{8}$ to leave $14\frac{2}{3}$?

34. By how much does $84\frac{1}{6}$ exceed $17\frac{3}{4}$?

35. Of the weight of the earth's atmosphere $\frac{231}{1000}$ is oxygen. What fraction of the weight of the earth's atmosphere do its other constituents aggregate?

EXERCISE 75

Simplify :

1. $\frac{2}{3}$ of $\frac{5}{6}$ of $2\frac{4}{7}$.
2. $\frac{3}{4}$ of $\frac{9}{10}$ of $4\frac{4}{9}$.
3. $\frac{7}{8}$ of $1\frac{2}{5}$ of $\frac{20}{49}$.
4. $\frac{4}{11}$ of $\frac{2}{3}$ of $4\frac{1}{8}$.
5. $\frac{3}{8} \times \frac{3}{11} \times 4\frac{8}{9}$.
6. $\frac{9}{10} \times 1\frac{1}{2} \times 1\frac{1}{18}$.
7. $\frac{3}{13}$ of $4\frac{7}{8}$ of $1\frac{1}{7}$.
8. $\frac{11}{14} \times 5\frac{1}{3} \times 1\frac{10}{11}$.
9. $\frac{7}{16}$ of $1\frac{7}{8}$ of $1\frac{1}{15}$.
10. $2\frac{1}{7} \times 3\frac{1}{7} \times 2\frac{32}{33}$.
11. $5\frac{1}{2} \times 5\frac{1}{2} \times \frac{4}{121}$.
12. $2\frac{3}{4} \times 2\frac{3}{4} \times 1\frac{1}{15}$.
13. $7\frac{1}{8} \times \frac{16}{19} \times \frac{3}{8}$.
14. $9\frac{2}{7} \times 1\frac{8}{13} \times 1\frac{1}{6}$.
15. $1\frac{2}{3} \times 1\frac{2}{3} \times 1\frac{2}{3} \times 1\frac{2}{25}$.
16. $(\frac{3}{4} + \frac{1}{2}) \times (\frac{3}{4} - \frac{1}{2})$.
17. $(\frac{5}{6} + \frac{1}{3}) \times (\frac{5}{6} - \frac{1}{3})$.
18. $(\frac{9}{10} + \frac{1}{4}) \times (\frac{9}{10} - \frac{1}{4})$.
19. $\frac{2}{5}\ (3\frac{1}{3} + 1\frac{1}{4}) \times \frac{3}{8}$.
20. $\frac{2}{3}$ of $4\frac{1}{2} - \frac{1}{8}$ of $3\frac{1}{4} \times 3\frac{1}{5}$.
21. $3\frac{9}{11} \times (3\frac{1}{3} + \frac{5}{6} - \frac{20}{21})$.
22. $2\frac{1}{2} \times (1\frac{3}{4} + 2\frac{1}{2} + 1\frac{7}{8})$.
23. $7\frac{1}{2} \times 1\frac{7}{8} \times \frac{16}{225} \times (\frac{1}{2} + \frac{1}{3} + \frac{1}{4})$.
24. $1\frac{11}{12} \times 1\frac{12}{17} \times \frac{101}{58} \times (\frac{3}{4} + \frac{2}{3} - 1\frac{5}{12})$.
25. $2\frac{2}{3} \times 5\frac{1}{4} \times 1\frac{1}{2} \times (\frac{9}{10} + \frac{1}{4} - 1\frac{3}{20})$.
26. $3\frac{1}{2} \times 1\frac{1}{14} + 7\frac{1}{2} - (1\frac{1}{4} \times 1\frac{1}{2} \times 4\frac{1}{5})$.

EXERCISE 76

Divide :

1. $25\frac{2}{3}$ by $1\frac{5}{6}$.
2. $3\frac{1}{5}$ by $2\frac{2}{5}$.
3. $10\frac{1}{8}$ by $1\frac{35}{64}$.
4. $7\frac{5}{16}$ by $292\frac{1}{2}$.
5. $52\frac{1}{2}$ by $131\frac{1}{4}$.
6. $16\frac{1}{5}$ by $\frac{6}{7}$ of $6\frac{3}{4}$.
7. $\frac{3}{8}$ by $\frac{4}{9}$ of $1\frac{13}{32}$.
8. $\frac{1}{6}$ by $\frac{2}{7}$ of $1\frac{1}{4}$.
9. $\frac{37}{60}$ by $\frac{5}{14}$ of $1\frac{1}{6}$.
10. $2\frac{15}{16}$ by $1\frac{13}{72}$ of $1\frac{3}{5}$.

11. $1\frac{2}{3}$ by $\frac{3}{8}$ of $1\frac{3}{7}$.

12. $3\frac{3}{14}$ by $1\frac{8}{17}$ times $73\frac{4}{15}$.

13. $(\frac{9}{16} - \frac{1}{4})$ by $(\frac{3}{4} + \frac{1}{2})$.

14. $(1\frac{24}{25} - 1\frac{21}{100})$ by $(1\frac{2}{5} + 1\frac{1}{10})$.

15. $(3\frac{1}{16} - 2\frac{1}{4})$ by $3\frac{1}{4}$.

16. $11\frac{1}{9}$ by $(1\frac{5}{6} + 1\frac{1}{2})$.

17. If the dividend is $6\frac{1}{2}$ and the quotient $\frac{10}{13}$, find the divisor.

18. What must $10\frac{2}{3}$ be divided by to give as quotient $2\frac{2}{3}$?

19. If the dividend is $272\frac{1}{4}$ and the quotient $16\frac{1}{2}$, find the divisor.

20. Divide 43,560 by $272\frac{1}{4}$.

PERCENTAGE

The result of taking a per cent of a quantity is **percentage.** A percentage of a number is simply a fraction of the number. The central fact in percentage is that 1 % is $\frac{1}{100}$. Hence 100 % is equivalent to 1. What per cent is $\frac{19}{20}$ equivalent to?

$$\frac{19}{20} \text{ of } 1 = \frac{19}{20} \text{ of } 100\,\% = 95\,\%.$$

Example **1.** Find $2\frac{3}{4}$ % of 1789.

$$\begin{array}{ll}
17.89 & = 1\,\% \text{ of } 1789. \\
\underline{\quad 2\frac{3}{4}} & \\
35.78 & 2 \text{ times } 17.89. \\
8.945 & \frac{1}{2} \text{ of } 17.89. \\
\underline{4.4725} & \frac{1}{4} \text{ of } 17.89. \\
49.1975 & = 2\frac{3}{4} \text{ times } 17.89.
\end{array}$$

Example **2.** Find 3.6 % of 2992.

3.6 % is the same as $3.6 \div 100$, or .036. Therefore, 3.6 % of 2992 is the same as .036 of 2992, which is 107.712.

EXERCISE 77

Find:

1. 7 % of 184.
2. 9 % of 275.
3. 6 % of 213.
4. 8 % of 534.
5. 9 % of 3280.
6. 8 % of 3297.
7. 4 % of 615.
8. 7 % of 2630.
9. 9 % of 4280.
10. 10 % of 7850.
11. 11 % of 983.
12. 12 % of 2570.
13. $12\frac{1}{2}$ % of 928.
14. $23\frac{1}{3}$ % of 5220.
15. $16\frac{2}{3}$ % of 733.
16. $37\frac{1}{2}$ % of 828.
17. $62\frac{1}{2}$ % of 9200.
18. 60 % of 2855.
19. $4\frac{1}{2}$ % of 2280.
20. $3\frac{3}{4}$ % of 3066.
21. $6\frac{1}{2}$ % of 1820.
22. $5\frac{3}{4}$ % of 10,656.

23. Express the following fractions as per cents:

$\frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{8}, \frac{1}{16}, \frac{5}{32}, \frac{7}{16}$.

24. Express each of the following decimals as a per cent:

.04, .08, $.12\frac{1}{2}$, .0165, .002, .006, .0024.

25. A piece of coal taken from the Texas and Pacific Coal Company's mine contains moisture, 5.46 %; combustible matter, 35.66 %; carbon, 49.17 %; ash, 9.71 %. Find the amount of each constituent in 2000 lb. of coal. Check your answer.

26. A specimen of lignite contains as follows: moisture, 29.07 %; combustible matter, 28.96 %; fixed carbon, 24.47 %; ash, 17.50 %. Find the amount of each constituent in a ton of lignite. Check.

27. Distilled water is composed of two gases, $11\frac{1}{9}$ % by weight being hydrogen, and $88\frac{8}{9}$ % by weight being oxygen. Find the weight of each gas that can be obtained from 10 lb. of water.

28. A bookkeeper receives a salary of \$1800 per annum. If he spends $62\frac{1}{2}\%$ of his salary, and saves the remainder, how much does he spend? How much does he save?

29. A gold-bearing ore contains $.5\%$ of gold. How many pounds Avoirdupois of gold would 2240 lb. of this ore yield?

30. A copper ore contains $5\frac{1}{2}\%$ of copper. Find the number of pounds of copper in 500 lb. of this ore.

31. A owns $16\frac{2}{3}\%$ of a boat valued at \$12,300. What is the value of A's share of the boat?

32. The estimated value of the exports of the United States for 1899 was \$1,275,000,000. The percentages of exports from United States ports for that year are given as follows: New York, 37.4%; Boston, 10.43%; Philadelphia, 5.05%; Baltimore, 8.9%; New Orleans, 6.4%; Galveston, 7.17%. Find the value of the exports from these cities.

To find what per cent one number is of another.

Example **1.** What per cent of 12 is 5?

5 is $\frac{5}{12}$ of 12.

$\frac{5}{12} = \frac{5}{12}$ of $100\% = 41\frac{2}{3}\%$

Example **2.** The value of the property of the Fort Worth and Denver City Railroad in Texas, as ascertained by the Railroad Commission of Texas, for the year 1906, was \$5,771,600, and the income from its operation was \$1,178,040. Find the rate per cent of income from operation.

$$\frac{\$1{,}178{,}040}{\$5{,}771{,}600} \text{ of } 100\% = 20.4\%.$$

To find what per cent one number is of another, find what fractional part the first number is of the second and multiply by 100%.

EXERCISE 78

1. What per cent of 15 is 12?
2. What per cent of 20 is 4? is 7? is 11? is 13?
3. What per cent of 40 is 2? is 8? is 12? is 17?
4. What per cent of 90 is 9? is 12? is 27? is $11\frac{1}{4}$?
5. What per cent of 120 is 15? is 18? is 45? is 80?
6. What per cent of 480 is 28.8? is 33.6? is 40?
7. What per cent of 1728 is 345.6? is 155.52? is 288?
8. What per cent of 231 is 21? is 77? is 34.65?
9. What per cent of 5280 is 88? is 440? is 330?
10. What per cent of a bushel is a quart?
11. What per cent of a mile is 8 rd.?
12. What per cent of an acre is a square rod?
13. What per cent of a chain is a yard?
14. What per cent of 1 sq. ch. is 1 sq. rd.?
15. What per cent of 1 gal. is 1 pt.?
16. What per cent of 1 sq. rd. is 7 sq. yd. 5 sq. ft. 9 sq. in.?
17. What per cent of a rod is 1 yd. 2 ft. 6 in.?
18. What per cent of 1 mi. is 1 knot?
19. What per cent of 1 mi. is an arc of 1′ measured on the 40th parallel of latitude? (1′ = 4670 ft.)
20. What per cent of 1 mi. is 22 yd.? is 176 yd.?
21. What per cent of 1 sq. mi. is the S.W. $\frac{1}{4}$ of N.E. $\frac{1}{4}$ of a section of land?
22. What per cent of a common year is 73 da.? is 219 da.? is 292 da.?
23. A meter is 39.37 in. What per cent of a meter is 1 yd.? What per cent of 1 yd. is 1 meter?

24. A kilometer is 1000 meters. What per cent of 1 mi. is 1 km.?

25. What per cent of 1 lb. Avoirdupois is 1 lb. Troy?

26. What per cent of 1 oz. Avoirdupois is 1 oz. Troy?

27. What per cent of the area of each of the following states consists of irrigated land?

	Acres Irrigated	Area in Square Miles
(*a*) California	1,446,114	158,360
(*b*) Colorado	1,611,270	103,925
(*c*) Louisiana	201,685	48,720
(*d*) Montana	951,054	146,080
(*e*) Nevada	501,168	110,700
(*f*) Oregon	388,110	96,030
(*g*) Utah	629,290	84,970
(*h*) Washington	135,470	69,180
(*i*) Wyoming	605,230	97,890

28. Find the increase per cent in the population of each of the following cities for the ten years from 1900 to 1910:

	Population in 1910	Population in 1900
Mobile	51,521	38,469
Little Rock	45,941	38,307
Los Angeles	319,198	102,479
Denver	213,381	133,859
Pensacola	22,982	17,747
Savannah	65,064	54,244
Springfield, Ill.	51,678	34,159
Evansville	69,647	59,007
Dubuque	38,494	36,297
Kansas City, Kan.	82,331	54,418
Lexington, Ky.	35,099	26,369
Kansas City, Mo.	248,381	163,752
Minneapolis	301,408	202,718

29. The total production of butter in the United States in 1899 is estimated at 1,430,000,000 lb. Of this quantity 16,002,000 lb. was exported. Find the per cent of the total production exported.

30. For the year 1899 the total production of cheese in the United States is estimated at 300,000,000 lb. Of this quantity 27,203,200 lb. was exported. Find the per cent of cheese exported.

31. The number of farms in the United States June 1, 1900, was 5,739,657. The number of farms operated by owners was 3,713,371. The number of farms operated by tenants was 752,920. The number of farms operated by share tenants was 1,273,366. Find the per cent operated by owners, by tenants, and by share tenants respectively.

Given a number as a per cent of some other number, to find the other number.

Example 1. If 15.5 % of a number equals 22.785, what is the number?

$$15.5\,\%\text{ of the number} = 22.785$$

$$1\,\%\text{ of the number} = \frac{22.785}{15.5}$$

$$100\,\%\text{ of the number} = \frac{22.785}{15.5} \times 100 = 147.$$

$\therefore$ the number is 147.

Three dots arranged thus, $\therefore$, are read, "therefore." As this symbol will be used frequently it should be remembered.

Example 2. If $3\frac{3}{4}\,\%$ of a number is 2934, what is the number?

$3\frac{3}{4}\% = \frac{3\frac{3}{4}}{100} = \frac{15}{400} = \frac{3}{80}.$

$\frac{3}{80}$ of the number $= 2934.$

$\frac{1}{80}$ of the number $= 978.$

$\frac{80}{80}$ of the number $= 78{,}240.$

$\therefore$ the number is 78,240.

EXERCISE 79

1. If 5 % of a number equals 185, what is the number?

2. If $12\frac{1}{2}$ % of a man's salary is $156, what is the man's salary?

3. If $37\frac{1}{2}$ % of a man's property is valued at $324, what is the value of his property?

4. The number 360 is equal to 5 % of what number? 6 % of what number? 8 % of what number? 9 % of what number?

5. The number 120 is equal to 3 % of what number? 4 % of what number? 5 % of what number? 6 % of what number? 8 % of what number? 12 % of what number?

6. $750 is 6 % of what sum of money? $6\frac{2}{3}$ % of what sum of money? $7\frac{1}{2}$ % of what sum of money? $12\frac{1}{2}$ % of what sum of money?

7. $108 is 30 % of what sum? 25 % of what sum? $33\frac{1}{3}$ % of what sum? $44\frac{4}{9}$ % of what sum?

8. $450 is $6\frac{1}{4}$ % of what sum? $6\frac{2}{3}$ % of what sum? $8\frac{1}{3}$ % of what sum? $12\frac{1}{2}$ % of what sum? $16\frac{2}{3}$ % of what sum?

9. $420 is $37\frac{1}{2}$ % of what sum? $62\frac{1}{2}$ % of what sum? $87\frac{1}{2}$ % of what sum?

10. Recently the commercial value of silver was 3% of the value of gold. Find the number of ounces of silver equivalent in value to 126 ounces of gold.

11. Of the population of the United States in 1900, 27,849,760 were married. This number was 36.5% of the population. Find the population in 1900.

12. The census of 1900 gives the number of married men in the United States as 14,003,798. This number was 35.9% of the number of males. Find the male population.

13. The census of the same year gives the number of married women in the United States as 13,845,963. This number was 37.2% of the entire number of females. Find the female population.

14. Of the number of illiterates above 10 years of age in the United States 15.5%, according to the census of 1900, can neither read nor write. This number is 955,840. Find the number of illiterates above 10 years of age in the United States in 1900.

15. Of the average value of the raw cotton exported from the United States in 5 years 50.4% went to England. If the export of raw cotton to England amounts in value to $107,500,000, find the average value of the raw cotton exported.

16. Of the sheep exported from the United States 86% are shipped to England. If the value of the sheep shipped to England in a certain year was $1,685,800, find the total value of the export of sheep for that year.

17. Of the pupils attending school in a certain city .6% are in the senior class of the high school. If the senior

class numbers 21, find the number of pupils attending school in that city.

18. In a certain city the number of pupils promoted at the end of the scholastic year was 2765. This number was 79% of the number of pupils in school. Find the number of pupils attending school in that city.

19. The foreign born population of a city is 10,668. This number is $3\frac{1}{2}\%$ of the population of the city. Find the population of the city.

20. Of the water of the Dead Sea 22.857% is saline material. If a quantity of Dead Sea water is evaporated, and the saline material left behind weighs 914.28 lb., what is the weight of the water before evaporation?

PROFIT AND LOSS

In actual business, gains and losses are reckoned as a per cent of the **cost**.

Example. How much does a person gain by buying 360 yd. of cloth at \$1.30 per yd. and selling it at a profit of 15%? What is the selling price per yard?

SOLUTION. $360 \times \$1.30 \times 15\% = \$70.20$, gain.

$$\begin{array}{rl} \$1.30 & = \text{cost per yd.} \\ .13 & = 10\% \text{ of } \$1.30 \\ .065 & = 5\% \text{ of } \$1.30 \\ \hline \$1.495 & = \text{selling price per yd.} \end{array}$$

Observe, selling price per yd. is 115% of cost price.

EXERCISE 80

1. Find the selling price of articles, the cost prices and rates per cent of profit being given as follows:

	Cost Price	Rate Per Cent of Profit
(*a*)	\$150	12 %
(*b*)	\$75	25 %
(*c*)	\$31	$7\frac{1}{2}$ %
(*d*)	\$215	$16\frac{2}{3}$ %
(*e*)	\$540	27 %
(*f*)	\$318	$8\frac{1}{3}$ %
(*g*)	\$512	$18\frac{3}{4}$ %
(*h*)	\$234	15 %
(*i*)	\$457	$12\frac{1}{2}$ %

Example. A dealer buys apples at \$1.75 per barrel, and sells them at \$2.10 per barrel. Find his gain per cent.

Solution. $\$2.10 - \$1.75 = \$.35.$

$\frac{\$.35}{\$1.75} = \frac{1}{5}$. The gain is $\frac{1}{5}$ of the cost. $\frac{1}{5} = 20\ \%$.

2. Find the rate per cent of profit or loss, if the cost prices and selling prices are given as follows:

	Cost Price	Selling Price
(*a*)	\$ .20	\$ .25
(*b*)	\$ .22	\$ .20
(*c*)	\$ .90	\$1.50
(*d*)	\$2.10	\$1.40
(*e*)	\$125	\$160

Given selling price and rate per cent of profit or loss, to find cost price.

Example **1.** A piano was sold for \$450 at a profit of $12\frac{1}{2}$ %. Find the cost price.

$(100\ \% + 12\frac{1}{2}\ \%)$ of cost $= 112\frac{1}{2}\ \%$ of cost $= \frac{9}{8}$ of cost.

$\frac{9}{8}$ of cost $= \$450.$

Therefore, cost $= \$450 \div \frac{9}{8} = \$400.$

Example 2. A dealer sells goods for \$200.56 at a loss of 8 %. Find the cost price.

$$(100\,\% - 8\,\%) \text{ of cost} = 92\,\% \text{ of cost.}$$
$$92\,\% \text{ of cost} = \$200.56.$$
$$\therefore \text{cost} = \frac{\$200.56}{92\,\%} = \$218.00$$

EXERCISE 81

1. Find the cost price, the selling price and rate per cent of profit being given as follows :

	Selling Price	Rate Per Cent of Profit
(*a*)	\$63	25 %
(*b*)	\$143	$8\frac{1}{3}$ %
(*c*)	\$54	8 %
(*d*)	\$189	17 %
(*e*)	\$205	$2\frac{1}{2}$ %
(*f*)	\$315	5 %

2. Find the cost, if the selling price and the rate per cent of loss are given :

	Selling Price	Rate Per Cent of Loss
(*a*)	\$41.30	10 %
(*b*)	\$87.50	20 %
(*c*)	\$90	30 %
(*d*)	\$45.50	50 %
(*e*)	\$59.90	$33\frac{1}{3}$ %
(*f*)	\$33.60	$16\frac{2}{3}$ %
(*g*)	\$55.80	20 %
(*h*)	\$253	$12\frac{1}{2}$ %

Example. A man sold a horse for \$126, thereby losing 20 %. What should have been the selling price to make a profit of 15 % ?

SOLUTION. The selling price is 80 % of cost.

To make a profit of 15 %, the selling price should be 115 % of cost. In this problem 80 % of a number is given and 115 % of it is required. Hence,

$$115 \times \frac{\$126}{80} = \$181.125 = \text{required selling price.}$$

EXERCISE 82

1. A sold a house lot for $ 3835 at a gain of 18 %. Find the cost.

2. By selling a piano for $ 270, a dealer loses 10 %. Find the cost of the piano.

3. By gaining 25 % of his capital, a merchant increases his capital to $ 4550. What was his original capital?

4. If goods are bought for $ 20 and sold for $ 22.50, what is the gain per cent?

5. If the cost is $ 48 and the selling price is $ 52, what is the gain per cent?

6. If the cost is $ 190 and the selling price is $ 152, what is the loss per cent?

7. If the cost is $ 145 and the selling price is $ 159.50, what is the gain per cent?

8. If the cost is $ 118 and the selling price is $ 128.62, what is the gain per cent?

9. If the selling price is $ 106.70 and the gain is 10 %, what is the cost?

10. If the selling price is $ 75.25 and the loss is $12\frac{1}{2}$ %, what is the cost?

11. If the selling price of a rug is $ 90 and the gain is 20 %, what is the cost of the rug?

12. If the selling price is $ 89.25 and the loss is 15 %, what is the cost?

13. If the selling price is $ 95 and the gain is $18\frac{3}{4}$ %, what is the cost? What is the profit?

14. By selling a horse for $ 168, a man gains 40 % of the cost. What is the cost of the horse?

15. By selling velvet at $ 4.55 a yard, a clothier makes a profit of $8\frac{1}{3}$ %. Find the cost per yard of the velvet.

16. At an auction a dealer buys goods at 20 % below the market price. If he sells these goods at the market price, what is his gain per cent?

17. A lawyer collects a debt. He charges 3 % for collection, and remits to his client, after deducting his fee, $ 952.54. Find the amount of the debt.

18. If property was sold for $ 11,778, at a loss of $2\frac{1}{2}$ %, what was the value of the property?

19. Two horses were sold for $ 200 each, one at a gain of 20 % and the other at a loss of 20 %. Did the seller gain or lose by the transaction, and how much?

20. A sells goods to B at a profit of 10 %, and B sells them to C at a profit of 15 %. If C paid $ 253 for the goods, find A's cost price.

21. A merchant increases his capital $18\frac{3}{4}$ %, and at the end of a second year he also increases his capital $18\frac{3}{4}$ %. If his capital is then $ 14,440, what was it at first?

22. The first year A adds 25 % to his capital, the next year he adds 25 % to the capital of the previous year, the third year he loses 40 % of his capital, and is then worth $ 10,800. How much capital did he begin with?

23. If it takes $ 81,415 for the running expenses of the schools of a city, and if 95 % of the tax levied for school

purposes goes to the support of the schools, what should be the amount levied for school purposes?

24. By selling tea at 65 ¢ per pound a merchant gained $ 118.75. If his gain was $62\frac{1}{2}$ %, how many pounds of tea did he sell?

25. By selling turkeys at $ 2.50 each a dealer makes a profit of 60 %. What would have been his gain per cent, had he sold the turkeys for $ 1.75 each?

26. By selling wine at $ 2.10 a gallon, a merchant makes a profit of 20 %. What would be his gain per cent, should he sell the wine for $ 2 a gallon?

27. A merchant mixes tea which cost him 60 ¢ per pound with tea which cost him 70 ¢ per pound in the proportion of 5 lb. of the former tea to 6 lb. of the latter. If he sells the mixture at 80 ¢ per pound, find his gain per cent.

28. A horse is sold for $ 145.50, at a loss of 3 %. At what price should the horse have been sold so as to make a profit of 10 %?

29. If oranges are bought at 5 for 3 ¢ and sold at 3 for 5 ¢, what is the gain per cent?

30. If 325 lb. of sugar are bought for $ 13, at what price per pound must it be sold to make a profit of 25 %?

31. If 15 yd. of silk are bought for $ 26.25, at what price per yard should it be sold to make a profit of 20 %?

32. By selling cloth at the rate of $17\frac{1}{2}$ ¢ a yard a clothier loses $12\frac{1}{2}$ %. What should the selling price per yard be so as to make a profit of 20 %?

33. A merchant buys butter at 18 ¢ per pound and sells 120 lb. for $ 25.20. What is his gain per cent?

34. A grocer buys cheese at 9 ¢ per pound and sells it at the rate of 8 lb. for $ 1. What is his gain per cent?

35. A merchant mixes 5 lb. of one kind of tea with 2 lb. of another. If the teas are worth respectively 68¢ and 75¢ per pound, what should be the selling price per pound so as to make a profit of 20 %?

36. If I buy a horse for \$80 and sell him for \$71, what is the loss per cent?

37. By selling a horse for \$83.30 I lost 15 %. What did the horse cost?

38. Two men, A and B, buy two horses, each paying the same price. A sells his horse for \$90 at a loss of $14\frac{2}{7}$ %. B sells his horse at a loss of 6 %. Find the selling price of B's horse.

39. A merchant buys 70 yd. of cloth at 50¢ a yd. and sells it at a gain of 15 %. He buys also 70 yd. of silk at 90¢ a yard. On both he gains 10 %. At what price per yard does he sell the silk?

40. I buy a quantity of barley and intrust it to an agent to sell. The agent sells it at an advance of 25 % on the cost. For his services he charges 2 % on the selling price. He takes his pay out of the money he receives and sends me the balance, which is \$539. Find the cost of the barley.

41. A fruit dealer buys a crate of oranges for \$2.50. He sells them at 2¢ each, making a profit of 60 %. How many oranges in the crate?

42. If goods are bought at 25 % below the retail price and sold at the retail price, what is the gain per cent?

43. A coal dealer buys 120 T. of coal at \$4 a ton. He sells $\frac{1}{2}$ of the coal at an advance of 25 %, $\frac{1}{3}$ of it at an advance of 50 %, and the remainder at an advance of 10 %. Find his entire profit and his gain per cent on the coal.

Example. How should goods be marked so that a dealer may give a discount of 20 % and still make a profit of 15 %?

SOLUTION. (100 % − 20 %) of marked price = 80 % of marked price.

$$\text{Cost} + 15\,\% \text{ of cost} = 115\,\% \text{ of cost.}$$

$$\therefore \quad 80\,\% \text{ of marked price} = 115\,\% \text{ of cost.}$$

$$\therefore \quad 1\,\% \text{ of marked price} = \frac{115\,\%}{80} \text{ of cost.}$$

$$\therefore \; 100\,\% \text{ of marked price} = \frac{115\,\%}{80} \times 100 = 143\tfrac{3}{4}\,\% \text{ of cost.}$$

The goods must be marked $43\frac{3}{4}$ % above cost.

44. (*a*) How should goods be marked so as to make a profit of 12 % after deducting 20 % from the marked price?

(*b*) How should they be marked so as to make a profit of 17 % after deducting 10 % from the marked price?

(*c*) How should they be marked so as to make a profit of 20 % after deducting 25 % from the marked price?

(*d*) How should they be marked so as to make a profit of 10 % after deducting 10 % from the marked price?

(*e*) By taking 30 % off the marked price a merchant neither gains nor loses. How were the goods marked?

COMMERCIAL DISCOUNTS

Commercial or **trade discount** is an allowance made on the list price of goods, or on the amount of a bill.

Discounts are reckoned as per cent.

Example. If watches are listed at $35 each, and a discount of 10 % is allowed, what is the cost of one of the watches?

SOLUTION. 10% of $\$35 = \$3.50 =$ the discount.

$\$35 - \$3.50 = \$31.50.$

$\therefore$ the cost is $31.50.

EXERCISE 83

Find the cost when the list prices and rates of discount are :

1. $60, 20% off.
2. $75, 30% off.
3. $15, 25% off.
4. $14, 20% off.
5. $24, 10% off.
6. $32, 15% off.
7. $68, $12\frac{1}{2}\%$ off.
8. $78, $16\frac{2}{3}\%$ off.
9. $92, $33\frac{1}{3}\%$ off.
10. $80, $37\frac{1}{2}\%$ off.
11. $152, $12\frac{1}{2}\%$ off.
12. $176, 25% off.
13. $88, 5% off.
14. $96, 5% off.
15. $125, 15% off.
16. $175, 2% off.
17. $213, 4% off.
18. $723, 6% off.
19. $3280, 8% off.
20. $2712, 7% off.
21. $5350, $8\frac{1}{3}\%$ off.
22. $2490, $6\frac{1}{4}\%$ off.
23. $3778, 9% off.
24. $5062, 12% off.
25. $885, 15% off.
26. $363, 12% off.
27. $689, 8% off.
28. $2034, 7% off.
29. $992, $7\frac{1}{2}\%$ off.
30. $2572, $4\frac{1}{2}\%$ off.

COMMERCIAL DISCOUNTS WHEN TWO OR MORE ARE ALLOWED

In some cases several discounts are allowed. If there are two or more discounts, the first is reckoned on the list or catalogue price ; the next is reckoned on the remainder after deducting the first discount ; the third is reckoned on the second remainder ; and so on.

Example 1. What is the cost, if the list price is $750, and discounts of 20 %, 15 %, and 10 % are allowed?

SOLUTION. First discount = 20 % of $750 = $\frac{1}{5}$ of $750 = $150. ∴ the first remainder = $750 − $150 = $600.

The second discount = 15 % of $600 = $90.

$600 − $90 = $510.

The third discount = 10 % of $510 = $51.

$510 − $51 = $459. Or

100 % − 20 % = 80 % ; 100 % − 15 % = 85 % ;

100 % − 10 % = 90 %.

90 % of 85 % of 80 % of $750 = $459.

Example 2. What single discount is equivalent to the three discounts in the above example?

SOLUTION. $80\% \times 85\% \times 90\% = .8 \times .85 \times .9 = .612 = 61.2\%$.

$$100\% - 61.2\% = 38.8\% \textit{ Ans.}$$

EXERCISE 84

1. A suit of clothes is marked $70, and is sold with discounts of 25 % and 10 % for cash. Find the selling price.

2. On a bill of $900 two discounts of 20 % and 15 % are allowed. What is the net amount of the bill?

3. If goods are marked $175 and sold for $122.50, what is the discount?

4. On a bill of $1500 discounts of 25 %, 15 %, and 6 % are allowed. Find the net cash amount of the bill.

5. A piano is listed at $450, with discounts of 20 % $12\frac{1}{2}$ %, and 10 %. Find the cost price to the purchaser.

6. Find the cash value of a bill of $320 with discounts of 15 %, 10 %, and 5 %.

7. Suppose you were offered a single discount of 45 % or two discounts of 30 % and 20 %, which would you take? What would be the difference in a bill of $1000?

8. A dealer buys a quantity of goods marked $550, with a discount of 20 %. If he sells the goods at 6 %, above the marked price, what is his gain per cent?

9. If I buy goods listed at $150, with a discount of 10 %, and sell them at 12 % above the marked price, find my gain per cent.

10. A bookseller buys 100 books marked $1.50 each, at a discount of 20 % and sells them at the marked price. What is his gain per cent?

11. A bookseller bought 100 books at $1.25 each. He made a profit of 20 % after giving a discount of $\frac{1}{4}$. At what price did he mark each book, and what was his profit?

12. Find the cost price in each case, if the list price and the rates of discount are as follows:

	List Price	Discounts
(*a*)	$480	20 %, 30 %, 10 %
(*b*)	$1000	40 %, 5 %, 4 %
(*c*)	$775	25 %, $\frac{1}{5}$
(*d*)	$880	$12\frac{1}{2}$ %, 4 %
(*e*)	$720	$\frac{1}{3}$, 16 %
(*f*)	$960	$12\frac{1}{2}$ %, 12 %
(*g*)	$1200	10 %, 8 %, 5 %
(*h*)	$1760	25 %, $12\frac{1}{2}$ %, $16\frac{2}{3}$ %

COMMISSION AND BROKERAGE

A very large part of the buying and selling of the produce of the country is done through commission merchants and brokers. A commission merchant is usually

intrusted with goods, such as butter, eggs, fruit, vegetables, etc., which are sent to him to be sold.

A broker finds purchasers for goods which may not be in his possession, but which he has been asked to sell. The distinction is not exact, but usually brokers deal in financial securities, such as stocks and bonds; while commission merchants deal in commodities, such as potatoes.

The fee which a commission merchant charges for his services, or which an agent charges who buys and sells land, or collects rents, is called a **commission.** The fee which a broker charges is called **brokerage.**

Commission and brokerage are usually reckoned as per cents of the buying price when goods or other commodities are bought, or of the selling price when they are sold.

The person who employs another person to buy, to sell, or to do other business is called a **principal.** The person who transacts business for a principal is called an **agent.** Commission merchants and brokers are agents.

Example 1. A commission merchant sold 400 boxes of oranges at \$2.75 per box, charging 5 % commission. What was his commission, and how much did he remit?

The price of 400 boxes of oranges @ \$2.75 = \$1100.

$5\,\%$ of \$1100 = \$55, the commission.

\$1100 − \$55 = \$1045, amount remitted.

Example 2. A commission merchant charges \$94.50 for selling 4500 bushels of wheat. Rate of commission $2\frac{1}{2}\,\%$. What is the price of wheat per bushel?

$2\frac{1}{2}\,\% = \frac{1}{40}$. Selling price = \$94.50 × 40 = \$3780.

Selling price of one bushel = $\$\frac{3780}{4500}$ = \$.84.

Example 3. A real estate agent remits to his principal \$9788.75, being the amount of the sales of four city lots

after deducting a commission of $4\frac{1}{2}\%$. Find the selling price of the four lots and the agent's commission.

SOLUTION. The selling price $- 4\frac{1}{2}\%$ of itself $= 95\frac{1}{2}\%$ of the selling price.

$95\frac{1}{2}\%$ of the selling price $= \$9788.75$.

$\therefore 1\%$ of the selling price $= \dfrac{\$9788.75}{95\frac{1}{2}}$.

$\therefore 100\%$ of the selling price $= \dfrac{\$9788.75}{95\frac{1}{2}} \times 100 = \$10,250$.

$\therefore \$10,250 - \$9788.75 = \$461.25$, the commission.

EXERCISE 85

1. What is the commission for selling 200 A. of land at \$40 an acre, the rate of commission being 4 %?

2. An agent charges 6 % for selling real estate. If he sells 350 A. of land at \$50 an acre, find his commission and the sum he remits to his principal. What per cent of the selling price does the principal receive?

3. If 200 boxes of oranges are sold at \$3.75 a box by a commission merchant who charges 6 %, what is the commission? How much is remitted to the principal?

4. If 800 bu. of wheat are sold on a commission of $\frac{1}{2}$¢ per bushel, what is the commission?

5. How many bushels of barley at 45¢ per bushel must a commission merchant sell at $2\frac{1}{2}\%$ commission to make an annual salary of \$1080?

6. If 480 bbl. of flour at \$3.90 a barrel are sold on commission at a rate of 3 %, find the commission. What is the net amount realized from the sale?

7. A broker sells 2500 lb. of beef at 12¢ a pound. What is his brokerage at $1\frac{1}{2}\%$?

8. Thirty-six hundred gallons of oil are sold by a broker at 45 ¢ per gallon. If he charges $3\frac{1}{2}$ % brokerage, find his brokerage and the amount remitted.

9. A commission merchant sells 1500 T. of hay at $15 per ton, and charges 5 % commission. Find his commission. How much does he remit? What per cent of the selling price does he remit?

10. A coal dealer receives a commission of 90 ¢ a ton for selling anthracite coal, and 80 ¢ a ton for selling bituminous coal. If he sells an equal quantity of each kind of coal, and if his entire commission is $1530, how many tons of each kind does he sell?

11. A broker charges 35 ¢ for selling a bale of cotton. How many bales must he sell to realize for himself $295.40? If a bale contains 500 lb., and the brokerage is equivalent to $\frac{7}{8}$ %, find the price of cotton per pound.

12. A commission merchant sells 1200 doz. eggs at 18 ¢ per dozen. Find his commission at 8 %.

13. If a commission of $70.98 is paid for buying 40 A. of land at $54.60 an acre, what is the rate per cent of commission?

14. If $284 is received for selling grain on a commission of $\frac{1}{8}$ ¢ per bushel, how many bushels of grain were sold?

15. If an agent charges 3 % for collecting debts, and in one month his commission from this source is $126, what is the amount collected?

16. A real estate agent's commission at $4\frac{1}{2}$ % is $351. Find the amount of his sales and the amount he remits to his principal.

17. A broker buys 10,000 bu. of wheat at $79\frac{1}{2}$¢, and sells the wheat the next day at $79\frac{7}{8}$¢, charging $\frac{1}{8}$¢ a bushel for buying and $\frac{1}{8}$¢ for selling. Find his commission.

18. An agent remits to his principal \$9184.50 as the net proceeds from the sale of 12,000 bu. of wheat. Find the agent's commission if he charges at the rate of $2\frac{1}{2}$%. Find also the selling price of wheat per bushel.

19. After deducting a commission of 3%, an agent remits \$1813.90 from the sale of oats at $46\frac{3}{4}$¢ per bushel. How many bushels of oats were sold?

20. When the market price of pork is 13.6¢ per lb., how many pounds can be bought for \$520.20, if 2% is charged for brokerage?

21. How many acres of land can be bought for \$4635 at \$30 an acre, by a real estate agent who charges a commission of 3%?

22. Find the commission and amount remitted to the principal on the sale of the following articles:

(*a*)	Turkey	1750 lb.	@	$12\frac{3}{4}$¢	8%	commission
(*b*)	Oranges	275 boxes	@	\$2.95	5%	commission
(*c*)	Apples	580 bbl.	@	\$1.85	8%	commission
(*d*)	Oysters	390 bbl.	@	\$1.45	5%	commission
(*e*)	Potatoes	1270 bu.	@	\$1.05	$7\frac{1}{2}$%	commission
(*f*)	Celery	560 bunches	@	\$ .85	10%	commission
(*g*)	Onions	240 bu.	@	\$1.25	8%	commission
(*h*)	Beans	960 bu.	@	\$2.15	5%	commission
(*i*)	Rice	3500 lb.	@	$5\frac{3}{4}$¢	5%	commission
(*j*)	Chickens	984 lb.	@	$11\frac{1}{2}$¢	9%	commission
(*k*)	Cotton	287 bales	@	\$49.	3%	commission
(*l*)	Peaches	315 boxes	@	\$1.35	9%	commission

INTEREST

Interest is money paid for the use of money.

The **principal** is the sum of money for the use of which interest is paid.

The **amount** is the sum of the principal and the interest.

Simple interest is reckoned as a per cent of the principal, usually, for one year. In most transactions, the year is considered as having 360 days, and the month 30 days.

To find the simple interest on a sum of money, **Multiply the principal by the rate to get the interest for 1 year, and this product by the time expressed in years.**

Example 1. Find the interest and the amount of \$1780 for 5 mo. at 7%.

SOLUTION: The interest on \$1780 for 1 year at 7% is $.07 \times \$1780$. As 5 mo. is $\frac{5}{12}$ of 1 yr., the interest for 5 mo. is $\frac{5}{12}$ of the interest for 1 yr. Therefore, the interest on \$1780 for 5 mo. at 7% is $\frac{5}{12} \times .07 \times \$1780 = \$51.92$.

The amount is the sum of the principal and the interest. Therefore, the amount of \$1780 for 5 mo. at 7% is $\$1780 + \$51.92 = \$1831.92$.

A concrete quantity which is contained an exact number of times in another concrete quantity is called an **aliquot part** of that quantity.

Thus, $2\frac{1}{2}$ yd. is an aliquot part of 10 yd.; \$2.75 is an aliquot part of \$11.

The principle of aliquot parts which is but another name for exact ratio is the most convenient and, outside of banks, the most widely used method of computing interest.

Example. Find the interest on \$780 for 1 yr. 2 mo. 10 da. at 7%.

Solution by Aliquot Parts

	\$780
	.07
	\$54.60 = int. for 1 yr.
2 mo. = $\frac{1}{6}$ of 1 yr.	9.10 = int. for 2 mo.
10 da. = $\frac{1}{6}$ of 2 mo.	1.52 = int. for 10 da.
	\$65.22 = int. for 1 yr. 2 mo. 10 da.

EXERCISE 86

Find the interest on:

1. \$700 for 1 yr. at 3 %; for 1 yr. at 5 %.
2. \$278 for 1 yr. at 6 %; for 1 yr. at 7 %.
3. \$598 for 1 yr. at 9 %; for 1 yr. at 8 %.
4. \$289 for 2 yr. at 4 %; for 2 yr. at 5 %.
5. \$1000 for 1 yr. 3 mo. at 6 %; for 1 yr. 5 mo. at 6 %.
6. \$1200 for 1 yr. 4 mo. at 8 %; for 1 yr. 5 mo. at 7 %.
7. \$1500 for 1 yr. 7 mo. at 9 %; for 1 yr. 8 mo. at 8 %.
8. \$1600 for 2 yr. 3 mo. at 5 %; for 1 yr. 7 mo. at 6 %.
9. \$2000 for 1 yr. 6 mo. at 4 %; for 1 yr. 9 mo. at 5 %.
10. \$3500 for 10 mo. at 7 %; for 7 mo. at 8 %.
11. \$156.40 for 1 yr. at $4\frac{1}{2}$ %; at $5\frac{1}{2}$ %.
12. \$185.50 for 1 yr. at $4\frac{1}{2}$ %; at 7 %.
13. \$375.60 for 5 mo. at 6 %; for 4 mo. at 5 %.
14. \$928.40 for 2 mo. at 8 %; for 3 mo. at 6 %.
15. \$735.60 for 3 mo. at 7 %; for 2 mo. at 8 %.
16. \$1200 for 2 mo. at 8 %; for 5 mo. at 6 %.
17. \$1350.50 for 5 mo. at 5 %; for 4 mo. at 7 %.
18. \$393.80 for 7 mo. at 4 %; for 5 mo. at 3 %.
19. \$385.40 for 8 mo. at 5 %; for 7 mo. at 6 %.
20. \$934.54 for 9 mo. at 8 %; for 8 mo. at 9 %.

21. $2713.64 for 10 mo. at 9 %; for 7 mo. at 8 %.

22. $3800 for 11 mo. at 7 %; for 10 mo. at 6 %.

23. $2825 for 10 mo. at 7 %; for 9 mo. at 5 %.

24. $2700 for 1 yr. 1 mo. at 6 %, for 14 mo. at 5 %.

25. $3280 for 1 yr. 3 mo. at 5 %; for 8 mo. at 4 %.

26. $4500 for 8 mo. 15 da. at 6 %; for 7 mo. at 7 %.

27. $329.50 for 7 mo. 10 da. at 8 %.

28. $982 for 9 mo. 18 da. at 6 %.

29. $545 for 10 mo. 25 da. at 8 %.

30. $775.24 for 11 mo. 24 da. at 7 %.

Example. Find the interest on $384.42 from Jan. 11, 1907, to April 30, 1907, at 7 %.

SOLUTION. First, find the difference between the two dates.

Yr.	Mo.	Da.
1907	4	30
1907	1	11
	3	19

$384.42
.07
$26.9094 = int. for 1 yr.

3 mo. = $\frac{1}{4}$ of 1 yr.	6.727	= int. for 3 mo.
15 da. = $\frac{1}{6}$ of 3 mo.	1.121	= int. for 15 da.
3 da. = $\frac{1}{5}$ of 15 da.	.224	= int. for 3 da.
1 da. = $\frac{1}{3}$ of 3 da.	.075	= int. for 1 da.
	$8.147	= int. for 3 mo. 19 da.

Ans. $8.15.

EXERCISE 87

Find the interest on :

1. $450 from Jan. 12 to April 18 at 6 %.

2. $783 from Feb. 14 to April 24 at 8 %.

3. $2385 from Mar. 6 to Nov. 11 at 6%.
4. $3950 from July 4 to Dec. 6 at 6%.
5. $4280 from Aug. 31 to Nov. 18 at 8%.
6. $7335 from July 5 to Sept. 14 at 7%.
7. $3280 from Jan. 8 to July 5 at 4%.
8. $4592.40 from Jan. 10 to May 4 at 3%.
9. $384.75 from Mar. 10 to Aug. 14 at 10%.
10. $327.50 from April 6 to July 3 at 9%.
11. $935 from Jan. 1 to April 30 at $4\frac{1}{2}$%.
12. $3540 from Jan. 10 to Oct. 5 at $4\frac{1}{2}$%.
13. $1382.60 from Feb. 8 to Nov. 4 at 5%

EXERCISE 88

Find the amount of :

1. $800 for 1 yr. 2 mo. 15 da. at 4%

   HINT. Find the interest and add it to the principal.

2. $580 for 3 yr. 3 mo. at 6%.
3. $750 for 10 mo. 15 da. at 8%.
4. $327.60 for 11 mo. 12 da. at 9%.
5. $326.54 for 8 mo. 8 da. at 5%.
6. $739.90 for 5 mo. 11 da. at 4%.
7. $843.90 for 6 mo. 14 da. at 8%.
8. $325 for 9 mo. 12 da. at $4\frac{1}{2}$%.
9. $982 for 8 mo. 16 da. at 5%.
10. $375 for 7 mo. 14 da. at 6%.
11. $1280 from Jan. 10 to July 14 at 8%.
12. $3580 from Jan. 14 to Aug. 18 at 8%.
13. $1500 from March 11 to July 19 at 6%.
14. $4350 from Aug. 14 to Dec. 17 at 7%.

15. \$1200 from Jan. 1 to July 8 at 7 %.

16. \$480 for 9 mo. 15 da. at 5 %.

17. \$800 for 6 mo. 18 da. at 6 %.

18. \$550 for 3 mo. 15 da. at 8 %.

19. \$650 for 5 mo. 12 da. at 6 %.

20. \$850 for 8 mo. 15 da. at 5 %.

21. \$980 for 7 mo. at 6 %.

22. \$2329 for 1 yr. 7 mo. 16 da. at 5 %.

23. \$3278 for 2 yr. 10 mo. 11 da. at 7 %.

24. \$2594 for 1 yr. 5 mo. 10 da. at 3 %.

25. \$978 for 1 yr. 1 mo. 20 da. at 6 %.

26. \$1857 for 1 yr. 5 mo. 18 da. at 5 %.

27. \$903.53 for 1 yr. 3 mo. at 5 %.

EXERCISE 89

Example 1. How many sq. yd. in 1 sq. mi.?

1 sq. mi. $= 1760 \times 1760 \times 1$ sq. yd. $= 3{,}097{,}600$ sq. yd.

Example 2. A lot in the form of a rectangle contains 16 A., and is 48 rd. wide. Find its length.

SOLUTION. $48 \times \text{length} = 16 \times 160$. (160 sq. rd. = 1 A.)

Therefore, $\text{length} = \dfrac{16 \times 160}{48} = \dfrac{160}{3} = 53\frac{1}{3}$. *Ans.* $53\frac{1}{3}$ rd.

Example 3. A rectangular plot of ground 55 yd. by 11 yd. produces 2 bu. of buckwheat. How much will 1 A. produce at this rate?

SOLUTION. 55×11 sq. yd. produce 2 bu.

$$1 \text{ sq. yd. produces } \frac{2}{55 \times 11} \text{ bu.}$$

$$4840 \text{ sq. yd. produce } 4840 \times \frac{2}{55 \times 11} \text{ bu.} = 16 \text{ bu.}$$

1. The area of a rectangle is 12,500 sq. ft. and one side is 100 ft. Find its other dimension.

2. The area of a rectangle is $30\frac{1}{4}$ sq. yd. and its length is $5\frac{1}{2}$ yd. Find its width.

3. A lot contains 16 A., and its length is 64 rd. Find its width.

4. The area of a room is 252 sq. ft., and its length is 18 ft. Find its width.

5. A rectangular piece of ground 15 yd. by 12 yd. yields 6 bu. of potatoes. Find the yield of an acre.

6. A rectangular tract of land 30 yd. by 11 yd. yields 2 bu. of oats. At this rate how much will 1 A. yield?

7. A rectangular tract of land 50 yd. by 15 yd. yields 3 bu. of corn. At this rate how much will 1 A. yield?

8. The base of a triangle is 84 yd. and its area is 840 sq. yd. Find its height.

9. Find the altitude of a triangle having for base 34 yd. and area 289 sq. yd.

10. Given the area of a triangle 1024 sq. yd. and base 64 yd., find its altitude.

11. The area of the walls of a room is 560 sq. ft. and the room is 16 ft. by 12 ft. Find the height of the room.

12. The width of a rectangle is 3.9 ft. and its area is 17.55 sq. ft. Find the length.

13. The area of a hall is 497 sq. ft., its length is 71 ft. Find its width.

14. A street 1 mile long contains an area of $5\frac{1}{3}$ acres. Find the width of the street in feet.

EXERCISE 90

1. The inside dimensions of a box car are 36 ft. by 8 ft. by 8 ft. 6 in. Find the number of cubic feet in the car, the number of bushels it will hold, allowing $1\frac{1}{4}$ cu. ft. to the bushel, and the weight of the carload of corn.

SOLUTION. $36 \times 8 \times 8\frac{1}{2} = 2448 =$ number cu. ft. in car.
$2448 \div 1\frac{1}{4} = 2448 \times .8 = 1958.4 =$ number bu. in car.
1958.4×56 lb. $= 109670.4$ lb. $=$ weight of corn in car.

2. The dimensions of a box car are 32 ft. by 8 ft. by 7 ft. Find the number of cubic feet in the car, the number of bushels it will hold, allowing $1\frac{1}{4}$ cu. ft. to the bushel, and the weight of the carload of oats.

3. A refrigerator car is 28 ft. 9 in. long, 7 ft. 6 in. wide, and 8 ft. high. Find the number of cubic feet in it. The capacity of this car is 64,000 lb. How many dressed turkeys, averaging $12\frac{1}{2}$ lb. each, will the car hold, allowing 4000 lb. for ice?

4. A car 100,000 lb. capacity, length 40 ft., width 8 ft. 6 in., height 8 ft., inside dimensions, is loaded with wheat. Find the number of cubic feet in the car. Find the weight of the wheat that will fill it. What fraction of this weight is the capacity of the car?

5. A car 40 ft. long, 8 ft. 6 in. wide, 8 ft. high, is loaded with 4-ft. wood. How many cords does this car contain? What is the value of the wood at $4.75 per cord? What will it cost to ship the wood from Sublime to San Antonio at $1.50 per cord?

6. The electric railroad from Rochester to Avon, New York, is 19 miles long. The rails used in its construction weigh 80 lb. to the yard. Find in tons the weight of the rails of this railroad.

EXERCISE 91

Example 1. Find the weight in pounds Avoirdupois of 5000 silver dollars. A silver dollar weighs $412\frac{1}{2}$ grains.

SOLUTION. 1 lb. (Troy) = 5760 grains.
1 lb. (Avoirdupois) = 7000 grains.
1 dollar weighs $412\frac{1}{2}$ grains.
5000 dollars weigh $5000 \times 412\frac{1}{2}$ grains.

5000 dollars weigh as many pounds as the number of times that 7000 grains are contained in $5000 \times 412\frac{1}{2}$ grains.

$$\frac{5000 \times 412\frac{1}{2}}{7000} = 294\tfrac{9}{14} \text{ lb. Av.}$$

2. Find the weight in pounds Troy of 1,000,000 silver dollars. Find also the weight in pounds Avoirdupois.

3. A silver dollar is 90 % silver. Find the weight of pure silver in one silver dollar.

4. The commercial value of a silver dollar is the value of the pure silver it contains. What is the commercial value of a silver dollar when pure silver is worth 66¢ per oz. (Troy oz.)?

5. A 10-dollar gold piece weighs 258 grains. Find the weight of 10,000 dollars in gold. Find also the weight of 1,000,000 dollars in gold.

6. Gold coins contain 90 % pure gold. Find the weight of pure gold in a 10-dollar gold piece.

7. The alloy in a gold coin neither adds to nor takes from the value of the coin. Compute the value of 480 grains of pure gold. (Remember that 90 % of 258 gr. is gold worth $10.)

8. The bullion value of the pure silver in a silver dollar at the average price of silver for the year 1905 was $.472. Find the value of 1 oz. Troy of pure silver.

9. In 1906 the value of an ounce of pure silver was \$.6769, and the value of an ounce of pure gold was \$20.67. How many ounces of silver could be bought for one ounce of gold at those prices?

SOLUTIONS OF PROBLEMS

THE UNIT METHOD

Example 1. If 17 bu. of wheat cost \$15.30, find the cost of 15 bu.

SOLUTION. 17 bu. cost \$15.30.

$$1 \text{ bu. costs } \frac{\$15.30}{17}.$$

$$15 \text{ bu. cost } 15 \times \frac{\$15.30}{17} = \$13.50.$$

Example 2. If 25 men do a piece of work in 18 days, how long will it take 20 men to do the same work?

SOLUTION. 25 men take 18 da. to do the work.

$\therefore$ 1 man takes 18 da. $\times$ 25 to do the work.

$\therefore$ 20 men take $\frac{1}{20}$ of 18 da. $\times 25 = 22\frac{1}{2}$ days.

EXERCISE 92

1. If 19 bu. of wheat cost \$14.25, find the price of 11 bu. of wheat.

2. Seven men can dig a trench in 16 da. How long will it take 10 men to do the same work?

3. If 11 sheep cost \$71.50, find the cost of 17 sheep at the same rate.

4. If 14 men can do a piece of work in 11 da., how long will it take 21 men to do the same work?

5. The earth revolves on its axis 15° in 1 hour. Through how many degrees does it revolve in 23 minutes?

6. Twelve horses plow a field of 47 acres in 7 days. How many acres will 8 horses plow in the same time?

7. A pole 30 ft. high casts a shadow 24 ft. long. Find the length of the shadow cast by a pole 70 ft. high.

8. How long will it take 20 men to pave a street which 15 men pave in 15 days?

9. A sum of money yields \$40.30 interest in 125 days. What interest will the same sum yield in 75 days?

10. A train runs 40 miles in 1 hr. and 20 min. How far will it run in 2 hr.?

11. If $3\frac{1}{2}$ acres of land are worth \$259, what is the value of $2\frac{1}{2}$ acres?

12. If $\frac{2}{3}$ of an acre of land is worth \$89, how much is a tract measuring $2\frac{1}{2}$ acres worth?

13. A train runs 45 miles an hour. Find how many feet it will go in 1 minute; in 1 second.

14. When the rate of a train is 36 miles an hour, what is its rate in feet per second?

15. Sound travels 1 mile in 5 seconds. How long will it take sound to travel 4400 feet?

16. Light travels from the sun to the earth, a distance of 92,790,000 miles, in 8 minutes and 18 seconds. Find its rate per second.

17. If $\frac{2}{3}$ of a clerk's yearly salary is \$900, what is his salary per month?

18. If $11\frac{1}{2}$ yards of carpet can be bought for a certain sum of money, how many yards can be bought for the same sum when the price of carpet falls 8 %?

MISCELLANEOUS TOPICS

WORK AND TIME

Example 1. A can do a piece of work in 6 da., and B can do the same piece of work in 8 da. In what time can A and B do the work together?

ANALYTICAL SOLUTION. A does $\frac{1}{6}$ of the work in 1 da. B does $\frac{1}{8}$ of the work in 1 da.

A and B together do $(\frac{1}{6}+\frac{1}{8})$ of the work in 1 da.

A and B do $\frac{7}{24}$ of the work in 1 da.

A and B do $\frac{1}{24}$ of the work in $\frac{1}{7}$ of 1 da.

A and B do $\frac{24}{24}$ of the work in $\frac{24}{7}$ of 1 da.

$\therefore$ A and B do the work in $3\frac{3}{7}$ da.

Example 2. A cistern has three pipes. The first pipe fills the cistern in 12 hr., the second, in 15 hr., and the third empties it in 10 hr. In what time will the cistern be filled, if all three pipes run together, and the cistern is empty when the pipes start running?

ANALYTICAL SOLUTION. The first pipe fills $\frac{1}{12}$ of the cistern in 1 hr.

The second pipe fills $\frac{1}{15}$ of the cistern in 1 hr.

The third pipe empties $\frac{1}{10}$ of the cistern in one hour.

The first two pipes would fill $(\frac{1}{12}+\frac{1}{15})$ of the cistern in 1 hr. $\frac{1}{12}+\frac{1}{15}=\frac{3}{20}$. But during each hour an amount equal to $\frac{1}{10}$ of the capacity of the cistern runs out through the third pipe. So the amount left is the difference between what runs in during the hour and what runs out. This is $\frac{3}{20}-\frac{1}{10}$ which equals $\frac{1}{20}$.

the three pipes fill $\frac{1}{20}$ of the cistern in 1 hr.

the three pipes fill $\frac{20}{20}$ of the cistern in 20 hr.

$\therefore$ the cistern is filled in 20 hr.

Example 3. A and B do a piece of work in $3\frac{11}{15}$ hr.; A alone, in 7 hr. In what time does B do the work?

SOLUTION. A and B together do $\dfrac{1}{3\frac{11}{15}}$ of the work in 1 hr.; *i.e.* A and B together do $\frac{15}{56}$ of the work in 1 hr.

But A alone does $\frac{1}{7}$ of the work in 1 hr.

B alone does $(\frac{15}{56} - \frac{1}{7})$ of the work in 1 hr.

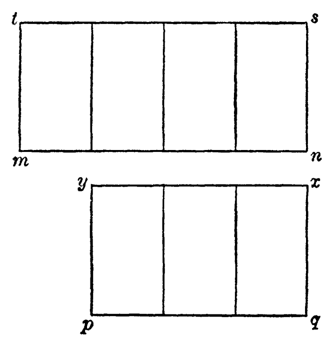
B alone does $\frac{1}{8}$ of the work in 1 hr.

$\therefore$ B does the work in 8 hr.

Example 4. A and B can mow a field in 21 hr. A can do $\frac{3}{4}$ as much work as B. Find the time in which each does the work.

SOLUTION. Represent B's work by the rectangle *mnst*. Divide it into four equal parts. Then A's work is equal to three of these parts.

Represent A's work by the rectangle *pqxy* formed of three parts equal to the parts in rectangle *mnst*. Then the two rectangles together represent as much as the whole field.



If it takes A 21 hrs. to mow three of the parts, or $\frac{3}{7}$ of the field, it will take him $\frac{1}{3}$ of 21 hrs., or 7 hrs., to mow one part, or $\frac{1}{7}$ of the field. If it takes him 7 hrs. to mow one part, or $\frac{1}{7}$ of the field, it would take him 7×7 hrs., or 49 hrs., to mow the seven parts, or $\frac{7}{7}$ of the field, which is the whole field.

Similarly, if it takes B 21 hrs. to mow 4 parts, or $\frac{4}{7}$ of the field, it will take him $\frac{21}{4}$ hrs. to mow $\frac{1}{7}$ of the field,

and $7 \times \frac{21}{4}$ hrs., or $36\frac{3}{4}$ hrs., to mow $\frac{7}{7}$, or the whole field.

Therefore,

$$\text{A's time} = 49 \text{ hrs.}$$
$$\text{B's time} = 36\tfrac{3}{4} \text{ hrs.}$$

EXERCISE 93

1. If a person can do a piece of work in 7 da., what part of the work can he do in one day? If he can do the work in $4\frac{1}{2}$ da., what part of the work can he do in one day?

2. If B can copy a manuscript in $5\frac{3}{4}$ hr., how much can he copy in 1 hr.?

3. John travels $\frac{5}{16}$ of the distance between two cities in 1 hr. How many hours will it take him to travel the remainder of the distance?

4. A can do a piece of work in 3 hr., B in 5 hr., and C in 5 hr. How long will it take the three working together to do the work?

5. A can do a piece of work in 4 hr., B in 5 hr., and A, B, and C together in $1\frac{1}{2}$ hr. How long would it take C alone to do the work?

6. A, B, and C can do a piece of work in 6, 8, and 10 da., respectively. If they begin the work together, what part of the work remains to be done at the end of the second day?

7. A, B, and C can build a fence in 10, 15, and 20 hr. respectively. They work together for 4 hr., when B quits. In what time can A and C finish the work?

8. A cistern has two pipes. One can fill it in 20 min., and the other can empty it in 30 min. If the cistern is

empty, in what time can it be filled, if both pipes begin to flow at the same instant?

MOTION IN THE SAME DIRECTION, OR IN OPPOSITE DIRECTIONS

Example 1. A starts to overtake B, who is 100 yd. ahead of him. A travels 11 yd. to B's 9 yd. How far must A travel in order to overtake B?

SOLUTION. A gains on B 2 yd. in every 11 yd. he goes.

∴ A gains on B 1 yd. in every $5\frac{1}{2}$ yd. he goes.

∴ A gains on B 100 yd. in every 550 yd. he goes.

Ans. 550 yd.

Example 2. A freight train moving at the rate of 18 mi. an hour is 78 mi. ahead of a passenger train moving in the same direction at the rate of 30 mi. an hour. Find the distance the passenger train must run to overtake the freight train.

SOLUTION. In 1 hr. the passenger train gains on the freight $(30 - 18)$ mi., *i.e.* 12 mi.

∴ in $(78 \div 12)$ hr. the passenger train will overtake the freight train, *i.e.* in $6\frac{1}{2}$ hr.

In $6\frac{1}{2}$ hr. the passenger train goes 30 mi. $\times\ 6\frac{1}{2} = 195$ mi.

EXERCISE 94

1. Two ships leave New York for Glasgow, one on Monday morning at 9 o'clock, and the other on the following morning at 9 o'clock. Their rates were 15 and 21 miles an hour respectively. How far from New York City will the second ship overtake the first?

2. Dallas and Galveston are 315 mi. apart. A train leaves Dallas for Galveston at 8 o'clock A.M. at the rate of 30 mi. an hour. At the same time a train leaves Gal-

veston for Dallas at the rate of 33 mi. an hour. How far will the trains be from Dallas when they meet?

3. Paris, Texas, is 584 mi. from St. Louis. A passenger train leaves Paris for St. Louis at 6.50 P.M. Three hours later a freight train leaves St. Louis for Paris. When and where will they meet, the rates being respectively 24 mi. and 16 mi. per hour?

4. A man walking at the rate of 4 mi. an hour is overtaken by a train 88 yd. long, and is passed in 10 sec. Find the rate of the train.

5. A train going at the rate of 40 mi. an hour passes in 6 sec. a man walking in the same direction at the rate of 4 mi. an hour. What is the length of the train?

6. Two trains start from the same station and travel in the same direction. The first train leaves at 7 A.M., and the second train at 9 A.M. How many miles from the station will the second train overtake the first if the rate of the first train is 30 mi. per hour and the rate of the second train is 45 mi. per hour?

REVIEW

EXERCISE 95

1. Multiply the sum of $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, by $1\frac{2}{13}$, and divide the product by $\dfrac{2\frac{1}{2}}{3\frac{3}{4}}$.

2. What number taken from $20\frac{1}{2}$ leaves as a remainder $4\frac{2}{3}$?

3. The difference between two numbers is $1\frac{1}{2}$, and the less is $4\frac{5}{6}$. Find the other number.

4. Which is the greater, $\frac{3}{4}$ of $4\frac{4}{9}$, or $\frac{2}{3}$ of $4\frac{3}{4}$?

5. From $\frac{7}{8}$ of the sum of $\frac{1}{2}$, $\frac{1}{5}$, $\frac{1}{8}$, $\frac{3}{40}$, take the sum of $\frac{7}{16}$, $\frac{7}{40}$, $\frac{7}{64}$.

6. From $\frac{3}{4}$ of $(\frac{1}{3} - \frac{1}{6} + \frac{8}{9})$ take $(\frac{1}{4} + \frac{2}{3} - \frac{1}{8})$.

7. Express in pounds the difference between .0125 of a ton and $\frac{3}{4}$ cwt.

8. What fraction having 48 for denominator is equivalent to .1875?

9. Find in feet the value of $\frac{15}{16}$ of a mile.

10. Express $\frac{2}{3}$ yd. as the decimal of a rod.

11. Express $\frac{3}{5}$ of 95 lb. as the decimal of $1\frac{1}{4}$ cwt.

12. Express $27\frac{3}{4}$ lb. as the decimal of a ton.

13. Take $\frac{4}{5}$ T. from $1\frac{1}{2}$ T., and express your result in pounds.

14. From $\frac{7}{8}$ of a right angle take 33° 45′.

15. From $\frac{1}{5}$ of a circumference take $\frac{1}{6}$ of the circumference and express your answer in degrees.

16. $\frac{2}{5}$ of a bushel is what part of $1\frac{3}{5}$ bu.?

17. An estate is left to A, B, and C. A gets $\frac{1}{2}$ of the estate, B $\frac{1}{5}$ of the estate, and C the remainder. What part of the estate does C get? If C's share is $450, what is the value of the estate? Find A's share and B's share.

18. A man spends $\frac{2}{5}$ of his salary on board, $\frac{1}{3}$ on clothing, $\frac{1}{5}$ on rent. He saves the remainder, amounting to $125. Find his salary.

19. Subtract $\frac{1}{8}$ from 2.1, and divide the remainder by .25.

20. The dividend is 3.562 and the quotient is .3125. Find the divisor.

21. Express $5.24 as a decimal of $100.

22. After giving away $\frac{1}{9}$, $\frac{1}{12}$, and $\frac{17}{36}$ of his money, a man has left \$392.95. How much money had he at first?

23. The third part of a number exceeds the fifth part of the same number by 15. What is the number?

24. The sixth part and the eighth part of a number together make $66\frac{1}{2}$. What is the number?

25. If $\frac{1}{3}$ and $\frac{2}{5}$ of a farm are together worth \$1650, what is $\frac{1}{4}$ of the remainder worth?

26. Subtract the product of $\frac{1}{2}$ and $\frac{2}{9}$ from their sum.

27. What number divided by $3\frac{3}{5}$ gives $1\frac{1}{12}$ for the quotient?

28. Find the value of $\frac{7}{9}$ of $1\frac{3}{14}$ of \$19.80.

29. A man owns $\frac{2}{3}$ of a boat and sells $\frac{3}{4}$ of his share for \$750. At this rate, find the value of the boat.

30. If 6 men do a piece of work in 9 days, how long will it take 4 men to do the same work?

31. If 15 men pave a street in 16 days, how long will it take 40 men to pave the same street?

32. If 18 men remove an embankment in 12 da., how long will it take 24 men to remove the embankment?

33. Two trains start at the same time from two stations 840 mi. apart, and travel toward each other, one train going at the rate of 35 mi. an hour, and the other of 25 mi. an hour. In how many hours will they meet?

34. If a man performs $\frac{3}{7}$ of a piece of work in 15 da., in how many days more will he complete the work?

35. Add $\frac{1}{2}$, $\frac{1}{4}$, $\frac{3}{8}$, $\frac{5}{16}$, and $\frac{7}{20}$. Express the sum as a decimal. Check by reducing to decimals and adding.

36. What is the smallest number which, added to the sum of $\frac{1}{3}$, $\frac{1}{5}$, and $\frac{5}{9}$, will make the final result an integer?

37. If $\frac{3}{5}$ of a barrel of sugar is sold and afterward 40 lb. are sold, how many pounds of sugar were in the barrel originally, supposing it still contains 90 lb.?

38. By selling a piano at $\frac{7}{10}$ of its cost, a dealer loses \$98. Find the cost of the piano.

39. If 45 sq. rd. of land cost \$18, find the cost of 1 A. Find also the cost of $3\frac{3}{4}$ A.

40. If $\frac{3}{4}$ of a clerk's salary per year is \$675, find his salary per month. If his expenses average \$48.89 per month, how much will he save per year? How long will it take him to save \$652.75?

41. If $3\frac{1}{3}$ A. of land are worth \$119, find the value of $2\frac{1}{2}$ A. How much is a rectangular strip of this land $\frac{3}{4}$ of a mile long and 33 feet wide worth?

42. How much is a plot of ground 80 ft. by 60 ft. worth, if an acre is worth \$55? If an acre is worth \$121?

43. Find the weight of a piece of coal in the shape of a rectangular solid, $1\frac{1}{2}$ ft. by $1\frac{1}{4}$ ft. by 10 in. A cubic foot of coal weighs $81\frac{1}{4}$ lb.

44. A man's property is assessed at \$4550. If he pays 40 ¢ on every \$100 for school tax, how many dollars school tax does he pay?

45. How much taxes will be paid on real estate worth \$9580, if the tax is at the rate of \$1.27 on \$100?

46. A man invests \$4800 and gains \$540. How much does he gain on every dollar invested? How much does he gain on every \$100?

47. If an investment of \$9600 produces a gain of \$1056, find the gain on \$1; also on \$100.

48. When \$8400 produces a profit of \$1092, how much does \$1 produce? \$100?

49. The school tax in a city is 2 mills on the dollar. The assessed valuation of the property is \$17,294,000. Find the total tax levied for school purposes. If .95 of this total is collectible, find the amount collected.

50. Find the tax on \$33,254,000 at 4 mills on the dollar.

51. A man insures his dwelling for \$5450. If he pays \$11.50 on every \$1000, how much does he pay all together?

52. Find the cost of insuring a house valued at \$7840, if the rate of insurance is \$1½ on every \$100?

53. If \$47.50 is paid to insure a boat valued at \$9500, how much is paid on \$1? on \$100?

54. The distance from New York City to Plymouth, England, is 2962 knots. The steamship *Deutschland* sailed from Plymouth to New York in July, 1900, in 5 da. 15 hr. 45 min. Find, in knots,* its rate per hour. Find also its rate in miles per hour.

55. In September, 1910, the steamship *Mauretania* made the voyage from Queenstown to New York in 4 da. 10 hr. and 40 min., the distance being 2780 knots. Find in knots its rate per hour. Express the rate also in miles per hour.

56. The steamer *Kronprinz Wilhelm* made, in September, 1902, a voyage from Cherbourg to New York in 5 da. 11 hr. 57 min. Find its rate per hour, the distance from Cherbourg to New York being 3184 knots.

57. In May, 1900, a passenger train ran from Burlington to Chicago, 205.8 mi., in 3 hr. 8 min. 30 sec. Find its rate per hour.

* **Knot,** a nautical mile, 6080.27 ft.

58. The fastest time on record by a passenger train for a distance over 450 miles was made in October, 1895, on the Lake Shore and Michigan Southern Railroad, from Chicago to Buffalo, a distance of 510 mi., in 8 hr. 1 min. Find its rate per hour.

59. The run from London to Edinburgh, $393\frac{1}{2}$ mi., has been made in 7 hr. 45 min. Find the speed per hour.

60. The market quotations, Feb. 19, 1903, were: wheat, $78\frac{3}{4}$¢ per bushel; corn, $45\frac{5}{8}$¢ per bushel; oats, 34¢ per bushel. Find the price of 100 lb. of each of these commodities.

61. Market quotations of live stock sales are in dollars per 100 lb. Find the cost of:

(*a*) 44 cattle, average weight 1121 lb., @ $4.80.
(*b*) 132 cattle, average weight 1018 lb., @ $4.80.
(*c*) 24 cattle, average weight 915 lb., @ $4.20.
(*d*) 23 cattle, average weight 1060 lb., @ $3.90.
(*e*) 133 heifers, average weight 862 lb., @ $4.00.
(*f*) 69 calves, average weight 201 lb., @ $5.00.
(*g*) 82 hogs, average weight 188 lb., @ $6.20.

CHAPTER III

GENERAL REVIEW BY TOPICS

ADDITION

ADDITION is the process of combining two or more numbers to form a single equivalent number. The result is the **sum.**

Only numbers of the same kind can be added. Thus, 5 yd. and 7 yd. may be added; but 5 yd. and $7 cannot be added.

5 ft. and 7 in. may be added, provided the 5 ft. is changed to inches, or the 7 in. changed to feet. The sum in one case is 67 inches, in the other case, $5\frac{7}{12}$ feet.

Example **1.** Add 279, 514, 928, 763.

$$\begin{array}{r} 279 \\ 514 \\ 928 \\ 763 \\ \hline 2484 \end{array}$$

The process is 3, 11, 15, 24; 24 units = 2 tens and four units.

Write the 4 units and carry the 2 tens.

2, 8, 10, 11, 18; 18 tens = 1 hundred and 8 tens.

Write 8 and carry 1.

1, 8, 17, 22, 24; write 24.

Example **2.** Add $2.79, $5.14, $9.28, $7.63.

$$\begin{array}{r} \$2.79 \\ 5.14 \\ 9.28 \\ 7.63 \\ \hline \$24.84 \end{array}$$

The process is the same as in Example 1.

Example 3. Add 6 ft. 8 in., 3 ft. 6 in., 5 ft. 4 in.

FT.	IN.
6	8
3	6
5	4
15	6

The process is 4, 10, 18; 18 in. = 1 ft. 6 in. Write 6 in., carry 1 ft.
1, 6, 9, 15. Write 15 ft.

Example 4. Add $6\frac{2}{3}$, $3\frac{1}{2}$, $5\frac{1}{3}$.

$$\begin{array}{r} 6\frac{2}{3} \\ 3\frac{1}{2} \\ 5\frac{1}{3} \\ \hline 15\frac{1}{2} \end{array}$$

Change $\frac{2}{3}$, $\frac{1}{2}$, and $\frac{1}{3}$ to equivalent fractions having 12 for denominator.

$$\frac{2}{3}+\frac{1}{2}+\frac{1}{3}=\frac{8+6+4}{12}=\frac{18}{12}=1\frac{1}{2}.$$

Write $\frac{1}{2}$, carry 1. 1, 6, 9, 15. Write 15.

Observe the same principle pervades the four examples. Units of the same name are placed in the same column, and the columns added separately.

An example in addition may be *checked* by *adding the columns in reverse order*. This is not an absolute test.

EXERCISE 96

Add:

(1)	(2)	(3)	(4)	(5)
4792799	7998992	5998476	5879824	6876894
8399384	8409499	9208503	9473473	9219479
7207383	7294792	8392393	7777888	7328337
9476583	8514798	6555773	6456789	8474494
4728737	9998777	7918924	9875874	9318327
9219777	6666784	5729998	8294295	6438444
6444673	8542728	6329888	7318392	7473478
8299299	5293294	7774673	9299497	9888777
9218288	4445454	6428427	6713729	6666679
7666729	7778898	9444779	9873876	9218289

6. Add vertically and horizontally, and finally add the vertical and the horizontal totals:

392	965	873	599	222
876	329	888	307	399
543	707	393	448	937
878	538	427	388	542
929	928	925	394	234

7. The mileage of railroads in operation in the several states is given below for the years 1903, 1904, and 1905. Find the total mileage for each group of states.

Group and State	1903	1904	1905
New England			
Maine	2,004.79	2,029.89	2,091.12
New Hampshire	1,191.42	1,191.77	1,191.77
Vermont	1,057.84	1,056.96	1,063.20
Massachusetts	2,117.41	2,110.81	2,101.87
Rhode Island	209.84	209.84	209.84
Connecticut	1,025.90	1,020.12	1,020.12
Total			
Middle Atlantic			
New York	8,180.85	8,167.21	8,212.12
New Jersey	2,242.56	2,266.64	2,269.61
Pennsylvania	10,784.54	10,991.97	11,161.45
Delaware	333.63	334.86	333.60
Maryland	1,368.98	1,364.45	1,406.81
District of Columbia . . .	24.70	24.70	24.70
Total			
Central Northern			
Ohio	9,023.61	9,163.97	9,243.26
Michigan	8,459.65	8,467.76	8,521.46
Indiana	6,834.75	6,863.03	7,046.90
Illinois	11,502.38	11,742.10	11,959.09
Wisconsin	6,921.40	7,014.79	7,188.18
Total			

Group and State	1903	1904	1905
South Atlantic			
Virginia	3,833.09	3,823.67	3,862.11
West Virginia	2,565.49	2,820.82	2,966.05
North Carolina	3,790.73	3,913.86	4,015.58
South Carolina	3,112.48	3,146.24	3,184.19
Georgia	6,109.21	6,298.97	6,516.61
Florida	3,469.92	3,585.83	3,635.38
Total			
Gulf and Mississippi Valley			
Alabama	4,442.69	4,590.89	4,758.57
Mississippi	3,156.56	3,367.23	3,541.04
Tennessee	3,355.19	3,484.92	3,606.88
Kentucky	3,193.31	3,261.56	3,355.07
Louisiana	3,419.38	3,592.68	3,764.17
Total			
Southwestern			
Missouri	7,316.62	7,797.18	7,859.57
Arkansas	3,651.28	3,946.54	4,165.72
Texas	11,308.05	11,614.13	11,949.02
Kansas	8,810.50	8,841.09	8,874.58
Colorado	4,852.44	4,989.85	5,093.20
New Mexico	2,450.02	2,441.93	2,596.64
Indian Country	2,320.02	2,585.69	2,686.47
Oklahoma	2,359,52	2,635.64	2,836.19
Total			

Add:

8. $2\frac{1}{2}$, $3\frac{3}{4}$, $5\frac{5}{6}$.

9. $2\frac{3}{10}$, $3\frac{3}{5}$, $5\frac{1}{2}$.

10. $3\frac{1}{4}$, $2\frac{2}{3}$, $5\frac{7}{12}$.

11. $5\frac{5}{8}$, $9\frac{7}{12}$, $2\frac{1}{2}$.

12. $1\frac{3}{4}$, $5\frac{1}{2}$, $2\frac{1}{6}$.

13. $7\frac{2}{5}$, $6\frac{7}{10}$, $9\frac{7}{15}$.

14. $9\frac{9}{16}$, $5\frac{7}{8}$, $7\frac{1}{4}$.

15. $12\frac{2}{9}$, $11\frac{2}{3}$, $6\frac{1}{2}$.

16. $2^\circ\ 17'\ 50''$, $7^\circ\ 24'\ 30''$, $9^\circ\ 27'\ 37''$, $128^\circ\ 14'\ 43''$.

17. 2 ft. 9 in., 7 ft. 3 in., 9 ft. 11 in., 15 ft. 7 in.

18. 8 qt. 1 pt., 9 qt. 1 pt., 15 qt., 12 qt. 1 pt.

19. 4 gal. 2 qt., 7 gal. 3 qt., 9 gal. 1 qt., 8 gal. 3 qt.

20. 5 pk. 7 qt., 9 pk. 3 qt., 12 pk. 5 qt., 13 pk. 4 qt.

21. 3 bu. 3 pk., 9 bu. 2 pk., 7 bu. 1 pk., 4 bu. 3 pk.

22. 12 hr. 15 min., 15 hr. 8 min., 17 hr. 42 min., 5 hr. 13 min.

23. 5 da. 12 hr., 18 da. 17 hr., 13 da. 18 hr., 5 da. 3 hr.

24. 15 yd. 2 ft., 26 yd. 1 ft., 32 yd. 2 ft., 9 yd. 1 ft.

25. How many times does a clock strike in 24 hours?

26. I purchased goods amounting to $ 3.92, $ 5.58, $ 6.57, $ 6.30. In payment I gave a $ 50 bill. How much change did I receive?

27. The dimensions of a table are 8 ft. 3 in. and 3 ft. 7 in. How many feet in its perimeter?

28. The Galveston Sea Wall was constructed by Galveston County and the United States Government; the former built 3.5 miles, and the latter .87 mile. Find the total length of the sea wall.

In its construction there were used 1150 carloads of cement, 6100 carloads of crushed rock, 1400 carloads of round piling, 475 carloads of sheet piling, 4300 carloads of riprap, and 6 carloads of reënforcing rods. How many carloads of material were used in its construction?

How many miles would the cars extend if placed end to end, allowing 39.6 ft. to a car?

29. The locomotives operating between Clarion Junction and Freeman, Ohio, weigh 268,000 lb. each. Express this weight in tons.

SUBTRACTION

Subtraction is the inverse of addition.

To subtract 7 from 16 is to find a number which added to 7 will make 16.

Example 1. Subtract 63 from 92.

92
63
29

PROCESS. 3 and 9 are 12; write 9, carry 1. 1 and 6 are 7, 7 and 2 are 9; write 2.

Example 2. Subtract 6.3 from 9.2.

9.2
6.3
2.9

The process is the same as in Example 1.

Example 3. Subtract 6 hr. 3 min. from 9 hr. 2 min.

HR.	MIN.
9	2
6	3
2	59

PROCESS. 3 min. and 59 min. make 1 hr. and 2 min.; write 59 min., carry 1 hr. 1 hr. and 6 hr. are 7 hr. 7 hr. and 2 hr. are 9 hr.; write 2 hr.

Example 4. Subtract $6\frac{3}{5}$ from $9\frac{2}{5}$.

$9\frac{2}{5}$
$6\frac{3}{5}$
$2\frac{4}{5}$

PROCESS. $\frac{3}{5}$ and $\frac{4}{5}$ are $1\frac{2}{5}$; write $\frac{4}{5}$, carry 1. 1 and 6 are 7, 7 and 2 are 9; write 2.

Example 5. From 75,218 take the sum of 4799, 3928, 9476, 8873.

75218
4799
3928
9476
8873
48142

PROCESS. 3, 9, 17, 26; 26 and 2 are 28. Write 2, carry 2. 2, 9, 16, 18, 27; 27 and 4 are 31. Write 4, carry 3. 3, 11, 15, 24, 31; 31 and 1 are 32. Write 1, carry 3. 3, 11, 20, 23, 27; 27 and 8 are 35. Write 8, carry 3. 3 and 4 are 7. Write 4. The remainder is 48,142.

This example shows the practical value of this method of subtraction. (Austrian Method.)

EXERCISE 97

1. The values of manufactured articles exported from the United States for the years ending June 30, 1897 and June 30, 1907 are given below. Find the increase in value of the exports of each article, or group of articles. Check your work by finding the difference between the total value of these exports for each of the two years, and comparing this difference with the sum of the separate differences.

ARTICLE	1897	1907
Iron and steel, manufactures of	$57,497,872	$181,530,871
Copper, manufactures of	31,621,125	88,791,225
Wood, manufactures of	35,679,964	79,704,395
Oils — mineral, refined	56,463,185	78,228,819
Leather and manufactures of	19,161,446	45,476,960
Cotton, manufactures of	21,037,678	32,305,412
Agricultural implements	5,240,686	26,936,456
Naval Stores	9,214,958	21,686,752
Carriages, cars, and other vehicles	9,952,033	20,513,407
Chemicals, drugs, dyes, and medicines	8,792,545	18,220,630
Instruments and apparatus	3,054,453	14,661,455
Paper and manufactures of	3,333,163	9,856,733
Paraffin and paraffin wax	4,957,096	9,030,992
Fibers, manufactures of	2,216,184	3,308,112
India rubber, manufactures of	1,926,585	7,428,714
Furs and skins	3,284,349	7,139,221
Books, maps, engravings, etc.	5,647,548	5,813,107
Tobacco, manufactures of	5,025,817	5,735,613
Brass and manufactures of	1,171,431	4,580,455
Gunpowder and other explosives	1,555,318	4,082,402
Paints, pigments, and colors	944,536	3,391,988
Soap	1,136,880	3,806,097
Musical instruments	1,276,717	3,252,063
Nickel and manufactures of	726,789	3,218,862
Clocks, watches	1,770,402	3,160,272
Coke	547,046	3,013,088
Glass and glassware	1,208,187	2,604,717
All other articles	19,799,642	47,295,739

Find the difference between :

2. 200 and .02.
3. 400 and 1.37.
4. $75 and 73¢.
5. $700 and $2.84.
6. $100 and $1.75.
7. $1000 and 5¢.
8. $324.80 and $100.99.
9. $70.73 and $19.94.
10. $403.05 and $92.89.
11. $60.52 and $23.87.
12. 100 and .01.
13. 6.29 and 2.9924.
14. 5.001 and 4.0073.
15. 7.2 and 2.77.
16. 11 and 1.5.
17. 17.3 and 11.9.

18. The product of 6.715×6.715 and the product of $.285 \times .285$.

19. $7\frac{2}{3}$ and $4\frac{11}{12}$.
20. $18\frac{5}{9}$ and $7\frac{5}{6}$.
21. $9\frac{5}{6}$ and $4\frac{3}{4}$.
22. $21\frac{2}{3}$ and $11\frac{13}{18}$.
23. $7\frac{3}{11}$ and $2\frac{1}{2}$.
24. $8\frac{3}{16}$ and $5\frac{9}{10}$.
25. $9\frac{5}{12}$ and $3\frac{7}{8}$.
26. $6\frac{9}{16}$ and $3\frac{2}{3}$.
27. $10\frac{3}{11}$ and $7\frac{1}{4}$.
28. $19\frac{2}{3}$ and $8\frac{5}{8}$.
29. $12\frac{7}{15}$ and $9\frac{9}{10}$.
30. $23\frac{11}{14}$ and $12\frac{16}{21}$.

31. 5 ft. 7 in. and 4 ft. 9 in.
32. 17 ft. 3 in. and 12 ft. 8 in.
33. 19 ft. 1 in. and 9 ft. 4 in.
34. 27 ft. 3 in. and 18 ft. 4 in.
35. 9 lb. 2 oz. and 4 lb. 7 oz.
36. 17 lb. 6 oz. and 5 lb. 11 oz.
37. 33 lb. 2 oz. and 18 lb. 8 oz.
38. 12 hr. 10 min. and 9 hr. 24 min.
39. 90° and 34° 14′ 15″.
40. 180° and 115° 4′ 50″.
41. 180° and the sum of 56° 16′, and 92° 18′.

42. 15 pk. 3 qt. and 3 pk. 7 qt.

43. 23 pk. 5 qt. and 13 pk. 6 qt.

44. From 40,000 take the sum of 3211, 4711, 5283, 9438.

45. From 50,580 take the sum of 19,311, 12,218, 1273, 5559.

46. From 18,900 take the sum of 3419, 3428, 4584, 2293.

47. A man owns two houses worth respectively $2390 and $4575; he has deposited in the bank $3280; he owes two notes for $783 and $870. How much is he worth?

48. The area of the British Isles is 120,975 square miles; the area of Texas is 265,780 square miles. By how many square miles does the area of Texas exceed the area of the British Isles?

*** 49.** The population of the Chinese Empire is 433,553,000; of the British Empire, 363,900,000; of the Russian Empire, 141,000,000; of the United States, exclusive of colonial possessions, 84,150,000; of Germany, 60,478,000. How many more people in the United States than in Germany? In the British Empire than in Russia, United States, and Germany combined? By how many does the population of China exceed the population of Russia, United States, and Germany together?

50. The areas of Maine, New Hampshire, Vermont, Massachusetts, Rhode Island, and Connecticut in square miles are respectively: 33,040, 9305, 9565, 8315, 1250, 4990. The area of California is 158,360 square miles. By how many square miles does the area of California exceed the area of the six New England states?

* These figures are for 1906.

51. In going from Galveston to Chicago by rail, a distance of 1410 miles, a man travels the first day 345 miles; the next day, 201 miles; the third day, 290 miles. How far is he from Chicago at the end of the third day?

MULTIPLICATION

If one factor of the product is multiplied by a number, and the other factor divided by the same number, the product will be unchanged.

Thus, $84 \times 20 = 1680.$

$420 \times 4 = 1680.$ Here 84 is multiplied by 5, and 20 is divided by 5.

Example 1. Multiply 3782 by 234.

$$\begin{array}{rl} 3782 & \\ \underline{234} & \\ 15128 & = 4 \times 3782 \\ 11346 & = 30 \times 3782 \\ \underline{7564} & = \underline{200 \times 3782} \\ 884988 & = 234 \times 3782 \end{array} \qquad \text{or} \qquad \begin{array}{rl} 3782 & \\ \underline{234} & \\ 7564 & = 200 \times 3782 \\ 11346 & = 30 \times 3782 \\ \underline{15128} & = \underline{4 \times 3782} \\ 884988 & = 234 \times 3782 \end{array}$$

To multiply integers, write the multiplier under the multiplicand, then multiply the multiplicand by each digit of the multiplier, placing the first figure of each partial product directly under the digit of the multiplier producing it, and add the partial products.

Example 2. Multiply 43,800 by 23.

$$\begin{array}{r} 43800 \\ \underline{23} \\ 131400 \\ \underline{876} \\ 1007400 \end{array}$$

Multiply the digits and place the result properly. Bring down the zeros into the first partial product and the result.

Example 3. Multiply 5364 by 2700.

$$\begin{array}{r} 5364 \\ 2700 \\ \hline 3754800 \\ 10728 \\ \hline 14482800 \end{array}$$

Multiply the digits and bring down the zeros into the first partial product and the result.

When either the multiplicand or the multiplier ends in zero, multiply the digits and bring down the zeros.

Example 4. Multiply 17.32 by .47.

$$\begin{array}{r} 17.32 \\ .47 \\ \hline 12\ 124 \\ 69\ 28 \\ \hline 8.1404 \end{array}$$

PROCESS. The numbers are multiplied as if both were integers; then beginning at the right of the product four places are pointed off, that is the number of decimal places in multiplicand and multiplier combined. This may be readily seen by multiplying the multiplier by 100 and dividing the multiplicand by 100.

Thus, $17.32 \times .47 = \frac{17.32}{100} \times .47 \times 100 = .1732 \times 47 = 8.1404.$

Example 5. Multiply $4\frac{7}{8}$ by $2\frac{2}{3}$.

$$4\tfrac{7}{8} = \tfrac{39}{8}$$

$$2\tfrac{2}{3} = \tfrac{8}{3}$$

Therefore, $4\frac{7}{8} \times 2\frac{2}{3} = \frac{39}{8} \times \frac{8}{3} = \frac{\overset{13}{\cancel{39}} \times \cancel{8}}{\cancel{8} \times \cancel{3}} = 13.$

Example 6. Multiply 5 gal. 2 qt. 1 pt. by 9.

GAL.	QT.	PT.
5	2	1
		9
50	2	1

PROCESS. 9 times 1 pt. = 4 qt. 1 pt.; write 1 pt., carry 4 qt. 9 times 2 qt. = 18 qt. 18 qt. + 4 qt. = 22 qt. = 5 gal. 2 qt.; write 2 qt., carry 5 gal. 9 times 5 gal = 45 gal. 45 gal. + 5 gal. = 50 gal.

PARTICULAR SHORT METHODS OF MULTIPLICATION

$5 = \frac{1}{2}$ of 10	$75 = 100 - \frac{1}{4}$ of 100
$25 = \frac{1}{4}$ of 100	$875 = 1000 - \frac{1}{8}$ of 1000
$125 = \frac{1}{8}$ of 1000	$99 = 100 - 1$
$.16\frac{2}{3} = \frac{1}{6}$	$97 = 100 - 3$

Example 1. Multiply 97.3 by 125.

As $125 = \frac{1}{8}$ of 1000 we may multiply 97.3 by 1000 and divide by 8.

Thus, $125 \times 97.3 = \dfrac{1000}{8} \times 97.3 = \dfrac{97300}{8} = 12162.5.$

Example 2. Multiply 29.374 by 993.

As $993 = 1000 - 7$, we may multiply 29,374 by 1000 and subtract 7 times 29.374 which will give the same result as multiplying by 993.

Thus,
$$\begin{array}{rcr} 29.374 \times 1000 &=& 29374. \\ 29.374 \times \quad 7 &=& 205.618 \\ \hline 29.374 \times \ 993 &=& 29168.382 \end{array}$$

EXERCISE 98

1. Multiply each of the following numbers by 10 :
   234, 350.2, 25.07, .127, .0788, 1.003.

2. Multiply the following numbers by 100 :
   505, 67.5, 27.28, 5.347, .07954, .00392.

3. Multiply the following numbers by 1000 :
   728, 96.4, 12.87, 1.732, .0139, .00782.

4. Multiply the following numbers by 10,000 :
   318, 25.4, 19.96, 18.832, 27.796, .012.

5. **Find** in the shortest possible way the following **products**:

(*a*) 2780×99; 9218×998; 7215×999.

(*b*) 2.79×25; 3.18×125; 243×875.

(*c*) $78 \times .16\frac{2}{3}$; $90 \times .33\frac{1}{3}$; 297×9998.

6. Multiply 5280 by 5280; 1020 by 1020.

7. Multiply 7309 by 256; 9417 by 735.

8. The estimated production and value of the following cereal crops as given in the Annual Report of the Department of Agriculture for the year 1906 are as follows:

Cereals	Yield per Acre	Value per Bushel
	Bushels	Cents
Corn	30.3	39.9
Wheat	15.5	66.7
Oats	31.2	31.7
Rye	16.7	58.9
Barley	28.3	41.5
Buckwheat	18.6	59.6

Find the value of the yield per acre of each of these cereal crops.

9. The number of bales of cotton produced in Texas in the season 1904–05 was 2,598,949, and in 1903–04, 3,214,133. Allowing 500 lb. to a bale, how many more pounds of cotton were produced in the latter year than in the former.

10. The estimated production and value per ton of the hay crop for the year 1906 are given on page 193.

Find the value of the yield per acre in each of the states.

State	Yield per Acre	Price per Ton
	Tons	
New Hampshire	1.15	$12.50
Massachusetts	1.31	17.00
Connecticut	1.17	15.00
New York	1.28	12.10
New Jersey	1.32	15.95
Pennsylvania	1.30	13.40
Maryland	1.26	13.50
Virginia	1.25	15.50
South Carolina	1.46	15.25
Georgia	1.65	15.75
Alabama	1.95	13.30
Louisiana	1.93	11.50
Tennessee	1.51	13.45
Kentucky	1.35	13.25
Illinois	.98	12.50
Minnesota.	1.70	5.50
Kansas	1.28	6.25
Colorado	2.50	9.50
Utah	4.00	7.50
Idaho	2.95	8.00
California	1.85	11.25

11. A piece of coal taken from the mine at Coos Bay, Oregon, had the following composition by weight:

Moisture	= .1042
Combustible matter	= .4221
Fixed carbon	= .4318
Ash	= .0419

Find the amount of each in 87 tons of this coal; in 783 tons. Check your answers.

12. Find to the nearest cent the value of each of the following articles:

(*a*) $25\frac{1}{2}$ bu. corn @ $42\frac{1}{2}$¢ per bu.
(*b*) $12\frac{9}{10}$ bu. wheat @ $69\frac{1}{2}$¢ per bu.
(*c*) $28\frac{2}{5}$ bu. oats @ $25\frac{1}{2}$¢ per bu.
(*d*) $16\frac{3}{4}$ bu. rye @ $60\frac{1}{4}$¢ per bu.
(*e*) $20\frac{1}{2}$ bu. barley @ $47\frac{3}{4}$¢ per bu.
(*f*) $4\frac{3}{4}$ lb. wool @ 61¢ per lb.
(*g*) 497 lb. cotton @ $11\frac{3}{8}$¢ per lb.
(*h*) 512 lb. cotton @ $10\frac{7}{8}$¢ per lb.

13. The inside dimensions of the floor of a box car are 40 ft. $\frac{1}{8}$ in. and 8 ft. 6 in. Find the perimeter of the floor.

14. The inside dimensions of the floor of a refrigerator car are 28 ft. $9\frac{1}{2}$ in. and 8 ft. $1\frac{1}{4}$ in. Find its perimeter.

15. Multiply 5 yd. 2 ft. by 8; 9 ft. 8 in. by 7.

Find the product of:

16. 9 lb. 4 oz. by 5; 16 lb. 11 oz. by 9.

17. 3 hr. 20 min. 30 sec. by 6; 7 hr. 17 min. by 9.

18. 53° 12′ by 10; 68° 12′ 18″ by 5.

19. 4 pk. 7 qt. by 6; 7 pk. 3 qt. by 8.

20. 5 gal. 2 qt. by 9; 9 gal. 3 qt. by 12.

DIVISION

Division is the inverse of multiplication.

To divide 84 by 7 means to find a number which multiplied by 7 gives 84.

If the divisor and dividend are both multiplied by the same number, the quotient remains unchanged.

Thus,
$$96 \div 8 = 12.$$
$$(96 \times 6) \div (8 \times 6) = 12.$$

Example 1. Divide 2483 by 7.

$$7\overline{)2483}$$
$$354\tfrac{5}{7}$$

PROCESS. 7 is contained in 24 hundreds 3 hundred times, remainder 3 hundreds; 3 hundred = 30 tens. 30 tens and 8 tens = 38 tens. 7 is contained in 38 tens 5 tens times, remainder 3 tens; 3 tens = 30 units. 30 units and 3 units = 33 units. 7 is contained in 33 4 times, with a remainder of 5.

Example 2. Divide .437 by 1.92.

$$\begin{array}{r} .2276^{+} \\ 192\overline{)43.7} \\ 38\ 4 \\ \hline 5\ 30 \\ 3\ 84 \\ \hline 1\ 460 \\ 1\ 344 \\ \hline 1160 \end{array}$$

PROCESS. Move the decimal point two places to the right in divisor and dividend; this multiplies both by 100. Then write each quotient figure directly above the right-hand figure of the partial dividend which produces it. Write the decimal point in the quotient above the decimal point in the dividend.

Example 3. Divide $3\frac{2}{3}$ by $6\frac{1}{2}$.

$$3\tfrac{2}{3} \div 6\tfrac{1}{2} = \tfrac{11}{3} \div \tfrac{13}{2} = \tfrac{11}{3} \times \tfrac{2}{13} = \tfrac{22}{39}.$$

PROCESS. Change the mixed numbers to simple fractions, invert the divisor and multiply.

Example 4. To how many long tons are 3.30693 short tons equivalent?

$$\frac{3.30693 \times \overset{100}{\cancel{2000}}}{\underset{112}{\cancel{2240}}} = \frac{330.693}{112} = 2.9526.$$

To check an example in division, multiply the quotient by the divisor.

EXERCISE 99

1. The estimated acreage, production, and value of the potato crop by states for the year 1905 are as follows:

State	Acreage	Production	Farm Value
	Acres	Bushels	Dollars
New Hampshire	19,700	2,367,000	1,704,000
Rhode Island	6,490	811,200	722,000
Delaware	7,680	714,000	421,200
North Carolina	25,900	1,993,000	1,355,000
Florida	4,110	308,200	369,900
Mississippi	5,860	644,900	548,200
West Virginia	34,400	3,025,100	1,754,500
Michigan	242,000	16,203,000	9,073,700
Illinois	149,000	11,186,000	7,494,600
Missouri	86,100	7,059,000	3,882,614
North Dakota	25,400	2,415,000	917,800
Nevada	2,800	336,700	276,100

Find the number of bushels yielded per acre in each state, and the average price per bushel in cents.

Divide correct to four decimal places:

2. 128.016 by 420.
3. 2.3774 by 7.8.
4. 10.4987 by 3.2.
5. .77087 by .479.
6. .02734 by .044.
7. .035936 by .0888.
8. 1.57899 by .639.
9. 60.247 by 78.8.
10. 5.0748 by 3.88.

Divide:

11. 21 by $1\frac{3}{11}$.
12. 48 by $2\frac{2}{7}$.
13. 42 by $1\frac{5}{16}$.
14. 72 by $3\frac{3}{8}$.
15. $72\frac{1}{2}$ by $4\frac{1}{7}$.
16. $93\frac{3}{7}$ by $62\frac{2}{7}$.
17. $17\frac{3}{4}$ by $9\frac{31}{36}$.
18. $2\frac{2}{5}$ by $3\frac{9}{11}$.
19. $8\frac{1}{10}$ by $4\frac{8}{25}$.
20. $2\frac{11}{20}$ by $2\frac{32}{35}$.

Find the value of:

21. $\frac{2}{3} \times 1\frac{1}{4} \div \frac{3}{4} \div \frac{7}{8}$.

Solution. Arrange the work in the form of a continued product of fractions. Always use cancellation when possible.

$$\frac{2}{3} \times 1\tfrac{1}{4} \div \frac{3}{4} \div \frac{7}{8} = \frac{2}{3} \times \frac{5}{\cancel{4}} \times \frac{\cancel{4}}{3} \times \frac{8}{7} = \frac{80}{63} = 1\tfrac{17}{63}.$$

22. $\frac{3}{4} \times 1\frac{3}{5} \div 2\frac{2}{5} \times \frac{9}{10}$.

23. $\frac{4}{5} \times 1\frac{2}{3} \div 1\frac{1}{6} \div 3\frac{1}{2}$.

24. $\frac{7}{8} \times 2\frac{2}{3} \times 1\frac{1}{5} \div 1\frac{3}{4}$.

25. $1\frac{9}{10} \div 3\frac{1}{3} \div 3\frac{1}{6} \times \frac{1}{2}$.

26. $1\frac{2}{3} \div (\frac{7}{9} \div 4\frac{1}{2}) \times 4\frac{2}{3}$.

27. $\frac{2}{9}$ of $1\frac{2}{7}$ of $3\frac{1}{2} \div \frac{3}{8}$ of $1\frac{1}{4}$.

Hint. Observe that the divisor is $\frac{3}{8}$ of $1\frac{1}{4}$.

$$\frac{2}{9} \times 1\tfrac{2}{7} \times 3\tfrac{1}{2} \div \left(\frac{3}{8} \times 1\tfrac{1}{4}\right) = \frac{2}{\cancel{9}} \times \frac{\cancel{9}}{\cancel{7}} \times \frac{\cancel{7}}{\cancel{2}} \times \left(\frac{8}{3} \times \frac{4}{5}\right) = \frac{32}{15} = 2\tfrac{2}{15}.$$

28. $\frac{3}{4}$ of $1\frac{3}{5}$ of $2\frac{1}{2} \div \frac{2}{3}$ of $1\frac{2}{7}$.

29. $\frac{11}{12}$ of $1\frac{5}{7}$ of $\frac{7}{11} \div \frac{6}{13}$ of $\frac{2}{3}$.

30. $\frac{5}{6}$ of $4\frac{4}{5} \times \frac{9}{20} \div \frac{1}{2}$ of $\frac{7}{9}$.

31. $\frac{15}{16}$ of $3\frac{5}{9}$ of $\frac{2}{3} \div \frac{3}{4}$ of $\frac{5}{6}$.

Express in long tons:

32. 3.36 T., 6.6139 T., .00992 T., 4.4092 T.

Express in short tons:

33. 1.9684 long T., 6.8894 long T., .004921 long T.

Express in Troy pounds:

34. 5 lb. Avoirdupois, 13.228 lb. Avoirdupois, 6613.87 lb. Avoirdupois.

Express in Avoirdupois pounds:

35. 7 lb. Troy, 13.396 lb. Troy, 5.358 lb. Troy.

36. Express 1 ft. as a decimal of 1 mi.

37. Express 1 rd. as a decimal of 1 mi.

38. Express 1 sq. rd. as a decimal of 1 A.

39. Express 1 A. as a decimal of 1 sq. mi.

40. Express 1 sq. yd. as a decimal of 1 A.

41. Express 1 lb. as a decimal of 1 ton.

42. A lot is 40 by 120 feet. How many such lots make 40 acres?

43. How many barrels of $31\frac{1}{2}$ gallons each will a rectangular tank 12 ft. by 8 ft. and 5 ft. deep hold? (Allow $7\frac{1}{2}$ gal. to a cubic foot.)

44. At $12\frac{1}{2}$¢ per pound, how many pounds of coffee will $7500 buy?

45. Find the cost of boring an artesian well 1400 feet deep at $4 a linear foot for the first 900 feet, $4.50 per linear foot for the next 200 feet, $5 per linear foot for the next 100 feet, $5.50 per linear foot for the next 100 feet, and $6 per linear foot for the remainder.

46. The rails of the Great Western Railway, England, weigh $97\frac{1}{2}$ lb. per yard. Find in tons the weight of the rails required to construct 1 mile of this railway.

LONGITUDE AND TIME

In order to understand this subject it is well to have clearly in mind a picture of the earth as a ball rotating on its axis from west to east. The ends of the axis are called **poles**. Then imagine the sun as a fixed object at a distance from the earth but opposite a part of the earth about halfway between the poles. It is easy to see that as the earth rotates, any place on its surface reaches a position which is nearer the sun than at any other times, and passes on, being completely hidden from the sun during part of the

time. This movement of a place, from its position nearest the sun until it reaches the same position again, occupies one day. The time of day when a place is nearest the sun is noon.

Imagine a circle drawn around the earth through the poles and also through this place that we selected. Such a circle is called a **meridian circle.** Either half of a meridian circle, extending from one pole to the other, is called a **meridian.** Now we can see that all places on the same meridian will be nearest the sun at the same time, *i.e.* their noons will be the same; but the places east of this meridian will have noon before the places on the meridian, while places west of the meridian will not have noon until after places on the meridian. If all places set their clocks at 12 o'clock for their own noon, there will of course be the same difference of time throughout the day as there is in the time of their noon.

Time reckoned by each locality from its own noon is called **local time.**

Through every place there is a meridian.

Places situated on the same meridian have the same local time.

Imagine a circle around the earth halfway between the poles. This circle is called the equator.

The equator and the meridian circles are sometimes called great circles because each of them divides the globe into two equal parts.

Distances on the earth are often measured in degrees of these great circles.

The **latitude** of a place is its distance north or south of the equator measured in degrees of the meridian circle through the place.

The **longitude** of a place is the number of degrees on the

equator included between the meridian passing through the given place and some meridian accepted as a standard from which to measure. This standard meridian is called the **prime** meridian or **principal** meridian.

The meridian through any particular place may be used as the **prime** meridian. The meridians through the observatories of Greenwich, Washington, Paris, Madrid, Rome, Stockholm, Pulkova, and Lisbon have been used as prime meridians by the nations to which these cities belong. The International Geodetic Congress, which met at Washington in 1884, recommended that the meridian passing through the observatory at Greenwich, a suburb of London, be the prime meridian. This recommendation is now generally adopted by the great nations of the world. The meridian of Greenwich is taken as prime meridian in this book.

The earth rotates on its axis from west to east. Consider two places not on the same meridian; for example, New York City and St. Louis. New York being farther east will come sooner under the influence of the sun's rays. Therefore, when it is noon in New York City it is before noon in St. Louis. Since the earth's motion is uniform, and furthermore, since

in 24 hr. the earth rotates $360°$;
$\therefore$ in 1 hr. the earth rotates $15°$;
$\therefore$ in 1 min. the earth rotates $15'$;
$\therefore$ in 1 sec. the earth rotates $15''$.

A difference of $15°$ of longitude corresponds to a difference of 1 hr. of time. A difference of $15'$ of longitude corresponds to 1 min. of time. A difference of $15''$ of longitude corresponds to 1 sec. of time.

Hence, **to convert difference of longitude into difference of time, divide by 15.**

EXERCISE 100

1. When it is noon at London, the longitude of London being taken as 0°, what is the time at New Orleans, 90° W.?

2. When it is 9 o'clock A.M. on the meridian 75° W., what is the time on the meridian 90° W.?

3. The longitude of Denver is 105° W. When it is 3 o'clock P.M. in Denver, what is the time in London?

4. Two places differ in longitude by 20°. What is their difference in time?

5. A person travels east 15°. What change must he make in the time indicated by his watch so that it may indicate local time? Supposing he goes the same distance west, what change must be made in the time indicated by his watch?

6. When it is noon in London, what is the longitude of the places in which it is 4 o'clock P.M.? 5 o'clock A.M.?

7. When it is 2 o'clock P.M. in Washington, what is the time in places 30° W. of Washington? in places 75° E. of Washington?

8. What is the difference in longitude between places which differ in time by 2 hr. 30 min.? by 4 hr. 10 min.?

9. The longitude of New York is about 75° W. If a person travels from Denver to New York, having his watch set according to Denver local time, will his watch be fast or slow when he reaches New York, and how much?

10. To how many hours does a difference of 80° in longitude correspond?

11. What difference in longitude corresponds to a difference of 4 hr. 20 min. in time?

12. A person living on the 90th meridian W. wishes to send a telegram to a bank in New York City, directing the bank to pay on the same day a sum of money. Up to what hour in the afternoon may he do this, allowing 30 minutes for the transmission of the telegram? (New York banks close at 3 P.M.)

13. To what difference in time does a difference of 1° of longitude correspond? Suppose two places situated so that one is directly east of the other at a distance of 1° of longitude. At which place will the sun rise earlier? How much earlier?

LONGITUDES OF CITIES REFERRED TO IN THIS CHAPTER

Austin,	97° 44′ W.	Galveston,	94° 47′ W.
Baltimore,	76° 37′ W.	Havana,	82° 21′ 30″ W.
Bangor,	68° 47′ W.	Honolulu,	157° 52′ W.
Bismarck,	100° 47′ W.	Louisville,	85° 46′ W.
Boston,	71° 3′ 50″ W.	Melbourne,	144° 58′ 32″ E.
Brisbane,	153° 2′ E.	Manila,	120° 58′ 3″ E.
Buenos Ayres,	58° 22′ 14″ W.	Mexico City,	99° 6′ 39″ W.
Charleston,	79° 52′ 58″ W.	Montreal,	73° 33′ 4″ W.
Chicago,	87° 40′ W.	New Orleans,	90° 3′ 28″ W.
Cincinnati,	84° 24′ W.	New York,	74° 0′ 24″ W.
Constantinople,	29° 0′ 50″ E.	Norfolk,	76° 17′ 22″ W.
Detroit,	83° 3′ W.	Paris,	2° 20′ 15″ E.
Dublin,	6° 20′ 30″ W.	Pekin,	116° 29′ E.
Pensacola,	87° 16′ 6″ W.	St. Petersburg,	30° 19′ 40″ E.
Philadelphia,	75° 9′ 3″ W.	San Francisco,	122° 24′ 32″ W.
Portland,	122° 40′ W.	Savannah,	81° 5′ 25″ W.
Providence,	71° 24′ 20″ W.	Tientsin,	117° 11′ 44″ E.
Rome,	12° 28′ 40″ E.	Tokyo,	139° 44′ 30″ E.
St. Louis,	90° 16′ W.	Washington,	77° 0′ 36″ W.

Example 1. Find the difference between the longitudes of Austin and Honolulu.

SOLUTION. Honolulu, $157^\circ\ 52'$ W.
Austin, $97^\circ\ 44'$ W.
$60^\circ\ 8'$

$\therefore$ Honolulu is $60^\circ\ 8'$ farther west than Austin.

Example 2. Find the difference between the longitudes of Galveston and Constantinople.

SOLUTION. Galveston, $94^\circ\ 47'$ W.
Constantinople, $29^\circ\ 0'\ 50''$ E.

Here, the places are on opposite sides of the prime meridian. By going east from Galveston $94^\circ\ 47'$, one arrives at the prime meridian, and by going $29^\circ\ 0'\ 50''$ still farther east, he arrives at the meridian of Constantinople. Hence, the difference between the longitudes is $(94^\circ\ 47' + 29^\circ\ 0'\ 50'') = 123^\circ\ 47'\ 50''$.

To find the difference in the longitudes of two places: (1) Subtract their longitudes, if the places are on the same side of the prime meridian. (2) Add their longitudes, if the places are on opposite sides of the prime meridian.

EXERCISE 101

Find the difference in longitude between:

1. Baltimore and Bismark.
2. Bangor and Detroit.
3. Boston and Havana.
4. Buenos Ayres and Chicago.
5. Charleston and Constantinople.
6. Cincinnati and Honolulu.
7. Cincinnati and Melbourne.

8. Havana and Rome.
9. Louisville and St. Petersburg.
10. Constantinople and Tientsin.
11. Paris and Pekin.
12. Norfolk and Paris.
13. Montreal and Mexico City.
14. Pensacola and Portland.
15. St. Louis and St. Petersburg.
16. Savannah and Dublin.
17. San Francisco and Dublin.

Example 1. Find the difference in local time between Boston and Portland, Ore.

Portland	122°	40′		W.	
Boston	71°	3′	50″	W.	
15)	50°	36′	10″		
	3	22	25	*Ans.*	3 hr. 22 min. 25 sec.

Example 2. Find the difference in local time between Washington and Manila.

Washington,	77°	0′	36″	W.	
Manila,	120°	58′	3″	E.	
15)	197°	58′	39″		
	13	11	54.6	*Ans.*	13 hr. 11 min. 54.6 sec.

To find the difference in local time between two places, longitude being given, divide their difference in longitude by 15, and express the quotient as hours, minutes, and seconds, corresponding to the given degrees, minutes, and seconds.

EXERCISE 102

Find the difference in the local time of :

1. Mexico City and Montreal.
2. Philadelphia and San Francisco.
3. Philadelphia and Dublin.
4. Norfolk and Tientsin.
5. Chicago and Tokyo.
6. St. Louis and Rome.
7. Austin and St. Petersburg.
8. Savannah and Paris.
9. Washington and Brisbane.
10. Cincinnati and Manila.
11. Havana and Louisville.
12. Rome and Manila.
13. New Orleans and Portland.
14. Providence and St. Petersburg.
15. Montreal and Tokyo.
16. Bangor and Melbourne.
17. Baltimore and Buenos Ayres.

Example 1. When it is noon, February 22, in St. Louis, it is 15 min. 6 sec. past three o'clock A.M., Feb. 23, in Adelaide, Australia. Find the longitude of Adelaide.

SOLUTION. The time difference between St. Louis and Adelaide is

$$\begin{array}{lrrr} & 15 \text{ hr.} & 15 \text{ min.} & 6 \text{ sec.} \\ \text{Multiply by 15,} & & & 15 \\ \hline & 228^\circ & 46' & 30'' \end{array}$$

∴ Adelaide is 228° 46′ 30″ E. of St. Louis. Longitude of St. Louis is 90° 16′ W. ∴ longitude of Adelaide = (228° 46′ 30″ − 90° 16′′) E. = 138° 30′ 30″ E.

To find the difference in longitude between two places, the difference in time being given, multiply their difference in time by 15, and express the product as degrees, minutes, and seconds, corresponding to the given hours, minutes, and seconds.

EXERCISE 103

Calculate the longitude of each of the following cities, the time difference between New York City and each of them being given:

1. Berlin, 5 hr. 49.5 min.
2. Brussels, 5 hr. 13.4 min.
3. Calcutta, 10 hr. 49.2 min.
4. Edinburgh, 4 hr. 43.2 min.
5. Hamburg, 5 hr. 35.8 min.
6. London, 4 hr. 55.9 min.
7. Madrid, 4 hr. 41.1 min.
8. Vienna, 6 hr. 1.2 min.

9. The time difference between London and Amherst, Mass., is 4 hr. 50 min. 3 sec. Find the longitude of Amherst.

10. Find the difference in the time of sunrise between two points in the same latitude which differ in longitude by 39° 20′.

STANDARD TIME

The use of local time was a cause of confusion, especially after railroads became important. In 1883 a system was adopted in the United States by which the country was

divided into sections such that all places in the same section use the same time, which differs by just one hour from the time used in the next section. This system has now extended throughout the world. Time reckoned according to this system is called standard time.

As 15° of longitude correspond to one hour of time, the system of Standard Time is generally based upon a division of the earth into sections 15° of longitude in width, each section using the time of the meridian through its center; but it is found convenient to make the boundaries of these divisions somewhat irregular, as is shown by the map.

REFERENCE TABLE

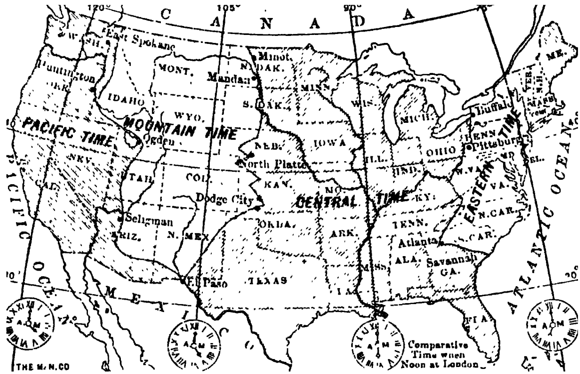
STANDARD MERIDIANS AND PLACES USING THEM

0°. Great Britain, Spain, Belgium, Holland, France.
15° E. Germany, Austria, Italy, Denmark, Norway.
30° E. South Africa, Egypt, Turkey.
$82\frac{1}{2}$° E. British India (since July 1, 1905).
$97\frac{1}{2}$° E. Burma (since July 1, 1905).
120° E. West Australia, eastern coast of China, Philippine Islands.
135° E. Japan.
$142\frac{1}{2}$° E. South Australia.
150° E. Victoria, Queensland, New South Wales.
$172\frac{1}{2}$° E. New Zealand.
60° W. Newfoundland and Eastern Canada.
75° W. Eastern belt of the United States, Chile.
90° W. Central belt of the United States.
105° W. Mountain belt of the United States.
120° W. Pacific belt of the United States.
135° W. Alaska.

150° W. Tahiti.

$157\frac{1}{2}$° W. Hawaiian Islands.

Russia does not use standard time.



STANDARD TIME IN THE UNITED STATES.

EXERCISE 104

1. Mariners carry on board ships chronometers which keep Greenwich time. When it is noon, local time, the chronometer indicates 4 hr. 48 min. P.M. What is the longitude of the ship?

2. When it is 10 o'clock P.M., March 2, in Washington, what is the standard time in Manila? Melbourne? Berlin?

3. When it is 2 o'clock A.M., standard time, in Denver, what is the standard time of London? Manchester? Glasgow? Tientsin? Constantinople?

4. A telegram is sent from Madrid to Washington at 9 o'clock A.M. Allowing 1 hr. for transmission, when will it reach Washington?

5. At noon, local time, a chronometer indicates 11 o'clock P.M. What is the longitude?

6. A telegram is sent from Galveston to London at 10 o'clock P.M. When will it be received, allowing 2 hr. for transmission?

7. When it is 2 o'clock A.M. in Washington, standard time, what is the time in New Zealand? Tahiti? British India?

8. The San Francisco earthquake occurred April 18, 1906, at 5 A.M. When should the news have reached London? Berlin? Tokyo? Adelaide? (allowing one hour for transmission).

9. When it is noon in Paris, France, what is the time in Denver? Natal? Calcutta? Wellington (New Zealand)?

10. When it is 9 o'clock A.M. in Madras, what is the time in St. John's, Newfoundland? Chicago? Sitka?

11. When it is noon in the Hawaiian Islands, what is the time in Cairo (Egypt)? Perth (Western Australia)?

THE LANGUAGE OF MATHEMATICS, RATIO, PROPORTION, PARTNERSHIP

By **mathematics** is understood those branches of knowledge which deal with quantity. Arithmetic, algebra, geometry, surveying, etc., are included in the term *mathematics*.

Mathematics has a **language** of its own.

The word **eight** conveys a definite idea to the mind; the figure eight conveys the same idea. The words **eight squared** convey a definite idea to the mind; 8^2 conveys the same idea. The words **three fourths of sixteen** convey an idea; $\frac{3}{4} \times 16$ conveys the same idea. Similarly, the words, **the quotient of seventy-two divided by eight,** convey an idea; $\frac{72}{8}$ conveys the same idea. The language of mathematics gives abbreviated forms for stating what

would occupy much more space if expressed in words; but one must be able to express in words the meaning conveyed by mathematical language.

The sum of three and four is seven. This is written in the language of mathematics as: $3 + 4 = 7$, which is usually read: "Three plus four equal seven."

The difference between eighteen and seven is eleven. In the language of mathematics, this is written as: $18 - 7 = 11$, which is read: "Eighteen minus seven equal eleven."

It is often convenient to represent a large number by an abbreviation, or to use an abbreviation to represent an unknown number, such as the answer to a problem. Letters are usually used for this purpose.

For example: Of the 24 hr. in a day, a boy spends $\frac{1}{4}$ at school, $\frac{1}{6}$ at play, $\frac{3}{8}$ in sleep, 2 hr. at meals, and the rest at work. How many hours does he work?

For convenience, let x represent the number of hours he works, and let y represent the number of hours he does not work.

Then, $x = 24 - y$,

and $y = \frac{1}{4}$ of 24, plus $\frac{1}{6}$ of 24, plus $\frac{3}{8}$ of 24, plus 2.

Therefore, $y = 6 + 4 + 9 + 2 = 21$.

As $x = 24 - y$, and $y = 21$,

then $x = 24 - 21 = 3$.

Therefore, the boy works 3 hr.

In using letters to represent numbers, any letter may be used to represent any number; but in any one problem each letter represents the same number in all parts of the problem.

The expression $a + b$ means the sum of the numbers represented by a and b; but it is read simply as: **a plus b**.

The expression $x - y$ means the difference obtained by subtracting the number represented by y from the number represented by x; but it is read simply as: **x minus y.**

In writing the product of numbers represented by letters, the letters are placed next each other without any sign between. Thus, instead of writing $a \times b \times c$ to represent the product of the numbers represented by a, b, and c, we write merely abc, and read it as: "a, b, c."

Division of numbers represented by letters may be indicated by the division sign, or by writing the letters as numerator and denominator of a fraction. Thus, the number represented by a, divided by the number represented by b, is written $a \div b$, or, $\frac{a}{b}$. The expression, $a \div b$ is read: "a divided by b." The expression, $\frac{a}{b}$, is read in the same way.

This use of letters gives very much abbreviated forms for stating rules. Thus, the rule for computing interest is: Multiply the principal by the rate to get the interest for 1 year, and this product by the time expressed in years. If the principal is represented by p, the rate by r, the time in years by t, then i equals p times r times t. So the rule becomes $i = prt$.

Similarly, if F represents the area of a rectangle, b the base, and a the altitude, the rule for computing the area of a rectangle becomes

$$F = ba.$$

A figure written before a letter indicates multiplication. Thus, $5\,a$ means 5 times a, which means five times the number represented by a. $4\,b$ means four times b, which means four times the number represented by b. $5\,a$ is read: **five a**, and $4\,b$ is read: **four b.**

A figure written before a letter to indicate multiplication is called a **coefficient.** When the coefficient is 1, it need not be written.

Thus, in the expressions $5a$, $4b$, $9xy$, $56mn$, x, the coefficients are, 5, 4, 9, 56, 1.

If a number is multiplied by itself, the result is called the **square** of that number.

Thus: $5 \times 5 = 25$. 25 is the square of 5. 5×5 may also be indicated as 5^2. $5^2 = 25$, is read: **5 squared** or **5 square equals** 25.

A figure written just to the right of a number and above it is called an **exponent.**

An exponent indicates how many times the number is to be used as a factor.

If a number is used three times as a factor; the result is called the **cube** of the number.

Thus, $5 \times 5 \times 5 = 125$; 125 is the **cube** of 5. $5 \times 5 \times 5$ may be written 5^3, and is read: **5 cube or 5 cubed.**

The result of multiplying a number by itself any number of times is called a **power** of the number.

Square and **cube** are special names for the second power and third power of a number. Above the third power the word **power** is used. Thus, 5^6 is read: five raised to the sixth power. This may also be read: the sixth power of five; or, five with the exponent six.

This method is especially useful when the number is represented by a letter. Thus, $y^2 = y \times y$, or yy, and is read: y square. y^3 means $y \times y \times y$, or yyy, and is read: y cube. y^4 means $y \times y \times y \times y$, or $yyyy$, and is read: y to the fourth power. y^5 means $y \times y \times y \times y \times y$, or $yyyyy$, and is read: y to the fifth power.

Example 1. If $a = 5$, $b = 3$, what is the value of $a + b$? $a - b =$? $4a =$? $3b =$? $2a - 3b =$?

SOLUTION. $a + b = 5 + 3 = 8$. $a - b = 5 - 3 = 2$. $4a = 4 \times 5 = 20$. $3b = 3 \times 3 = 9$. $2a - 3b = 2 \times 5 - 3 \times 3 = 1$.

Example 2. If $a = 7$, what is the value of a^2? a^3? $3a^2$? $4a^3$?

$a^2 = a \times a = 7 \times 7 = 49$. $a^3 = a \times a \times a = 7 \times 7 \times 7 = 343$.

$3a^2 = 3 \times a \times a = 3 \times 7 \times 7 = 147$. $4a^3 = 4 \times a \times a \times a = 4 \times 7 \times 7 \times 7 = 1372$.

EXERCISE 105

If $a = 4$, what is the value of $4a$? $7a$? $11a$? $13a$? $17a$? $19a$? $27a$? $\frac{1}{2}a$? $\frac{1}{4}a$?

If $a = 5$, what is the value of a^2? a^3? a^4? $2a^2$? $3a^2$? $2a^3$? $4a^2$? $a + a^2$? $a^2 + a^3$?

If $a = 3$, $b = 2$, what is the value of $a + b$? $a - b$? $2a + b$? $a + 2b$? $2a - b$? $2a - 3b$? $3a - 2b$?

If $a = 6$, $b = 3$, what is the value of $6a + 3b$? $3a + 5b$? $6a - 36$? $5a - 10b$? $7a - 5b$? $3a^2$? $a^2 + b^2$? $a^2 - b^2$?

If $x = 5$, $y = 6$, what is the value of xy? $2xy$? $3xy$? x^2y? xy? $\frac{1}{3}x^2$? $\frac{1}{4}y^2$?

If $x = 9$, $y = 4$, what is the value of $2x^2 - y^2$? $x^2 + 2y^2$? $3x^2 + y^2$? $2x^2 + 3y^2$? $y^2 - x^2$?

If $a = 10$, $b = 7$, find the value of ab, $5ab$, $ab + a^2$, $2ab + b^2$, $ab + 2b^2$.

EXERCISE 106

1. What is the sum of two times a number and three times the same number? What is the sum of $2x$ and $3x$?

2. What is the sum of $4x$ and $3x$? of $8a$ and $3a$? of $5b$ and $2b$? of $6b$ and $4b$?

3. What is the difference between $8x$ and $3x$? $6x$ and $2x$? $11b$ and $7b$? $8a$ and a?

4. Add $5x$ and $7x$; $4x$ and $9x$; $9b$ and $6b$; $10y$ and $6y$; $12x$ and $4x$.

5. Subtract $4x$ from $9x$; $8x$ from $14x$; $9x$ from $16x$; $7x$ from $13x$; $5ab$ from $8ab$.

6. Find the difference between $11y$ and $2y$; $5ab$ and ab; $7ab$ and $4ab$; $12ab$ and $2ab$.

Notice that addition may be indicated between unlike letters, but that only those terms can be combined in which exactly the same letters are used. This is like the addition of compound numbers, except that in the case of letters we have no system of reduction.

Add the following:

7.
$$\begin{array}{l} 5 \text{ ft. } 4 \text{ in.} \\ 3 \text{ ft. } 2 \text{ in.} \\ \underline{1 \text{ ft. } 5 \text{ in.}} \end{array}$$

8.
$$\begin{array}{l} 5 \text{ ft.} + 4 \text{ in.} \\ 3 \text{ ft.} + 2 \text{ in.} \\ \underline{1 \text{ ft.} + 5 \text{ in.}} \end{array}$$

9.
$$\begin{array}{r} 5a + 4b \\ 3a + 2b \\ \underline{a + 5b} \end{array}$$

10.
$$\begin{array}{r} 4x + 7y \\ 3x + 9y \\ \underline{7x + 5y} \end{array}$$

11.
$$\begin{array}{r} a^2 + 3ab + b^2 \\ 6a^2 + 2ab + 7b^2 \\ \underline{4a^2 + 8ab + 4b^2} \end{array}$$

12.
$$\begin{array}{r} 5xy + 7ab + 8b \\ 3xy + 2ab + 4b \\ \underline{9xy + 6ab + 3b} \end{array}$$

Subtract:

13.
$$\begin{array}{l} 8 \text{ ft. } 7 \text{ in.} \\ \underline{5 \text{ ft. } 4 \text{ in.}} \end{array}$$

14.
$$\begin{array}{l} 8 \text{ ft.} + 7 \text{ in.} \\ \underline{5 \text{ ft.} + 4 \text{ in.}} \end{array}$$

15.
$$\begin{array}{r} 8a + 7b \\ \underline{5a + 4b} \end{array}$$

16.
$$\begin{array}{r} 11x + 13y \\ \underline{6x + 7y} \end{array}$$

A statement that two expressions are equal is called an **equation**.

Written in the language of mathematics, an equation consists of two parts joined by the sign, $=$, denoting equality. Thus, $4+5=9$ is an equation; $3a-4b+c=x+y$, is an equation. The part of an equation at the left of the sign of equality is called the **first member** of the equation; the part of an equation at the right of the sign of equality is called the **second member** of the equation.

What is the product of a and a? $a \times a = a^2$.

What is the product of a and a^2? $a^2 = a \times a$.

$\therefore a \times a^2 = a \times a \times a = a^3$.

What is the product of a^2 and a^3? $a^2 = a \times a$; $a^3 = a \times a \times a$.

$\therefore a^2 \times a^3 = (a \times a) \times (a \times a \times a) = a^5$.

What is the product of a^4 and a^3?

$$a^4 = a \times a \times a \times a.$$
$$a^3 = a \times a \times a.$$

$\therefore a^4 \times a^3 = (a \times a \times a \times a) \times (a \times a \times a) = a^7$.

What is the product of $4a^2$ and $5a^3$?

$$4a^2 = 4 \times a \times a.$$
$$5a^3 = 5 \times a \times a \times a.$$
$$\begin{aligned}\therefore 4a^2 \times 5a^3 &= 4 \times a \times a \times 5 \times a \times a \times a \\ &= 4 \times 5 \times a \times a \times a \times a \times a \\ &= 20a^5.\end{aligned}$$

EXERCISE 107

1. $a \times 2a = ?$
2. $2a \times a^2 = ?$
3. $3a^2 \times a = ?$
4. $3a^2 \times 2a^2 = ?$
5. $4a \times a^3 = ?$
6. $7a^2 \times 2a^2 = ?$
7. $3a \times 5a^3 = ?$
8. $9a^2 \times 2a^3 = ?$
9. $4a^3 \times a^2 = ?$
10. $5x^3 \times 4x = ?$
11. $6x^2 \times x^3 = ?$
12. $5b \times b^2 = ?$
13. $4b^2 \times 3b = ?$
14. $5b^3 \times 4b^3 = ?$
15. $2b^2 \times 5b = ?$
16. $4y^2 \times 3y^3 = ?$
17. $7z^2 \times 5a = ?$
18. $4y \times 8b^3 = ?$

If the multiplicand has more than one term, each term is multiplied by the multiplier.

Thus; multiply $7a + 4b$ by 3.

$7a + 4b$	3 times $7a$ gives $21a$.
3	3 times $4b$ gives $12b$.
$21a + 12b$	The product is $21a + 12b$.

19. Multiply $3a + 4b$ by 2.

20. Multiply $7x + 5y$ by 5.

21. Multiply $8a + 7b$ by $3a$.

22. Multiply $4x + 9y$ by $7x$.

23. Multiply $6x + 3y$ by x.

24. Multiply $12a + 5b$ by a.

If the multiplier has more than one term, each term of the multiplicand is multiplied by each term of the multiplier, and these partial products are added.

Thus: multiply $5a + 2b$ by $3a + 4b$.

$$\begin{array}{l} 5a + 2b \\ 3a + 4b \\ \hline 15a^2 + 6ab \\ \qquad\quad 20ab + 8b^2 \\ \hline 15a^2 + 26ab + 8b^2 \end{array}$$

Write the multiplier under the multiplicand. Multiplying by $3a$ gives $15a^2 + 6ab$. Multiplying by $4b$ gives $20ab + 8b^2$. Addition of these partial products gives, for the answer, $15a^2 + 26ab + 8b^2$.

Notice that the $20ab$ is combined with the $6ab$; but that there is no other term in a^2 to combine with the $15a^2$, nor any other term in b^2 to combine with the $8b^2$.

25. Multiply $6a + 4b$ by $2a + 3b$.

26. Multiply $3a + 5b$ by $4a + 7b$.

27. Multiply $4x + 9y$ by $7x + 2y$.

28. Multiply $x + y$ by $2x + 3y$.

What is the quotient when a^3 is divided by a?

$$a^3 \div a = \frac{a^3}{a} = \frac{\cancel{a} \times a \times a}{\cancel{a}} = a \times a = a^2.$$

Here cancellation is utilized.

What is the quotient of $8\,a^3$ by $2\,a$?

$$8\,a^3 \div 2\,a = \frac{8\,a^3}{2\,a} = \frac{\overset{4}{\cancel{8}} \times \cancel{a} \times a \times a}{\cancel{2} \times \cancel{a}} = 4 \times a \times a = 4\,a^2.$$

EXERCISE 108

Find the following quotients:

1. $\frac{8\,a}{2\,a}$.
2. $\frac{9\,a}{3}$.
3. $\frac{12\,a}{4}$.
4. $\frac{16\,a^2}{4\,a^2}$.
5. $\frac{12\,a^3}{4\,a}$.
6. $\frac{21\,a^3}{7\,a^2}$.
7. $\frac{28\,a^4}{14\,a^2}$.
8. $\frac{18\,a^4}{3\,a^3}$.
9. $\frac{24\,a^4}{8\,a^2}$.
10. $\frac{11\,a^4}{a}$.
11. $\frac{22\,a^5}{2\,a^2}$.
12. $\frac{16\,a^5}{4\,a^3}$.
13. $\frac{32\,a^6}{16\,a^4}$.
14. $\frac{24\,a^5}{8\,a^3}$.
15. $\frac{25\,a^4}{5\,a^2}$.
16. $\frac{35\,x^4}{5\,x^3}$.
17. $\frac{39\,x^5}{13\,x}$.
18. $\frac{42\,x^4}{6\,x^3}$.
19. $\frac{45\,x^4}{9\,x}$.
20. $\frac{50\,x^5}{10\,x^4}$.
21. $\frac{60\,x^5}{10\,x^5}$.
22. $\frac{26\,x^5}{13\,x}$.
23. $\frac{33\,b^2}{11\,b}$.
24. $\frac{48\,b^4}{16\,b}$.
25. $\frac{46\,b^5}{23\,b^2}$.

RATIO

The **ratio** of one number to another number is the quotient obtained by dividing the first number by the second. Thus, the ratio of 3 to 7 is obtained by dividing 3 by 7.

The ratio of 3 to 7 is written $3:7$. When the quotient of $3:7$ is written $\frac{3}{7}$, the expression $\frac{3}{7}$ is a fraction.

The ratio $7:3$ is called the **inverse ratio** of 3 to 7.

The **inverse ratio** of two numbers is the ratio of the second to the first. It may be obtained by inverting the fraction which expresses the ratio of the two numbers.

If letters are used to represent the numbers, the ratio is obtained in the same way as with figures. Thus, the ratio of a to b is expressed as $a:b$, or $\frac{a}{b}$. The inverse ratio of a to b is expressed as $b:a$, or $\frac{b}{a}$. The inverse ratio of y to 5 is expressed as $5:y$, or $\frac{5}{y}$.

EXERCISE 109

1. What is the ratio of 2 ft. to 6 ft.?

2. What is the ratio of 4 in. to 1 yd.? of 3 in. to 1 yd.? of 1 yd. to 1 rd.? of $\frac{1}{2}$ rd. to 1 rd.?

3. What is the ratio of 80 A. to 1 sq. mi.? of 120 A. to 1 sq. mi.? of $\frac{1}{2}$ A. to 2 A.?

4. What is the ratio of the distance traveled by two trains in the same time, if the rate of the first train is 20 mi. per hour, and the rate of the second train is 30 mi. per hour?

5. If A walks at the rate of $2\frac{1}{2}$ mi. per hour, and B walks at the rate of 5 mi. per hour, what is the ratio of A's time to B's time in going any given distance?

6. What is the ratio of the time that 8 men take to do a piece of work to the time that 6 men take to do the same piece of work?

7. If you ride in a carriage at the rate of 7 mi. an hour and walk back the same distance at the rate of 3 mi. an hour, what is the ratio of the time in the carriage to the time walking?

8. A line 36 in. long is divided into two parts in the ratio 4 : 5. Find the length of each part. $4+5=9$. One part is $\frac{4}{9}$ of 36, the other $\frac{5}{9}$ of 36.

9. A person rides in a carriage a certain distance at 7 mi. an hour, and walks back at the rate of 3 mi. an hour. If he returns in 90 min., how far did he go?

Divide 90 in the inverse ratio 7 : 3; that is, in the ratio 3 : 7. $\frac{3}{10}$ of $90=27$; $\frac{7}{10}$ of $90=63$. In 63 min. he walks $\frac{63}{60}\times 3$ mi. $= 3.15$ mi. In 27 min. he rides $\frac{27}{60}\times 7$ mi. $=$ 3.15 mi.

10. I can buy two kinds of matting for 40¢ and 50¢ a yard respectively. If I spend the same amount of money in the purchase of the two kinds of matting, what is the ratio of the number of yards of matting of the first kind to the number of yards of the second kind bought?

11. Divide 15 in the ratio 2 : 3.

12. Divide 20 in the ratio 3 : 7.

13. Divide $1 in the ratio 18 : 7.

14. Divide 1 mi. in the ratio 7 : 9.

15. Divide 1 in the ratio 9 : 11.

16. Divide 1 gal. in the ratio 1 : 3.

17. Divide $1 in the inverse ratio 9 : 16.

18. Divide 22 yd. in the inverse ratio 3 : 8.

19. Divide $1000 in the inverse ratio 3 : 5.

PROPORTION

A statement indicating that two ratios are equal is called a **proportion.**

Illustrations: $2:3=4:6$. (1) $9:15=12:20$. (2)

These proportions may be written in fractional form by writing each ratio as a fraction.

Thus, $\frac{2}{3}=\frac{4}{6}$. $\frac{9}{15}=\frac{12}{20}$.

If these fractions are each reduced to lowest terms, the equality of ratios in each proportion is easily seen. Thus,

$$\tfrac{2}{3} = \tfrac{4}{6} \quad \text{becomes } \tfrac{2}{3} = \tfrac{2}{3},$$

and

$$\tfrac{9}{15} = \tfrac{12}{20} \quad \text{becomes } \tfrac{3}{5} = \tfrac{3}{5}.$$

Four quantities are said to be in **proportion,** or to be **proportional** when the ratio of the first to the second equals the ratio of the third to the fourth.

The numbers, 2, 3, 4, 6 are in proportion for, $\frac{2}{3} = \frac{4}{6}$. This proportion may be written:

$$\tfrac{2}{3} = \tfrac{4}{6}, \text{ or } 2:3 = 4:6, \text{ or } 2:3::4:6.$$

It is read: 2 is to 3 as 4 is to 6.

The first and fourth terms of a proportion are called the **extremes,** and the second and third terms are called the **means,** of the proportion.

In a proportion the product of the extremes is equal to the product of the means.

Thus, in the proportion $2:3 = 10:15$, $2 \times 15 = 30$, and $3 \times 10 = 30$. $\therefore 2 \times 15 = 3 \times 10$.

This property of a proportion enables us to find any term of a proportion, if three of the terms of the proportion are known.

Example 1. Find x in the proportion $x:4 = 9:6$.

SOLUTION. The product of the extremes is equal to the product of the means.

$$\therefore 6x = 36. \qquad x = 6.$$

Example 2. Find x in the proportion $10:35 = x:42$.

SOLUTION. Since the product of the means is equal to the product of the extremes,

$$35x = 10 \times 42.$$

$$\therefore \quad x = 10 \times \tfrac{42}{35} = 12.$$

EXERCISE 110

If x stands for the unknown term in each of the following proportions, find its value.

1. $2:3=6:x$.
2. $3:4=6:x$.
3. $15:25=12:x$.
4. $12:20=18:x$.
5. $14:21=x:27$.
6. $21:27=x:45$.
7. $35:84=x:72$.
8. $20:48=x:96$.
9. $16:24=x:33$.
10. $20:32=x:72$.
11. $25:45=x:99$.
12. $45:126=x:154$.
13. $57:133=x:126$.
14. $68:85=x:75$.
15. $36:x=52:65$.
16. $28:x=36:63$.
17. $27:x=15:50$.
18. $15:x=21:77$.
19. $28:x=36:81$.
20. $25:x=45:72$.
21. $35:x=30:48$.
22. $x:81=16:72$.
23. $x:99=26:117$.
24. $x:65=24:52$.
25. $x:112=45:144$.

Example 1. If 7 bu. of wheat cost \$5.25, find the cost of 11 bu. of wheat at the same rate.

Solution. 7 bu. cost \$5.25,
11 bu. cost x.

The answer is greater than 5.25 in the same proportion that 11 is greater than 7.

$$x:5.25=11:7.$$
$$7x=11\times 5.25.$$
$$x=\tfrac{11}{7}\times 5.25=8.25. \quad \textit{Ans. } \$8.25.$$

Example 2. If 12 men pave a street in 15 da., how long will it take 9 men to pave a street of the same area?

Solution. It will take 9 men longer than it takes 12 men, and the number of days in which 9 men can do the

work that 12 men do in 15 days will bear the same relation to 15 that 12 does to 9.

$$12:9=x:15$$
$$9x=15\times 12$$
$$x=\frac{15\times 12}{9}=20.$$

To solve a problem in proportion, find the first relation that the answer sought bears to a given quantity of the same kind. This relation will be equal to the relation of two other like quantities given. These four quantities will then form a proportion which can be solved by placing the product of the means equal to the product of the extremes.

It is well to state a problem so that one can see all around it; *e.g.*

12 men do the work in 15 da.
9 men do the work in x da.
$x=15\times \frac{12}{9}$,

as the answer is obviously more than 15 da.

EXERCISE 111

1. If 20 men earn \$ 450 in a given time, how much will 30 men earn in the same time?

2. If 15 bu. of corn cost \$7.20, what will 48 bu. of corn cost?

3. If 12 A. of land cost \$456.90, what will 16 A. of the same land cost?

4. If 4 men can do a piece of work in 15 da., how long will it take 6 men to do an equal amount of work?

5. If 18 head of cattle cost \$1450, what will 27 head of cattle cost at the same rate?

6. If a train goes 400 mi. in 12 hr., how long will it take to go 560 mi.?

7. If 8 masons build a wall in 15 da., how long will it take 6 masons to build a wall of the same size?

8. If 18 horses consume 14 bu. of corn in a week, how much will 24 horses consume in the same time?

9. If 18 horses plow a tract of land in 13 da., how long will it take 26 horses to plow the same tract?

10. How long will it take 126 sheep to eat a quantity of feed which will last 105 sheep 30 da.?

11. A garrison consisting of 1200 men has provisions for 16 da. How many men must be sent away so that the provisions may last 24 da.?

12. A garrison consisting of 1400 men has provisions for 27 da. If the garrison is reënforced by 400 men, in how many days will the provisions be consumed?

13. If I can buy a dozen turkeys for \$20.50, how many turkeys can I buy for \$30.75?

14. If the interest on \$750 for 4 mo. is \$12.50, what is the interest on \$39.60 for the same time?

15. If an arc of $12''$ on the 40th parallel of latitude is 933.92 ft., find the length of $1°$ on the 40th parallel of latitude.

16. If an arc of $30'$ on the circumference of a wheel is $1\frac{1}{4}$ in., find the length of the circumference of the wheel.

17. A fly wheel 63 ft. in circumference makes 150 revolutions per minute. How many feet does a spot on its rim travel in one second?

18. A train is running at 50 miles an hour. This speed is 25 % greater than usual. Find its usual speed.

EXERCISE 112

1. If 6 horses plow a field in 9 days, how long will it take 9 horses to plow the same field?

2. If a train runs in $3\frac{3}{4}$ hours between two stations at the rate of 18 miles an hour, how long will it take a train whose speed is 30 miles an hour to make the same run?

3. If 5 acres of land sell for \$423, at this rate what will be the selling price of 7 acres?

4. If 22 yd. of cloth are bought for a sum of money, how many yards may be bought for the same sum when the price falls 12%?

5. Eight horses consume a quantity of corn in 24 days. How long should the same quantity of corn last 12 horses?

6. The minute hand of a clock goes 360° in 1 hour. How many degrees does it go in 22 minutes?

7. An arc of 75° is 4 ft. 6 in. How many feet in the circumference of the circle?

8. If $\frac{11}{200}$ of the number of miles from Paris to Turin is $27\frac{1}{2}$, what is the entire distance separating the cities?

9. If $\frac{11}{15}$ of the number of miles from New York City to Panama is 1727, how far is Panama from New York?

10. Given .9 of the distance from London to Constantinople as 1827 mi., how many miles is it from the former to the latter?

11. If $\frac{13}{55}$ of the distance from Hamburg to Vienna is 143 mi., find the distance between these cities.

12. In the year 1902, $\frac{11}{25}$ of the United States internal revenue receipts from tobacco amounted to \$22,852,687. Find the total internal revenue receipts from tobacco for that year.

13. In the year 1902, $\frac{13}{81}$ of the excise tax in the United States on gross receipts under the War Revenue Law of 1898 amounted to $117,221. Find the total tax on gross receipts in 1902.

14. In the year 1902, $\frac{9}{20}$ of the United States internal revenue receipts from the tax on oleomargarine amounted to $1,325,021.40. Find the total receipts from this source.

15. The mark is the unit of money in Germany ; $\frac{3}{17}$ of its value in our currency is 42 mills. Express the value of a mark in dollars.

16. The yen is the standard of value in Japan ; $\frac{7}{83}$ of its value is equivalent to 4 cents and 2 mills. Express in dollars the value of the yen.

17. In Venezuela, the monetary unit is the Bolivar ; $\frac{3}{5}$ of its value is equivalent to $.1158. Find its value in cents.

18. Thirty-two thirty-fifths of a meter is very nearly equivalent to 1 yd. Express the value of a meter in yards.

COMPOUND PROPORTION

If two or more ratios are multiplied together, the result is called the **ratio compounded** of these ratios. For example, the ratio compounded of the ratios $2:3$, $4:5$, $7:8$ is the ratio $2 \times 4 \times 7 : 3 \times 5 \times 8$, or $56:120$, or $7:15$.

In some problems the ratio that the answer bears to the given quantity of the same kind would be equal to any one of the ratios of several pairs of other given quantities if the other pairs were lacking. In such a case, the ratio of the answer to its like quantity is equal to the product

of the ratios of the several pairs of quantities arranged in their proper order.

A proportion in which the final result thus depends upon a ratio compounded of two or more ratios is called **a compound proportion.**

Example 1. If 15 men mow 90 A. in 12 da., how many acres will 12 men mow in 14 da.?

15 men in 12 da. mow 90 A.

12 men in 14 da. mow x A.

SOLUTION. Let x stand for the number of acres which 12 men mow in 14 da.

In the same number of days, 12 men would mow less than 15 men. $\therefore 15 : 12 = 90 : x$ is a proportion from which can be found the number of acres 12 men can mow while 15 men mow 90 acres.

A number of men can mow less in 12 da. than in 14 da. If 90 acres is the amount mowed in 12 da., the amount mowed in 14 da. can be found from the proportion, — $12 : 14 = 90 : x$. These two proportions give the compound proportion, $15 \times 12 : 12 \times 14 = 90 : x$.

$$\therefore x = \overset{6}{\cancel{90}} \times \frac{\cancel{12}}{\cancel{15}} \times \frac{14}{\cancel{12}} = 84.$$

Ans. 84 acres.

Example 2. If 24 men build a house in 18 da. of 10 hr. each, how many men will it take to build the same house in 30 da. of 8 hr. each?

24 men in 18 da. of 10 hr. build a house.

x men in 30 da. of 8 hr. build a house.

SOLUTION. To build the house in 30 da. of 10 hr., fewer men would be required than to build it in 18 da.

of 10 hr., and the number of men can be found from the proportion:

$$30 : 18 = 24 : x.$$

But it would require more men working 8 hr. a day than it would working 10 hr. a day to complete the house in the same length of time; and the number can be found from the proportion:

$$8 : 10 = 24 : x.$$

The compound proportion obtained from these two proportions is:

$$30 \times 8 : 18 \times 10 = 24 : x.$$

$$\therefore x = \overset{\cancel{3}}{\cancel{24}} \times \frac{18}{\underset{\cancel{3}}{\cancel{30}}} \times \frac{\cancel{10}}{\cancel{8}} = 18.$$

Ans. 18 men.

EXERCISE 113

1. If 12 horses plow 84 A. in 6 da., how many acres will 16 horses plow in $4\frac{1}{2}$ da.?

2. If 14 men pave a street 200 ft. long in 8 da., how many feet will 12 men pave in 7 da.?

3. If a man earns $117 in 3 mo. working 6 hr. a day, how much will he earn in 5 mo. working 8 hr. a day?

4. A garrison of 3650 men consumed in 30 da. 82.3 T. of food. How much food would be required for 7500 men for 1 yr. at the same rate?

5. If 8 masons build in 2 da. a wall 40 ft. long and 6 ft. high, what height of wall 30 ft. long can they build in 5 da.?

6. If 21 men complete a piece of work in 8 da. of $7\frac{1}{2}$ hr. each, in how many days of 10 hr. each can 18 men do the same work?

7. A wall is to be built in 10 da. by 30 men. After 2 da. 10 men are dismissed. In what time will the remaining 20 men finish the work?

8. If 4 men or 6 boys dig a trench in 12 da., in what time can 2 men and 9 boys dig it?

9. If 12 men mow 30 A. in 3 da. of 8 hr. each, how many hours a day must 16 men work to mow 48 A. in 4 da.?

10. If the interest on $100 for 1 yr. is $6, find the interest on $840 for 2 yr. 3 mo.

11. If 12 men working 7 hr. a day earn $227.50 in 20 da., how much will 15 men earn in 20 da., working 9 hr. each?

12. If 6 men mow $\frac{3}{5}$ of a meadow in $4\frac{1}{2}$ da., how long will it take 8 men to mow the remainder?

13. In 10 da. of 8 hr. each 9 horses can plow $\frac{2}{5}$ of a field. In how many days of 9 hr. each can the remainder of the field be plowed by 15 horses?

14. A marble block 3 ft. by 4 ft. by 5 ft. weighs 5.1 T. Find the weight of a marble block 7 ft. by 3 ft. by 10 ft.

15. A mason can build 3 yd. of a wall in 15 hr. How long will it take 9 masons to build 24 yd. of a wall which is one third higher than the other wall?

16. If a trough of water 12 in. × 24 in. × 36 in. weighs 10,368 oz. how long must a trough 12 in. by 24 in. be if the water weighs 3456 oz.?

PARTNERSHIP

Example 1. A, B, and C enter into partnership. A puts in \$840, B puts in \$350, and C puts in \$2000. A withdraws from the concern in 5 mo., C in 7 mo., and at the end of 8 mo. the profits are divided. If the entire profit is \$450, how shall this be divided among A, B, and C?

SOLUTION. Each partner's share in the business depends upon the amount of his investment, and the length of time for which the investment is made. In order to make comparison more easy, the investments are computed which could be invested for one month and produce the same results as the given investments.

A's investment of \$840 for 5 mo. equals an investment of $5 \times \$840$, or \$4200 for 1 mo.

B's investment of \$350 for 8 mo. equals an investment of $8 \times \$350$, or \$2800 for 1 mo.

C's investment of \$2000 for 7 mo. equals an investment of $7 \times \$2000$, or \$14,000 for 1 mo.

The total investment is equal in effect to an investment of $\$4200 + \$2800 + \$14,000 = \$21,000$ for 1 mo.

A's share in the business is $\frac{4200}{21000} = \frac{1}{5}$.

B's share in the business is $\frac{2800}{21000} = \frac{2}{15}$.

C's share in the business is $\frac{14000}{21000} = \frac{2}{3}$.

$\therefore$ A's share of the profits $= \frac{1}{5}$ of $\$450 = \$90$.

B's share of the profits $= \frac{2}{15}$ of $\$450 = \$60$.

C's share of the profits $= \frac{2}{3}$ of $\$450 = \$300$.

Example 2. A and B are partners, A having \$4 to B's \$5. A gets 15% of the entire profits for managing the business, and the remainder is divided proportionally to

their capitals. B receives as his share \$1700. How much does A receive?

SOLUTION. As A invests $\frac{4}{5}$ as much as B, his share of the profits is $\frac{4}{5}$ of B's share; but B's share is \$1700.

Therefore, A's share is $\frac{4}{5}$ of \$1700 = \$1360.

The total profit is \$1700 + \$1360 = \$3060.

But this is 85% of amount upon which A's 15% for managing the business was reckoned.

If \$3060 is 85%, one per cent is $\frac{1}{85}$ of \$3060, and 100 per cent equals $\frac{100}{85} \times$ \$3060 = \$3600.

Of this \$3600, B received \$1700.

Therefore A received \$3600 − \$1700 = \$1900.

EXERCISE 114

1. A, B, and C enter into partnership with capitals of \$3000, \$3750, and \$4500 respectively. At the end of the year they divide among themselves a profit of \$3000. Find each person's share.

2. Two partners, A and B, invest \$600 and \$1125. A's money remains in the business 6 mo., and B's 8 mo. If they make a profit of \$2100, find each person's share.

3. Two men rent a pasture for \$171; one puts in the pasture 30 cattle for 30 da., and the other 45 cattle for 18 da. How much rent should each pay?

4. A and B enter into partnership. A's capital is \$200 more than B's. Out of a profit of \$640, B gets \$280. Find A's and B's capital.

5. A and B enter into a partnership, A contributing \$6400 and B \$7200. At the end of three mo. A withdraws \$1600, and at the end of 5 mo. B withdraws \$1440. C

then enters into the partnership with a capital of \$4800. Seven months later a gain of \$2154 is divided among them. Find each person's share.

6. A, B, and C enter into partnership. A puts in \$1000, B \$1200, and C \$1800. At the end of 3 mo. C withdraws, and at the end of 10 mo. B withdraws. At the end of a year the profits are divided. If C gets \$135, how much do A and B receive?

7. Two men form a partnership. Their capitals are in the ratio 2 : 3. After 6 mo. the first man increases his capital by $\frac{1}{3}$ of itself, and the second man diminishes his capital by $\frac{1}{3}$ of itself. After 6 mo. more they divide their profits, amounting to \$1450. Find each partner's share.

8. A, B, and C form a partnership, each contributing \$3000. At the end of 6 mo. A adds \$1000 and B \$2500, but at the end of 7 mo. C withdraws \$2000 of his capital. If A gets from the year's profits \$2100, what ought B and C to get?

PERCENTAGE

What is **percentage**? How is it reckoned? What does **per cent** mean? What symbol is used for **per cent**? How is per cent usually written?

The per cent equivalent of the following fractions should be thoroughly fixed in mind:

$$\tfrac{1}{2},\ \tfrac{1}{3},\ \tfrac{2}{3},\ \tfrac{1}{4},\ \tfrac{3}{4},\ \tfrac{1}{5},\ \tfrac{2}{5},\ \tfrac{3}{5},\ \tfrac{4}{5},\ \tfrac{1}{6},\ \tfrac{5}{6},\ \tfrac{1}{8},\ \tfrac{3}{8},\ \tfrac{5}{8},\ \tfrac{7}{8},\ \tfrac{1}{12}.$$

Example **1.** The total value of imports into this country through the Atlantic ports for the year 1906 was \$974,562,800; of this 75.35 % came through New York City. Find the value of the imports through this city.

SOLUTION. $\$974{,}562{,}8\cancel{00} \times \frac{75.35}{1\cancel{00}} = \$9{,}745{,}628 \times 75.35$
$= \$734{,}333{,}069.80$, value of imports through New York.

EXERCISE 115

1. Write the equivalent per cents of the following decimals: $.04, .08, .075, .0525, .1666\frac{2}{3}$.

2. Express as decimals the following per cents:
$4\frac{1}{2}\%$, 15%, $12\frac{1}{2}\%$, $62\frac{1}{2}\%$, $6\frac{1}{4}\%$, $3\frac{3}{5}\%$.

3. Find 5 % of each of the following numbers:
2151, 366.7, 689.5, 7.188, 12.469.

4. Find 6 % of each of the following numbers:
5262, 520.7, 2.66, 3.097, 6.41, .783.

5. Find $4\frac{1}{2}\%$ of each of the following numbers:
4150, 1418, 7120, 43.43, 53.17, 2.42.

6. The Engineer's Year Book for the year 1906 gives the cost of railway construction in England as $194,660 per mile. The per cents of cost were as follows:

Land	10	Permanent way	$11\frac{1}{2}$
Fencing	$1\frac{1}{2}$	Sidings	3
Earthquakes	24	Junctions	1
Tunnels	12	Stations	$6\frac{1}{2}$
Viaducts and bridges	17	Maintenance	$\frac{1}{2}$
Accommodation works	2	Legal and engineering expenses	6
Culverts	5		

Find the cost of each of the above items of expense.

7. The value of the total imports to the United States for the year 1906 was $1,226,560,000. Of this value 79.45 % came through the Atlantic ports, 4.42 % through the Gulf ports, 1.38 % through the Mexican border ports, 5.41 % through the Pacific ports, 7.97 % through the northern border ports, 1.37 % through the interior ports.

Find the value of the imports through each of these divisions.

8. The value of the total exports of the United States for the year ending June 30, 1906, was $1,743,860,000. The per cents of total value by principal customs districts were as follows:

New York	34.81	Savannah	3.72
Boston	5.66	Puget Sound	2.82
New Orleans	8.63	Detroit	2.02
Galveston	9.54	Buffalo Creek	1.72
		Mobile	1.25
Philadelphia	4.73	Newport News	1.15
Baltimore	6.31	Wilmington	1.06
San Francisco	2.29	Pensacola	1.06

Find the values of the exports through these cities.

Given a quantity, to find its value when decreased by a per cent of itself, subtract the given per cent from 100 per cent and multiply the given number by the difference.

Example 1. In the year 1906 the state of Ohio produced 11,562,500 lb. of wool; this shrunk 50 % from scouring. Find the number of pounds of scoured wool.

SOLUTION. $100\% - 50\% = 50 = \frac{1}{2}$.

Therefore, the weight of wool left after scouring is one half the weight of the wool before scouring.

$$11{,}562{,}500 \times \tfrac{1}{2} = 5{,}781{,}250.$$

Ans. 5,781,250 lb.

EXERCISE 116

1. The wool production and per cent of shrinkage from scouring for the year 1906, as given by the Bulletin of National Association of Wool Manufacturers, for the states named are as given in the following table.

Find the number of pounds of scoured wool produced in each of the states.

State	Number Pounds Unwashed	Per cent of Shrinkage
Michigan	9,450,000	50
Minnesota	2,450,000	52
Alabama	568,750	40
Montana	35,815,000	65
Wyoming	32,849,750	68
Idaho	16,905,000	67
Oregon	15,300,000	70
California	13,125,000	67
Utah	12,350,000	65
New Mexico	15,950,000	62
Colorado	9,450,000	67
Arizona	4,420,000	66
Texas	9,360,000	66
Washington	4,887,500	70

Given a per cent of a number, to find the number.

Example 1. During the month of January the average daily attendance of a school was 414. This number was 92% of the school enrollment. Find the number enrolled.

SOLUTION. 92% of enrollment is given.
100% of enrollment is sought.
1% of the enrollment $= \frac{414}{92}$.

$\therefore$ enrollment $= \frac{414}{92} \times 100 = 450.00$.

If x stands for enrollment,

$$.92x = 414, \text{ therefore } x = \frac{414}{.92} = 450.$$

Example 2. A dealer sells an article for \$522 at a gain of 16%. Find the cost price.

SOLUTION. 116% of cost price is given.
100% of cost price is sought.
1% of the cost price $= \$\frac{522}{116}$.

$\therefore$ cost price $= \$\frac{522}{116} \times 100 = \$450.00.$

$$\text{or } 1.16\,x = \$522.$$

$$\therefore x = \$522 \div 1.16 = \$450.$$

EXERCISE 117

1. Find the number of which 79 is 4%.

2. In a certain town 60% of the grown people are married. If there are 2394 married people, how many grown people are in the town?

3. A man spends $320 for board. This sum is 40% of his income. Find his income.

4. A man spends 83% of his salary and saves $170. What is his salary?

5. A lot is sold for $3380 at a gain of $12\frac{1}{2}$%. Find the cost of the lot.

6. After a discount of $16\frac{2}{3}$% is given, a man pays $84 for a bill of goods. Find the amount of the bill.

7. The following table gives the amounts of taxes levied in certain states in 1902, and the tax rate per cent of assessed valuation in each. Find the assessed valuation of property in each of these states.

State	Levy	Rate per Cent
Maine	$ 6,855,776	1.95
Pennsylvania	58,269,455	1.49
South Carolina	3,736,344	1.91
Kansas	14,847,136	4.09
Tennessee	7,626,068	1.88
Washington	9,002,727	3.45
Texas	13,683,526	1.34

To express one number as a percentage of another number.

Example 1. The foreign population of Danish extraction according to the United States Census of 1890 and 1900 was 132,543 and 153,805.

Find the increase per cent during the ten years.

SOLUTION. $153805 - 132543 = 21262$, increase.

$\frac{21262}{132543} =$ fraction the increase is of population in 1890.

$\frac{21262}{132543} \times 100\,\% = 16.04\,\%$, increase per cent.

Example 2. A dealer buys goods at a discount of 40 % off the list price, and sells them at 16 % off the list price. Find his gain per cent.

SOLUTION

The list price = 100 % of itself.

The discount is 40 % of the list price.

$100\,\% - 40\,\% = 60\,\%$ of the list price = cost.

The selling price is 16 % below the list price.

$100\,\% - 16\,\% = 84\,\%$ of list price = selling price.

Gain = selling price − cost.

$84\,\% - 60\,\% = 24\,\%$ of list price = gain.

Rate of gain = gain ÷ cost = $24\,\% \div 60\,\% = \frac{24}{60}$.

Rate per cent of gain = rate of gain, changed to per cent.

$\frac{24}{60}$, changed to per cent, equals 40 %.

Rate per cent of gain = 40 %.

EXERCISE 118

1. The railway mileage of the world January 1, 1906, as given by a German statistician, was as follows:

Country	Miles	Country	Miles
Europe	192,251	North America . . .	253,098
Asia	50,593	South America . . .	32,859
Africa	16,538	Australasia	17,441

Find the per cent of the total railway mileage in each of the six continents.

2. The foreign-born population of the United States by countries for the years 1890 and 1900 was as follows:

Country	1890	1900	Country	1890	1900
Austria . .	123,270	275,910	Germany .	2,785,000	2,663,000
England . .	909,090	840,513	Ireland . .	1,871,500	1,615,500
France . . .	113,174	104,197	Scotland . .	242,200	233,500

Find the rate per cent of increase or decrease.

3. A dealer buys goods at a discount of 40% off the list price, and sells them at 2% below the list price. What per cent of profit does he make?

4. Eggs are bought at the rate of 5 for 4¢, and sold at the rate of 4 for 5¢. What per cent of profit is made?

5. A house lot is sold for $1560 at a profit of $120. Find the rate per cent of profit.

6. Meat is sold at 18¢ per pound at a profit of 20%. Find the cost price per pound.

7. If the butcher has to pay 1¢ per pound more for the meat, how must he sell it to make a profit of 25%?

8. A piano is sold for $470 at a loss of 6%. What would the gain per cent have been if the piano had been sold for $520?

9. A tradesman buys at a discount of 10%, and sells at an advance of 15% on the nominal cost price. Find his rate per cent of profit.

10. A book costs the publisher 60¢ for printing and publishing. At what price should he sell the book in order that he may make a profit of 20%, after paying the author 10% on the selling price?

11. What should be the selling price of an article which costs $15, so that a profit of 20% may be made after giving the dealer a discount of 10%?

12. A tradesman marks his goods at 25% above cost, but allows the customer 6% discount. What per cent of profit does he make?

13. Tea is sold at 60¢ per pound at a profit of $33\frac{1}{3}$%. If the total gain is $15, how much tea is sold?

14. A man buys a house for $4000 which he rents for $40 per month; his taxes are 3% on a valuation of $3000. What per cent does his money yield?

15. A merchant marks his goods 20% above cost. What discount does he give if he sells at cost?

INTEREST

What is **interest?** How is it reckoned? What is the **principal? Amount? Rate?**

How many days are commonly taken for a month? For a year?

Example **1.** Find the interest on $670 at 5% from Jan. 14 to Aug. 10.

SOLUTION.

	MO.	DA.
Aug. 10 =	8	10
Jan. 14 =	1	14
	6	26

	\$670	
	.05	
6 mo. = $\frac{1}{2}$ of 1 yr.	2)\$33.50	= int. for 1 yr.
	16.75	= int. for 6 mo.
20 da. = $\frac{1}{9}$ of 6 mo.	1.861	= int. for 20 da.
5 da. = $\frac{1}{4}$ of 20 da.	.465	= int. for 5 da.
1 da. = $\frac{1}{5}$ of 5 da.	.093	= int. for 1 da.
	\$19.17	= int. for 6 mo. 26 da.

EXERCISE 119

Find the interest and amount of:

1. \$728 for 1 yr. 6 mo. at 5 %.
2. \$670 for 1 yr. 6 mo. at 7 %.
3. \$1260 for 1 yr. 3 mo. at 8 %.
4. \$385 for 1 yr. 4 mo. 12 da. at 7 %.
5. \$2750 for 1 yr. 8 mo. at 3 %.
6. \$3345 for 1 yr. 4 mo. at 6 %.
7. \$783 for 1 yr. 1 mo. 10 da. at 4 %.
8. \$597 for 1 yr. 4 mo. 24 da. at 5 %.
9. \$3000 for 3 mo. 6 da. at 7 %.
10. \$940 for 1 yr. 4 mo. at 3 %.
11. \$1800 for 1 mo. 15 da. at 4 %.
12. \$2100 for 2 yr. 9 mo. at 4 %.
13. \$960 for 8 mo. 17 da. at 7 %.
14. \$2911.25 for 1 yr. 7 mo. 16 da. at 4 %.
15. \$1857 for 1 yr. 5 mo. 18 da. at 5 %.
16. \$2775 from May 1 to Dec. 19 at 4 %.
17. \$1770 from Jan. 10 to Oct. 5 at $4\frac{1}{2}$ %.

18. $1975.14 from Feb. 8 to Nov. 4 at $3\frac{1}{2}\%$.

19. $1218 from March 6 to Nov. 1 at $5\frac{1}{2}\%$.

20. $1788 from Feb. 14 to Dec. 20 at $7\frac{1}{2}\%$.

EXACT INTEREST

Interest reckoned on the basis of 365 days to the year is called **exact interest.** Exact interest is used by the United States Government and sometimes in business transactions.

Example. Find the exact interest on $2384.50 from Jan. 12 to July 5 at 5%.

SOLUTION. From Jan. 12 to July 5 there are $(19 + 28 + 31 + 30 + 31 + 30 + 5)$ days $= 174$ days.

$$\$2384.50 \times .05 \times \tfrac{174}{365} = \text{exact interest.}$$

$$\frac{\$2384.50 \times .05 \times 174}{365} = \$56.84, \text{ nearly.}$$

EXERCISE 120

Find the exact interest on:

1. $913 from Jan. 4 to Feb. 4 at 5%.
2. $731.11 from Jan. 14 to Jan. 28 at 7%.
3. $52.50 from Jan. 1 to April 28 at 7%.
4. $2745 from Feb. 1 to April 6 at 5%.
5. $1095.80 from March 6 to June 7 at 5%.
6. $1911.17 from March 1 to May 11 at 7%.
7. $1464.98 from Jan. 4 to May 30 at 6%.
8. $10,565.65 from May 13 to June 25 at 4%.
9. $834 from Feb. 5 to July 12 at 11%.
10. $3561.50 for 81 da. at 5%.

INVERSE QUESTIONS IN INTEREST

Example. What principal will produce \$78.75 interest in 75 days at $7\frac{1}{2}\%$?

\$100 produces \$7.50 in 1 yr., or 360 days.
In 75 days \$100 produces $\frac{75}{360}$ of \$7.50.
In 75 days \$1 produces $\frac{1}{100}$ of $\frac{75}{360}$ of \$7.50.

It will take as many dollars to produce \$78.75 in 75 days as the sum produced by \$1 in 75 days is contained times in \$78.75.

Therefore, \$78.75 is to be divided by $(\frac{1}{100}$ of $\frac{75}{360} \times$ \$7.50).

$$78.75 \div \left(\frac{1}{100} \times \frac{75}{360} \times 7.50\right) = \overset{10.5}{\cancel{78.75}} \times \overset{4}{\cancel{100}} \times \frac{\overset{120}{\cancel{360}}}{\underset{\cancel{3}}{\cancel{75}}} \times \frac{1}{\cancel{7.50}} =$$

$$10.5 \times 4 \times 120 = 5040.$$

\$5040 will produce \$78.75 in 75 days at $7\frac{1}{2}\%$.

EXERCISE 121

What principal will produce:

1. \$60 in $1\frac{1}{2}$ yr. at 8 %?
2. \$120 in 2 yr. at 5 %?
3. \$135 in 1 yr. 6 mo. at 9 %?
4. \$36 in 3 yr. at 5 %?
5. \$144 in 1 yr. 4 mo. at $4\frac{1}{2}\%$?
6. \$12 in 1 yr. at 4 %?
7. \$21 in 1 yr. at $3\frac{1}{2}\%$?
8. \$84 in 3 yr. 6 mo. at 3 %?
9. \$16.90 in 2 yr. 2 mo. at 4 %?
10. \$42 in 2 yr. 4 mo. at 4 %?

11. $25.50 in 6 mo. at 5 % ?

12. $5.40 in 4 mo. at 5 % ?

13. $6.75 in 9 mo. at 3 % ?

Example. In what time will $840 produce $57.40 interest at 5 % ?

$100 produces $5 int. in 1 yr.
$840 produces $42 in 1 yr.

It will take $840 as many years to produce $57.40 as the $42 which it produces in one year is contained times in $57.40.

$57.40 \div 42 = 1\frac{11}{30} = 1$ yr. 4 mo. 12 da.

$\therefore$ $840 produces $57.40 in 1 yr. 4 mo. 12 da.

EXERCISE 122

In what time will :

1. $1088.75 produce $87.10 interest at 8 % ?
2. $144 produce $21.60 interest at 5 % ?
3. $215 produce $6.45 interest at 5 % ?
4. $1160 produce $278.40 interest at 6 % ?
5. $810 produce $56.70 interest at 7 % ?
6. $312.50 produce $43.75 interest at 8 % ?
7. $2220 produce $216.45 interest at 3 % ?
8. $1400 produce $78.75 interest at 5 % ?
9. $480 produce $85.50 interest at $9\frac{1}{2}$ % ?
10. $3835 produce $345.15 interest at 8 % ?
11. $1380 produce $88.55 interest at $3\frac{1}{2}$ % ?
12. $5400 produce $267.75 interest at 7 % ?
13. $7630 produce $1335.25 interest at 6 % ?

Example. At what rate per cent will \$720 produce \$42.50 interest in 1 yr. 2 mo. 5 da.?

Changing 1 yr. 2 mo. 5 da. to years gives $\frac{85}{72}$ yr.

\$720 produces \$42.50 in $\frac{85}{72}$ yr.

In one year at the same rate \$720 would produce \$42.50 $\div \frac{85}{72} = 42.50 \times \frac{72}{85}$.

The interest on \$720 for 1 yr. at 1 % is \$7.20.

The interest for 1 yr. at the given rate is as many times the interest at 1 % as

$$42.50 \times \tfrac{72}{85} \div 7.20 = 42.50 \times \tfrac{72}{85} \times \frac{1}{7.20} = 5$$

$\therefore$ the rate is $5 \times 1\ \% = 5\ \%$.

EXERCISE 123

Find the rate per cent when the interest on —

1. \$750 for 1 yr. is \$45.
2. \$928 for 1 yr. is \$64.96.
3. \$880 for $1\frac{1}{2}$ yr. is \$79.20.
4. \$945 for 6 mo. is \$37.80.
5. \$828 for 8 mo. is \$38.64.
6. \$1200 for 1 yr. 3 mo. is \$90.
7. \$1800 for 9 mo. is \$67.50.
8. \$2400 for 8 mo. is \$64.
9. \$2500 for 9 mo. 18 da. is \$100.
10. \$3000 for 7 mo. 12 da. is \$90.
11. \$3750 for 4 mo. 15 da. is \$112.50.
12. \$2754 for 2 mo. 20 da. is \$55.08.
13. \$4846 for 6 mo. 20 da. is \$121.15.
14. \$1440 for 7 mo. 10 da. is \$52.80.

Find the rate per cent when —

15. \$1080 amounts to \$1123.20 in 8 mo.

16. \$1200 amounts to \$1270 in 10 mo.

17. \$1600 amounts to \$1640 in 6 mo.

18. \$2460 amounts to \$2574.80 in 9 mo. 10 da.

19. \$92 amounts to \$102.12 in 2 yr.

20. \$324 amounts to \$333.72 in 8 mo.

Example. What principal will amount to \$136.27 in 1 yr. 3 mo. 15 da. at 5%?

The interest on \$1 for 1 yr. 3 mo. 15 da. is $\$.0645\frac{5}{6}$.
∴ the amount of \$1 for 1 yr. 3 mo. 15 da. is $\$1.0645\frac{5}{6}$.
∴ the number of dollars in principal $= \$136.27 \div \$1.0645\frac{5}{6} = \$128$, nearly.

EXERCISE 124

What principal will amount to —

1. \$840 in 1 yr. at 5%?
2. \$749 in 1 yr. at 7%?
3. \$645 in 1 yr. at $7\frac{1}{2}$%?
4. \$903 in $1\frac{1}{2}$ yr. at 5%?
5. \$414 in 3 yr. at 5%?
6. \$255.30 in 2 yr. at $5\frac{1}{2}$%?
7. \$12,540.45 in 2 yr. 3 mo. at 4%?
8. \$168.35 in 7 mo. 6 da. at 6%?
9. \$618.67 in 1 yr. 4 mo. at 5%?
10. \$646.80 in 8 mo. at 4%?
11. \$776.07 in 1 yr. 1 mo. at $4\frac{1}{2}$%?
12. \$481.50 in 1 yr. at 7%?
13. \$432.55 in 11 mo. at 8%?
14. \$282.75 in 2 yr. 4 mo. 15 da. at $7\frac{1}{2}$%?

15. $2090.07 in 1 yr. 1 mo. 15 da. at 8%?

16. $2067.75 in 1 yr. 5 mo. at $10\frac{1}{2}$%?

17. $268.28 in 1 yr. 7 mo. at 6%?

18. $254.25 in 7 mo. 15 da. at $9\frac{1}{2}$%?

19. $25,346.25 in 1 yr. 3 mo. at 10%?

20. $843.70 in 8 mo. 3 da. at $7\frac{1}{2}$%?

EXERCISE 125

REVIEW

1. Find the interest on $4000 for 13 mo. 2 da. at 9%.

2. Find the interest on $256.30 for 4 mo. 9 da. at 7%.

3. Find the interest on $30.85 for 11 mo. 6 da. at 5%.

4. Find the interest on $653 for 2 mo. 16 da. at 4%.

5. Find the interest on $2105.60 for 84 da. at 5%.

6. Find the amount of $805 for 10 mo. at 8%.

7. Find the amount of $507 for 1 yr. 12 da. at 8%.

8. What principal will produce $20.83 interest in 5 mo. at 5%?

9. What principal will produce $17.50 interest in 9 da. at 5%?

10. Find the rate of interest when $500 produces $2.92 interest in 1 mo.

11. Find the rate of interest when $250 produces $7.30 in 5 mo.

12. How much must I invest at 5% interest to have an annual income of $1200 from my investment?

13. A man buys a house and lot and rents it for $40 a month. Taxes and insurance cost him $120 a year. If

his net receipts gives him a profit of 6 % on his investment, find the cost of the house and lot.

14. For how long a time must \$ 3000 be loaned at 5 % to produce \$ 20 interest?

15. For how long a time must \$ 2400 be loaned at 7 % in order to have principal and interest amount to \$ 2456?

16. For how long a time must a sum of money be loaned at simple interest at 8 % to produce in interest $\frac{1}{5}$ of itself?

17. For how long a time must a sum of money be loaned at simple interest at 6 % to produce in interest $\frac{1}{10}$ of itself?

18. Find the exact interest of \$ 1200 for 292 da. at 5 %.

19. Find the exact interest of \$ 7300 for 146 da. at 7 %.

20. The exact interest of \$ 10,800 at 5 % is \$ 324. Find the time.

21. In what time will \$ 260 amount to \$ 262.60 at 5 %?

22. What sum must be deposited in a savings bank which pays $3\frac{1}{2}$ % interest to produce semiannually \$ 8.75?

23. A man deposits his money in two banks. In one bank he has \$ 572 which pays $3\frac{1}{2}$ %. The other bank gives 4 % interest. If he receives as interest the same amount from both banks, how much money has he all together?

24. If I invest half my money at 6 % and the remainder at 4 %, and derive an income of \$ 650 annually, how much money have I invested?

REVIEW QUESTIONS

1. Define *principal*, *rate*, *per cent*, *interest*, *amount.*

2. How does exact interest differ from interest according to the common use of the term?

3. How do you find the interest of a sum of money at a given rate and for a given time?

4. If you were given the interest, the rate, and the time, how would you find the principal?

5. Given the principal, interest, and time, how would you find the rate?

6. Given the principal, the rate, and the interest, how would you find the time?

7. Given the principal, the amount and the rate, how would you find the time?

8. Given the principal, the amount, and the time, how would you find the rate?

9. Given the amount, the rate, and the time, how would you find the principal?

10. If you knew the interest of a sum of money for a given time at 6 %, how would you find the interest for the same sum for the same time at 5 %? at 4 %? at 8 %?

11. If you knew the interest at 4 %, how would you find the interest of the same sum at 7 %? at 3 %? at 5 %? at $3\frac{1}{2}$ %?

12. If you were given the interest of a sum of money for a number of days, how would you determine from this the exact interest of the same sum for the same number of days?

PROMISSORY NOTES

A written promise by one person to pay another person a sum of money on demand, or after a specified time, is called a **promissory note**.

The following are promissory notes written in standard form:

Galveston, Texas, March 1, 1912.	*No.* ....
Sixty days after date without grace, for value received, I promise to	*Due* $380.75

pay to the order of The City National Bank of Galveston, at its office in Galveston, Texas, Three hundred and eighty .... $\frac{75}{100}$ *Dollars, for value received, negotiable and payable without defalcation or discount, with interest at the rate of ten per cent per annum from maturity until paid and ten per cent additional on amount of principal and interest unpaid for attorney's fees, if placed in the hands of an attorney for collection.*

Demand, suit, protest and notice thereof waived.

John Mosley

Post Office address ....

Dallas, Texas, March 4, 1907.

$474.25

On demand I promise to pay to the order of

.... *John Anderson* ....

.... *Four Hundred Seventy-four* ... $\frac{25}{100}$ *Dollars*

at .... *the First National Bank* ....

with interest at 7%.

Value received. *James Rowe.*

No. 34. *Due* ....

The first of the above promissory notes is called a **time note**; the second is called a **demand note.**

The person who promises to pay is called the **maker.** John Mosley is the maker of the first note above.

The person to whom the money is to be paid is called the **payee.** The person who has legal possession of a note is called its **holder.**

The sum specified in a note is called its **face.** A time note is legally due on the date indicated. In some states 3 days more than are indicated in the note are allowed before the note is legally due. These days are called **days of grace.** The day on which a note is legally due is called the **day of maturity.**

A note made payable to the order of a person, or a note made payable to the bearer is **negotiable,** *i.e.* it may be transferred from one person to another person.

A note made payable to the payee only is **non-negotiable.**

When a note payable to the order of the payee is transferred, every holder before parting with it must *indorse* it, *i.e.* write his name on the back of it. Every *indorser* thus becomes liable for the payment of the note, if the maker fails to pay it. The holder in whose possession the note is at maturity presents it to the maker for payment. If the maker refuses to pay it, the holder engages a Notary Public to give to the indorser, or indorsers, a written notice of its non-payment. This notice is called a **protest.**

A protest must be sent within 60 days after the note is due, or before the next term of court, unless the note contains the words: "Demand, suit, protest and notice thereof waived"; otherwise the indorsers are not held responsible for the payment of the note.

An indorser who writes over his signature the words

without recourse is not held responsible for the payment of the note.

A note made payable to *the bearer* is negotiable without indorsement. In some states a note must contain the words *value received* in order to be legal.

If the words *with interest* are not in a note, no interest is charged. If, however, the note is not paid on the date of maturity, interest at the legal rate may be charged. If a note contains the words *with interest*, and no rate is specified, it is then understood that the note bears the rate of interest usually charged in the state where it is made.

When the time of payment is indicated in months, calendar months are understood.

A note drawn March 6, and payable two months after date, matures on May 6 in states where days of grace are not allowed, and on May 9 in states where days of grace are allowed. About one half of the states and territories allow 3 days of grace.

BANK DISCOUNT

New Orleans, La., Feb. 14, 1903.

$ 350.00.

Sixty days after date I promise to pay to the order of ---------------*Joseph Coan*-----------------

Three hundred fifty ---------------------------- $\frac{00}{100}$ *Dollars*

Value received.

Alonzo Ryan.

No. 33. Due April 15/18, 1903.

The above time note is negotiable when indorsed Supposing the payee, Joseph Coan, needs money, he can

sell the note to a bank. The sum the bank gives him for the note is called the **proceeds** of the note. The difference between the proceeds and the face of the note is called the **bank discount.**

The **bank discount** is always a rate per cent of the value of the note on its day of maturity, reckoned from the date of the sale of the note to the day of maturity.

The **bank discount** is then the interest on the maturity value of the note computed from the date of discount to the date of maturity. This time is called the **term of discount.**

The maturity value of the note minus the **bank discount** is the **proceeds** of the note.

The essential features of a note are the **face, maturity value, date of drawing, date of sale,** or **date of discount, rate of interest** the note bears, **rate of interest charged,** known as **rate of discount,** and **date of maturity.**

BANKERS' INTEREST

Banks charge interest for the exact number of days between dates, allowing 30 days to a month. Banks usually draw notes for 30, 60, or 90 days.

Example. Find the discount and the proceeds of the note (page 250), if it was discounted at 8 %, March 1, 1903.

SOLUTION. The bank charges discount for the 48 days from March 1 to April 18.

$$
\begin{array}{lrl}
 & \$350. & = \text{maturity value.} \\
 & .08 & \\
\hline
 & 28.00 & = \text{int. for 1 yr.} \\
45 \text{ da.} = \frac{1}{8} \text{ of 1 yr.} & 3.50 & = \text{int. for 45 da.} \\
3 \text{ da.} = \frac{1}{15} \text{ of 45 da.} & .23 & = \text{int. for 3 da.} \\
\hline
\text{bank discount} = & \$3.73 & = \text{int. for 48 da.}
\end{array}
$$

$$\$350 - \$3.73 = \$346.27 = \text{proceeds of the note.}$$

EXERCISE 126

Find the bank discount and the proceeds of the following indicated notes, allowing 3 days of grace in examples 1, 2, 10, 11, 12, 13, and no grace in the remaining: —

	DATE	TIME	FACE	DIS-C'TED	RATE OF DISC'T
1.	Jan. 12,	60 da.,	$600,	Feb. 13,	8 %.
2.	July 4,	60 da.,	$800,	Aug. 3,	8 %.
3.	Mar. 3,	90 da.,	$500,	Mar. 3,	6 %.
4.	April 5,	60 da.,	$700,	April 5,	6 %.
5.	May 7,	60 da.,	$600,	May 10,	10 %.
6.	May 9,	20 da.,	$900,	May 24,	8 %.
7.	May 30,	90 da.,	$750,	July 1,	6 %.
8.	June 5,	60 da.,	$450,	July 5,	6 %.
9.	Aug. 11,	30 da.,	$800,	Aug. 11,	6 %.
10.	Sept. 9,	30 da.,	$350,	Sept. 12,	10 %.
11.	Oct. 4,	60 da.,	$800,	Oct. 7,	10 %.
12.	Oct. 14,	60 da.,	$500,	Nov. 16,	12 %.
13.	Nov. 10,	90 da.,	$600,	Nov. 25,	8 %.
14.	Dec. 4,	90 da.,	$750,	Feb. 5,	6 %.
15.	Jan. 10,	45 da.,	$650,	Feb. 9,	7 %.
16.	Jan. 5,	60 da.,	$850,	Feb. 5,	9 %.
17.	Jan. 30,	75 da.,	$950,	Feb. 28,	9 %.
18.	July 10,	3 mo.,	$380,	July 12,	9 %.

COMPUTING DISCOUNT ON INTEREST-BEARING NOTES

Find the bank discount and the proceeds of a 90-day note for $250, dated Portland, Me., June 9, 1907, bearing interest at 6 %, and discounted July 8, 1907, at 8 %.

Step **1.** Find the maturity value of the note. Maine allows 3 days of grace. Hence, the interest will be computed for 93 days.

$$\begin{array}{r l} \$\,250 & \\ 3.87 & = \text{interest at } 6\,\% \text{ for 93 days.} \\ \hline \$\,253.87 & = \text{maturity value of the note.} \end{array}$$

Step **2.** Find the term of discount (exact number of days from July 8 to Sept. 10, the date of maturity).

Step **3.** Find the bank discount. This is reckoned on the maturity value of the note.

Int. on \$253.87 for 64 days at 8 % = \$3.61.

\$253.87 − \$3.61 = \$250.26, proceeds of note.

EXERCISE 127

Find the discount and the proceeds of the following indicated notes, allowing 3 days of grace in examples 2, 3, 6, 9, 10, 13, and no grace in the remaining: —

	Face	Date	Time	Rate of Int.	Rate of Disc't.	Disc't.
1.	\$350,	Jan. 1,	45 da.,	10 %,	12 %,	Feb. 1.
2.	\$395,	Jan. 10,	3 mo.,	6 %,	10 %,	Jan. 20.
3.	\$450,	Feb. 1,	30 da.,	6 %,	8 %,	Feb. 18.
4.	\$600,	Mar. 1,	60 da.,	8 %,	12 %,	Mar. 31.
5.	\$500,	Apr. 2,	60 da.,	6 %,	8 %,	Apr. 17.
6.	\$900,	Apr. 10,	30 da.,	6 %,	8 %,	Apr. 10.
7.	\$1000,	Apr. 15,	60 da.,	8 %,	10 %,	Apr. 15.
8.	\$750,	May 4,	60 da.,	6 %,	6 %,	May 5.
9.	\$800,	July 10,	30 da.,	6 %,	8 %,	July 26.
10.	\$400,	Aug. 15,	60 da.,	6 %,	6 %,	Aug. 15.
11.	\$850,	Nov. 11,	90 da.,	7 %,	10 %,	Nov. 12.
12.	\$900,	Dec. 12,	45 da.,	6 %,	6 %,	Jan. 13.
13.	\$1200,	Dec. 5,	3 mo.,	6 %,	10 %,	Jan. 15.

PARTIAL PAYMENTS

When payments of parts of the face are made on a note, from time to time, these payments are known as **partial payments.** These payments and their dates of payment are written on the back of the note.

There are several methods of computing the amounts due on such notes. The best known and most widely used are the *United States Rule* and the *Merchants' Rule.*

UNITED STATES RULE

Find the amount of the Principal until the time of the first payment, or until the sum of two or more payments equals or exceeds the interest.

Subtract the payment, or the sum of the two or more payments, from the amount.

Proceed with the remainder as a new principal. Continue in this manner until the date of settlement.

A note for $600.00 dated Aug. 10, 1902, was indorsed as follows :

1902, Dec. 15, $100.
1903, Feb. 12, $150.
1903, March 15, $150.

Find the amount due April 1, 1903.

SOLUTION.

DATES OF INDORSEMENT			TIME	VEEN DATES		PAYMENTS
yr.	mo.	da.	yr.	mo.	da.	
1902	8	10		4	5	$100
1902	12	15		1	27	$150
1903	2	12		1	3	$150
1903	3	15			16	
1903	4	1				

$\$600 \quad$ = first principal. $\quad \$600 \times .06 \times \frac{4\frac{1}{6}}{12} = \$12.50.$
12.50 = int. for 4 mo. 5 da.
$\$612.50$ = amt. for 4 mo. 5 da.
$\$100.$ = first payment.
$\$512.50$ = second principal. $\$512.50 \times .06 \times \frac{1.9}{12} = \$4.87.$
4.87 = int. for 1 mo. 27 da.
$\$517.37$ = amt. for 1 mo. 27 da.
$150.$ = second payment.
$\$367.37$ = third principal. $\$367.37 \times .06 \times \frac{1.1}{12} = \$2.02.$
2.02 = int. for 1 mo. 3 da.
$\$369.39$ = amt. for 1 mo. 3 da.
$150.$ = third payment.
$\$219.39$ = fourth principal. $\$219.39 \times .06 \times \frac{16}{360} = \$.59.$
$.59$ = int. for 16 da.
$\$219.98$ = amt. for 16 da. $\$219.98$. *Ans.*

NOTE. Problems of this type are not as common in business as they were some years ago. Payments are now usually made at equal intervals of time.

EXERCISE 128

1. A man borrows from a loan association $3000 at 8% interest. If he pays $100 at the end of every 3 mo. for 1 yr., find the amount due at the end of the year.

2. A man borrows from a building and loan company $2000 at 8% interest. He pays in monthly installments of $50 each. How much does he owe at the end of 6 mo.?

3. If you borrow $1800 at 6% interest and pay annually $600, how much do you owe at the end of 3 yr.?

4. If you borrow $1500 at 8% interest and pay semi-annually $300, how much do you owe at the end of 2 yr.?

5. If $2400 is borrowed at 8% interest, and paid in quarterly installments of $150 each, how much is due at the end of two years?

6. A man borrows $3000 at 8% interest, and pays in semiannual installments of $500 each. How much is due at the end of three years?

7. A man borrows $1800 at 6%, and pays $400 per year. How much is due at the end of five years?

8. If I borrow $2500 at 5% interest, and pay $500 a year, how much do I owe at the end of five years?

9. How much is due at the end of two years on a loan of $2500 at 5% interest when paid in semiannual installments of $600 each?

10. A note dated Jan. 15, 1904, for $4000 had the following indorsements: July 10, 1904, $700; Dec. 24, 1904, $600; June 18, 1905, $800; Nov. 13, 1905, $500; March 17, 1906, $900; Aug. 10, 1906, $400; Feb. 12, 1907, $400. How much was due May 28, 1907, at 6%?

THE MERCHANTS' RULE

The Merchants' Rule for computing interest on partial payments is used when settlement is made within one year from the date of the note. The rule is as follows:

I. Compute the amount of the face of the note until the date of settlement.

II. Compute the amount of each indorsement from its date until the date of settlement, and add these amounts.

III. Subtract the sum of the amounts of the indorsements from the amount of the face of the note. The remainder will be the balance due on the date of settlement.

Example. A note for \$500, with interest at 8%, dated Jan. 10, 1902, had the following indorsements: April 1, 1902, \$100; May 10, 1902, \$200; July 1, 1902, \$100. Find the balance due Nov. 1, 1902.

SOLUTION. From Jan. 10 to Nov. 1 is 9 mo. 21 da.
From April 1 to Nov. 1 is 7 mo.
From May 10 to Nov. 1 is 5 mo. 21 da.
From July 1 to Nov. 1 is 4 mo.

\$500 for 9 mo. 21 da. amounts to		\$532.33
\$100 for 7 mo. amounts to	\$104.67	
\$200 for 5 mo. 21 da. amounts to	207.60	
\$100 for 4 mo. amounts to	102.67	
The payments amount to		414.94
Balance due Nov. 1,		\$117.39

EXERCISE 129

1. A note for \$500, with interest at 6%, dated Jan. 2, 1902, had the following indorsements: March 2, 1902, \$100; May 5, 1902, \$150; July 10, 1902, \$200. Find the balance due Nov. 1, 1902.

2. A note for \$600, with interest at 8%, dated Feb. 1, 1902, was indorsed as follows: March 11, 1902, \$200; May 15, 1902, \$100; July 3, 1902, \$100. Find the balance due Aug. 1, 1902.

3. A note for \$800, with interest at 8%, dated Feb. 10, 1902, was indorsed as follows: March 15, 1902, \$200; April 10, 1902, \$100; June 3, 1902, \$200. Find the amount due Nov. 1, 1902.

4. A borrows \$1500 on Jan. 1, 1907, at 6% interest, and pays \$450 each quarter. How much does he owe when three payments are made?

ANNUAL INTEREST

> *Lincoln, Neb., Jan. 1, 1899.*
>
> *$800.00*
>
> *On demand I promise to pay to* ....
> ....*Jeremiah Hill*................*or order*
> *Eight hundred*........................ $\frac{00}{100}$ *Dollars,*
> *with interest annually at 8 %.*
>
> *Value received.* *William Harris.*
>
> *No. 92.* *Due*........

In some states if a note contains the words "with interest annually," and if the interest remains unpaid for a number of years, then the interest due at the end of each year bears simple interest until the date of settlement.

The *interest* by this method of reckoning is called **annual interest.** The interest we have up to this time considered is known as **simple interest.**

Example. How much is due on a note for $800, dated Jan. 5, 1898, and bearing interest annually at 8 %, if left unpaid, both in principal and interest, until March 10, 1903?

SOLUTION

Interest on $800 for 1 yr. at 8 % = $64.
Interest on $800 for 5 yr. 2 mo. 5 da. at 8 % = $331.56.

YR.	MO.	DA.
1903	3	10
1898	1	5
5	2	5

The first year's interest is due Jan. 5, 1899, and bears interest until March 10, 1903. The interest falling due

at the end of the second year bears interest until March 10, 1903, and so on.

$64 bears interest for	4 yr.	2 mo.	5 da.
$64 bears interest for	3 yr.	2 mo.	5 da.
$64 bears interest for	2 yr.	2 mo.	5 da.
$64 bears interest for	1 yr.	2 mo.	5 da.
$64 bears interest for		2 mo.	5 da.
Adding, $64 bears interest for	10 yr.	10 mo.	25 da.

Interest on $64 for 10 yr. 10 mo. 25 da. at 8 % = $55.82.
Interest on $800 for 5 yr. 2 mo. 5 da. at 8 % = $331.56.
Amount due = $800 + $331.56 + $55.82 = $1187.38.

EXERCISE 130

1. Find the amount due April 1, 1903, on a note for), dated July 15, 1899, and bearing interest annually at 7 %.

2. Find interest from March 6, 1898, to Jan 1, 1903, on note for $1400, interest payable annually at 8 %.

3. Find amount due Jan. 1, 1903, on a note for $1200, dated July 1, 1897, interest payable annually at 6 %.

4. Find amount due Jan. 1, 1903, on a note for $1000, dated Jan. 1, 1898, interest payable semiannually at 5 %.

5. Find the amount due at the end of 5 years on a coupon note, interest payable semiannually, if the face of the note is $700, and the rate of interest is 7 %.

COMPOUND INTEREST

Savings banks and other banks that give interest on deposits add the interest semiannually or annually to the amount deposited. This interest added bears interest

until the next date of balancing the depositor's account book. The next interest is added in the same way.

Suppose a person deposited in a savings bank \$250 and allowed it to remain there for 10 yr. The question might be asked, how much will principal and interest amount to at the end of that time? The accruing interest in this case is called **compound interest.**

Find the amount of \$500 for 2 yr. 6 mo. at 6 %, interest compounded annually; also find the compound interest.

SOLUTION

Principal	\$500
Interest for 1 yr. at 6 %	30
Principal at beginning of 2d yr.	\$530
Interest for 2d yr. at 6 %	31.80
Principal at beginning of 3d yr.	\$561.80
Interest for 6 mo. at 6 %	16.85
Amount for 2 yr. 6 mo. at 6 %	\$578.65

Since the amount of \$500 is \$578.65, the compound interest is \$578.65 – \$500.00, or \$78.65.

NOTE. Unless otherwise stated, interest is understood to be computed annually. If compounded semiannually, the rate must be considered one half the annual rate mentioned; if quarterly, one fourth, etc.

EXERCISE 131

Find the amount and the compound interest of:

1. \$800 for 2 yr. at 5 %.
2. \$2000 for 3 yr. at 4 %.
3. \$3000 for 4 yr. 6 mo. at 6 %.
4. \$5000 for 2 yr. at 6 %, payable quarterly.
5. \$10,000 for 2 yr. at 6 %, payable quarterly.

6. \$16,000 for 1 yr. 8 mo. at 4 %, payable quarterly.

7. Find the amount of \$2000 for 10 yr. at 4 % compound interest; at 4 % compounded semiannually.

The following table gives the amount of \$1 at compound interest:

Yr.	2 %	2½ %	3 %	4 %	5 %	6 %
1	1.020000	1.025000	1.030000	1.040000	1.050000	1.060000
2	1.040400	1.050625	1.060900	1.081600	1.102500	1.123600
3	1.061208	1.076891	1.092727	1.124864	1.157625	1.191016
4	1.082432	1.103813	1.125509	1.169859	1.215506	1.262477
5	1.104081	1.131408	1.159274	1.216653	1.276282	1.338226
6	1.126162	1.159693	1.194052	1.265319	1.340096	1.418519
7	1.148686	1.188686	1.229874	1.315932	1.407100	1.503630
8	1.171659	1.218403	1.266770	1.368569	1.477455	1.593848
9	1.195093	1.248863	1.304773	1.423312	1.551328	1.689479
10	1.218994	1.280085	1.343916	1.480244	1.628895	1.790848
11	1.243374	1.312087	1.384234	1.539454	1.710339	1.898299
12	1.268242	1.344889	1.425761	1.601032	1.795856	2.012196
13	1.293607	1.378511	1.468534	1.665073	1.885649	2.132928
14	1.319479	1.412974	1.512590	1.731676	1.979932	2.260904
15	1.345868	1.448298	1.557967	1.800943	2.078928	2.396558
16	1.372786	1.484506	1.604706	1.872981	2.182875	2.540352
17	1.400241	1.521618	1.652847	1.947900	2.292018	2.692773
18	1.428246	1.559659	1.702433	2.025817	2.406619	2.854339
19	1.456811	1.598650	1.753506	2.106849	2.526950	3.025599
20	1.485947	1.638616	1.806111	2.191123	2.653298	3.207136

EXERCISE 132

With the aid of the above table, find the amount at compound interest of:

1. \$2000 for 8 yr. at 4 %.
2. \$3000 for 10 yr. at 3 %.
3. \$5000 for 12 yr. at 4 %.
4. \$6000 for 10 yr. at 5 %.

5. \$8000 for 8 yr. at 5%, interest compounded semi-annually.

6. \$2250 for 6 yr. at 4%, interest compounded semi-annually.

7. \$10,000 for 10 yr. at 2%, interest compounded semi-annually.

EXCHANGE

The written order of one party to another, directing the payment of a specified sum of money, is called a **draft.**

The parties to a draft are **drawer**, **drawee**, and **payee.**

SIGHT DRAFT

New Orleans, La., March 10, 1903.

\$600.00

At sight, pay to the order of ----------------

---------------------*Jacob Siegel*------------ *the sum of*

-----*Six hundred*------------------------------$\frac{00}{100}$ *Dollars.*

And charge to the account of

Value received *William Thompson.*

To David Meyer,

New York, N. Y.

A time draft payable *after sight* matures as a promissory note.

Bank drafts form one of the most important mediums of exchange. If a merchant owes money in New York, Chicago, or in any other place, he can always purchase a draft from his home bank, and by transmitting this through the mails settle his indebtedness.

TIME DRAFT

Louisville, Ky., Aug. 10, 1902.

$1500.00

Thirty days after date pay to the order of

------------------------------Self------------------------------

------------Fifteen hundred------------------ $\frac{00}{100}$ Dollars.

And charge to the account of

Value received. Richard Weaver.

To Henry Morley,

No. 8. Minneapolis, Minn. Sept. 9/12.

BANK DRAFT

Galveston, Texas, March 10, 1903.

No. 72.

Hutchings, Sealy & Co , Bankers,

Pay to the order of -------- Frances Newman --------

$300.00 -----------(Three hundred)----------- $\frac{00}{100}$ Dollars.

H. O. Stein, Cashier.

To Seventh National Bank,

New York.

Banks usually charge a small sum for making such drafts, ten cents for small amounts and fifteen cents per hundred dollars.

In case of a draft for a large amount of money, such as several thousand dollars, the charge for the draft is reckoned as a fraction of one per cent of the face of the draft.

This charge is largely affected by the amount of money in the part of the country to which the draft is to be sent, the demand for money at the place where the draft is made, the cost of shipping the actual money, and other conditions. Owing to the effect of such conditions it is sometimes possible to buy a draft for less than its face value. When exchange costs more than its face value, it is said to be at a *premium;* when it costs less than its face value, it is said to be at a *discount.* Exchange is *at par* when the cost of a draft is its face value. Exchange is always quoted as a rate per cent, or as so many dollars on a \$1000. The exchange is always reckoned on the face value of the draft. Thus $\frac{1}{8}$% premium, or \$1.25 premium, means that the cost of a draft for \$100 is \$100$\frac{1}{8}$, and the cost of a draft for \$1000 is \$1001.25. The rate of exchange is generally a fraction of 1%.

Other forms of exchange are Express Money Orders, Postal Money Orders, Bank Checks, Travelers' Checks.

EXERCISE 133

1. What is the cost of exchange on a sight draft for \$400 at $\frac{1}{8}$% premium?

2. What is the cost of exchange on a sight draft for \$300 at $\frac{1}{4}$% premium?

3. Find the cost of a draft on New York for \$200 at $\frac{1}{8}$% premium.

4. Find the cost of a demand draft for \$500 at $\frac{1}{4}$% premium.

5. Find the cost of a sight draft for \$600 at $\frac{1}{4}$% premium.

6. Find how much must be paid for a sight draft for \$1000 at \$1.25 premium.

7. Find how much must be paid for a sight draft for \$2000 at \$2.50 premium.

8. Find the cost of remitting \$800 from Dallas to Chicago by means of a bank draft when exchange is $\frac{1}{4}\%$ premium.

9. Find the cost of a sight draft on Chicago for \$700, exchange being at $\frac{1}{4}\%$ discount.

10. Find the cost of a sight draft on New York for \$400 when exchange is $\frac{1}{4}\%$ discount.

11. Find the cost of exchange on a draft for \$1000, exchange being $\frac{3}{8}\%$ premium.

12. Find the cost of an express order for \$1200 at the rate of 30¢ per \$100.

13. If you had to remit \$200 to Chicago when exchange is $\frac{1}{4}\%$ premium, would you prefer to buy exchange, or to buy an express money order at the rate of 30¢ for \$100?

14. What is the cost of an express money order for \$1000, at 30¢ for \$100?

15. What is the cost of a sight draft for \$3000 at \$1.50 premium?

16. What is the cost of a sight draft for \$3000 at \$2.50 premium?

Example. What is the cost of a draft on Buffalo for \$800, payable in 30 da. at 6% interest, exchange being at the rate of $\frac{1}{4}\%$ premium?

SOLUTION.

The interest on \$800 for 30 da. at 6% = \$4.00.

Cost of exchange = $\frac{1}{4}\%$ of \$800 = \$2.00.

The cost of the draft = \$800 + \$2 − \$4 = \$798.

EXERCISE 134

Find the cost of the following sight drafts :

	Face	Rate of Exchange
1.	$2720,	$\frac{1}{4}$% premium.
2.	$3480,	$\frac{1}{2}$% premium.
3.	$5080,	$\frac{1}{8}$% premium.
4.	$6290,	$2.50 premium.
5.	$8290,	$1.25 discount.
6.	$9980,	$\frac{1}{4}$% discount.
7.	$5493,	$1.50 discount.
8.	$5280,	$1.50 premium.
9.	$6040,	$2.00 discount.
10.	$6090,	$1.25 premium.
11.	$5400,	$1.25 discount.
12.	$9870,	1% discount.
13.	$4500,	$\frac{1}{5}$% premium.
14.	$6920,	$\frac{1}{10}$% discount.
15.	$7780,	$\frac{1}{8}$% premium.
16.	$1234,	$\frac{1}{8}$% discount.

17. Find the cost of an express money order for $1400 at 30¢ for $100.

18. Find the cost of a draft for $10,000 at $\frac{1}{8}$% discount.

19. Find the cost of a draft for $600 payable in 30 da. without grace, the rate of interest being 6%, exchange being $\frac{1}{4}$% premium.

20. Find the cost of a draft, for $1000 payable in 30 da., exchange being at par and the rate of interest being 8%.

21. Find the cost of a 45 da. draft for $900, exchange being at $\frac{1}{4}$% discount, and the rate of interest being 6%.

FOREIGN EXCHANGE

The use of drafts between parties living in different parts of the same country is known as **domestic exchange.** Similar transactions between parties living in different countries constitute what is known as **foreign exchange.** Foreign drafts are known as **bills of exchange.** These are issued in duplicate, *i.e.* two are drawn exactly alike. The two bills are sent by different mails. As soon as one is paid, the other is void.

Usually exchange is a little above or a little below the *par of exchange*, *i.e.* intrinsic value of foreign coins.

Exchange on Great Britain is quoted at the number of dollars to £1; exchange on France is quoted at so many cents to the franc, or as so many francs to the dollar; exchange on Germany is quoted as so many cents to 4 marks, or as so many cents to a mark.

BILL OF EXCHANGE

No. 24896 *New York, N. Y., Jan. 4, 1903.*

£1200

At sight of this first of exchange (second of the same date and tenor unpaid) ---------- *pay to the order of* ---------- *William Phibbs* ----------

---------- *Twelve hundred pounds sterling* ----------

and charge to the account of

Value received. *Bernard Johnson.*

To Macmillan & Co., Ltd.,

London, England.

VALUE OF FOREIGN COINS IN UNITED STATES MONEY

(Proclaimed by the Secretary of the Treasury, July 1, 1910.)

Country	Standard	Monetary Unit	Value in U. S. Gold Dollar
Argentine Republic	Gold	Peso	$ 0.965
Austria-Hungary	Gold	Crown	.203
Belgium	Gold	Franc	.193
Bolivia	Silver	Boliviano	.389
Brazil	Gold	Milreis	.546
Canada	Gold	Dollar	1.000
Central America	Silver	Peso	.391
Chile	Gold	Peso	.365
China	Silver	Tael { Shanghai	.586
		Tael { Haikwan	.653
		Tael { Canton	.641
Columbia	Gold	Dollar	1.000
Costa Rica	Gold	Colon	.465
Denmark	Gold	Crown	.268
Ecuador	Gold	Sucre	.487
Egypt	Gold	Pound (100 piasters)	4.943
France	Gold	Franc	.193
Germany	Gold	Mark	.238
Great Britain	Gold	Pound Sterling	4.8665
Greece	Gold	Drachma	.193
Haiti	Gold	Gourde	.965
India	Gold	Pound Sterling	4.8665
Italy	Gold	Lira	.193
Japan	Gold	Yen	.498
Mexico	Gold	Peso	.498
Netherlands	Gold	Florin	.402
Newfoundland	Gold	Dollar	1.014
Norway	Gold	Crown	.268
Panama	Gold	Balboa	1.000
Peru	Gold	Libra	4.8665
Portugal	Gold	Milreis	1.080
Russia	Gold	Ruble	.515
Spain	Gold	Peseta	.193
Sweden	Gold	Crown	.268
Switzerland	Gold	Franc	.193
Turkey	Gold	Piaster	.044
Uruguay	Gold	Peso	1.034
Venezuela	Gold	Bolivar	.193

The Circulars of the Secretary of the U. S. Treasury issued every 3 months, state the legal equivalent of the monetary units in the principal countries. In countries which have adopted the silver standard, monetary units fluctuate in value, owing to changes in the market value of silver.

Example 1. Express \$100 in florins (Netherlands).

$\$.402 = 1$ florin.

$\therefore \$100 = \frac{100}{.402}$ of 1 florin $= 248.76$ florins.

EXERCISE 135

1. Express the value of £1 in French currency; in German currency; in Austrian currency.

2. Change to United States currency (*a*) 450 francs, (*b*) 550 pesetas, (*c*) 280 Norwegian crowns, (*d*) 900 marks, (*e*) 200 colons, (*f*) 720 Austrian crowns, (*g*) 450 Danish crowns.

3. Find in United States currency the difference in value between 1000 francs and \$198.

4. Find the value of \$1000 (*a*) in the money of Costa Rica; (*b*) in Austrian currency; (*c*) in Swedish currency.

5. What is the equivalent in United States money of 200,000 bolivars (Venezuela)?

6. What is the equivalent in United States money of 10,000 Mexican pesos?

7. Express \$1 in Peruvian monetary units.

8. Express \$1 in Norwegian monetary units.

9. Find in United States currency the difference between the value of 10,000 Japanese yen and \$5000.

10. What is the value of 10,000 Portuguese milreis?

ENGLISH MONEY

4 farthings = 1 penny ($d.$)
12 pence = 1 shilling ($s.$)
20 shillings = 1 pound sterling (£)
£1 = 20$s.$ = 240$d.$ = 960 farthings

The abbreviations £, $s.$, $d.$, are the initial letters of the names of the Roman coins, libra, solidus, denarius.

EXERCISE 136

1. What is the equivalent of 7$s.$ $5\frac{3}{4}d.$ in U. S. currency?

$$7s.\ 5\tfrac{3}{4}d. = \left(7 + \frac{5.75}{12}\right)s. = 7.479166s. = £.373958.$$

$$\$4.8665 \times .37396 = \$1.82.$$

2. The average freight rate on wheat per bushel for the year 1906 from Chicago to New York was:

By lake and canal 5.94¢.

By lake and rail 6.48¢.

By all rail delivered to steamer 8.10¢, and from New York to Liverpool $1\frac{7}{16}d.$

Find the cost of shipping 5 carloads of wheat averaging 79,000 lb. from Chicago to Liverpool.

(a) By lake and canal to seaboard and thence by steamer.

(b) By lake and rail to seaboard and thence by steamer.

(c) By rail to seaboard and thence by steamer.

3. Find the cost of shipping from New York to Liverpool 12,000 bu. of wheat at $1\frac{7}{16}d.$ per bu.

4. Find the cost of shipping 50 tons of sacked flour from Chicago to Liverpool at 10.11$d.$ per 100 lb.

In examples 5 and 6 take £1 = \$4.80.

5. Find the cost of shipping from Galveston to Liverpool: —

(*a*) 10,884,000 lb. of cotton at 1*s*. $6\frac{1}{2}$*d*. per 100 lb.
(*b*) 19,490 bu. of wheat at 3*d*. per bu.
(*c*) 37,920 bu. of corn at 5*d*. per bu.

6. Find the cost of shipping from St. Louis to Liverpool *via* New Orleans: —

(*a*) 6475 bbl. of flour at 1*s*. $8\frac{1}{2}$*d*.
(*b*) 97,330 bu. wheat at $7\frac{5}{8}$*d*.

EXERCISE 137

1. Find the cost of a draft on London for £1000, exchange being 4.872.

2. Find the cost of a bill of exchange for £550 15*s*., exchange being 4.85.

3. Find the cost of a bill of exchange for 5750 francs, exchange being 5.15, *i.e.* 5.15 francs = $1.

4. Find the cost of a draft on Manchester for £3500 6*s*. 8*d*. when exchange is quoted at $4.87\frac{1}{2}$.

5. Find the cost of a draft on Berlin for 1480 marks when exchange is quoted at $95\frac{1}{4}$, *i.e.* 4 marks = $95\frac{1}{4}$ ¢.

6. Find the cost of a draft on Vienna for 8500 crowns, rate of exchange being 1 crown = $.202.

7. Find the cost of a draft for 1500 lira when exchange is quoted at 19.3, *i.e.* 19.3 ¢ = 1 lira.

8. Find the cost of each of the following drafts:

	Face	Where Payable	Rate of Exchange
(*a*)	£928 10*s*.	Liverpool	4.85
(*b*)	£850 6*s*. 8*d*.	Belfast	$4.86\frac{1}{4}$
(*c*)	2500 marks	Berlin	.24
(*d*)	9280 francs	Havre	5.16
(*e*)	8500 lira	Naples	$.19\frac{1}{4}$
(*f*)	£584 13*s*. 4*d*.	Dublin	$4.85\frac{3}{4}$

Example. What is the face of a New York draft on Liverpool, which costs \$7297.50, when exchange is quoted at $4.86\frac{1}{2}$?

SOLUTION. $\$4.865 = £1.$

$$\$1 = \frac{£1}{4.865}.$$

$$\$7297.50 = \frac{£7297.50}{4.865} = £1500.$$

EXERCISE 138

Find the face of each of the following drafts:

	COST	WHERE PAYABLE	RATE OF EXCHANGE
1.	\$5835	London	$4.86\frac{1}{4}$
2.	\$1218.75	Newcastle	$4.87\frac{1}{2}$
3.	\$9063.05	Berlin	95.20
4.	\$3724.90	Paris	19.30
5.	\$718.24	Christiania	26.80
6.	\$2366.82	Belfast	$4.86\frac{1}{2}$
7.	\$1037	Paris	$5.18\frac{1}{2}$
8.	\$5664.80	Hamburg	23.80

STOCKS

A **company**, or **corporation**, is a number of persons associated under a state law for the purpose of transacting business.

The modern company, or corporation, has supplanted to a great degree the old-time partnership. Nowadays business is conducted on so extensive a scale that a company often numbers several thousand persons.

The money which a company invests in business is called its **capital**, or **stock**.

The stock of a company is usually divided into shares of $10, $50, or $100 each. The face value of a share in most large companies is $100.

These shares, as a rule, can be bought and sold. The buying and selling is usually done by agents called **stock-brokers.** The usual fee of a stockbroker is at the rate of $\frac{1}{8}$ % of the par value for buying a share and $\frac{1}{8}$ % of the par value for selling. A broker's fee is called **brokerage.**

If the business of a company is prosperous, a share may sell for more than its face value. The stock is then said to be at a **premium.** If the business is not prosperous, the value of a share in the market is likely to be less than its face value. The stock is then said to be at a **discount.**

When a share sells in the market for its face value, it is said to be **at par.** A quotation "10 % premium" means the same as 110 % of the face value. A quotation "10 % discount" means the same as 90 % of the face value.

The profits of a company, usually distributed to stock-holders annually or semi-annually, are called **dividends.** The dividend is generally so many dollars on a share, or a percentage of the face value of the stock. Thus, "4 % dividend" means $4 on a share whose face value is $100.

Stock is of two kinds, **common** and **preferred.** The **preferred** is given certain privileges by the charter of the corporation, of which privileges the most common is that of a fixed dividend. When the company does not earn enough to pay this dividend on the preferred stock, the dividend accumulates, and must be paid, together with succeeding overdue dividends, before any dividend is paid on the **common** stock. What is left of the earnings of the company, after expenses are paid and the dividend on the preferred stock, becomes a dividend on the **common** stock. Thus the common stock is not as sure of a dividend as the

preferred; but the dividend on the preferred stock is fixed in amount, while the dividend on the common stock is only limited by the earnings.

BONDS

When a city, state, or nation, or a corporation, wishes to borrow a large amount of money, it usually does this by issuing bonds. These bonds are very similar to the promissory notes given by individuals. They are promises to pay to the owner of the bond the amount specified in the bond after a specified number of years, and to pay interest at specified intervals at a rate named. Bonds are usually issued in \$1000 or \$5000 denominations, and are usually protected by a mortgage held by a trust company. In case the bonds are not paid, the property included in the mortgage can be sold for the benefit of the bondholders.

Bonds are of two kinds, **registered** and **coupon**. A **registered bond** is one which is recorded by number and by name of the owner, and it is not transferable except in writing and at the office of the treasurer. **Coupon bonds** are so called because they have interest slips attached to them, which are cut off as the interest falls due. These interest slips are payable to the bearer.

The following is quoted from a daily newspaper giving the New York stock market.

Stocks

	Sales	Highest	Lowest	Closing Bid
Atchison	30,200	82	$80\frac{1}{2}$	$81\frac{7}{8}$
Atchison pf.	1,800	$97\frac{1}{2}$	$97\frac{1}{8}$	$97\frac{3}{8}$
Baltimore & Ohio . .	14,400	93	$91\frac{3}{8}$	$92\frac{7}{8}$
Canadian Pacific . .	13,800	$129\frac{1}{2}$	$127\frac{5}{8}$	$129\frac{1}{2}$
Manhattan L	17,300	$142\frac{3}{4}$	$141\frac{3}{8}$	$142\frac{3}{4}$

	SALES	HIGHEST	LOWEST	CLOSING BID
Metropolitan St. Ry. .	21,100	$135\frac{1}{8}$	$131\frac{3}{8}$	134
Missouri Pacific . . .	30,800	$108\frac{1}{2}$	$106\frac{5}{8}$	$108\frac{3}{8}$
Pennsylvania	53,400	$144\frac{1}{8}$	$142\frac{1}{4}$	144
St. Paul	58,500	$168\frac{7}{8}$	$167\frac{1}{8}$	$168\frac{1}{4}$
Southern Pacific . . .	54,400	$63\frac{1}{4}$	$61\frac{1}{4}$	$62\frac{7}{8}$

Bonds

U. S. new 4s registered,	135	U. S. old 4s registered,	103
U. S. new 4s coupon,	136	Baltimore & Ohio 4s,	$102\frac{1}{2}$
U. S. old 4s coupon,	$109\frac{1}{2}$	Chicago B. & Q. new 4s,	$93\frac{1}{8}$

Notice that in stock quotations the fractions used are halves, quarters, eighths. Notice further that the variations in the prices of stock in the course of a day are considerable. There are instances where stocks have fallen in one day 50 % of their face value. Why the prices vary so much would take the proverbial Philadelphia lawyer to explain. In the above quotations the first column shows the number of shares sold; the second column gives the highest prices paid for the shares; the third gives the lowest prices paid; and the fourth gives the closing bids. In the course of the day above referred to there were several other prices than those given.

Example. Find the cost of 10 shares of stock at $107\frac{1}{2}$.

SOLUTION

Market price $= \$107\frac{1}{2}$.
Brokerage $= \$\frac{1}{8}$.
Cost of 1 share $= \$107\frac{5}{8}$ (*i.e.* $\$107\frac{1}{2} + \$\frac{1}{8}$).
Cost of 10 shares $= \$107\frac{5}{8} \times 10 = \$1076.25$.

EXERCISE 139

1. Give the premium or discount of each of the stocks quoted (pages 274 and 275).

2. What is the cost of 20 shares of Atchison at $80\frac{7}{8}$? at $81\frac{7}{8}$?

3. What is the cost of 40 shares of Baltimore & Ohio at $92\frac{7}{8}$?

4. What is the cost of 200 shares of Canadian Pacific at $145\frac{1}{4}$?

5. What is the cost of 25 shares of stock at $118\frac{1}{2}$?

6. Find the cost of 50 shares of Missouri Pacific at $106\frac{5}{8}$.

7. Find the cost of 100 shares of each of the stocks quoted (page 274), at the prices indicated in the fourth column.

Example. **1.** What sum should be received from the sale of 40 shares of Pennsylvania at $144\frac{1}{8}$?

SOLUTION

Market value of 1 share of stock = \$ $144\frac{1}{8}$.
Brokerage = \$ $\frac{1}{8}$.
Sum received from 1 share = \$ 144.
Sum received from 40 shares = \$ 144×40 = \$ 5760.

Example **2.** What profit is made by purchasing 50 shares of stock at $142\frac{1}{4}$ and selling them at \$ 144?

SOLUTION. Cost of 1 share = \$ $142\frac{1}{4}$ + \$ $\frac{1}{8}$ = \$ $142\frac{3}{8}$.
Receipts from 1 share = \$ 144 − \$ $\frac{1}{8}$ = \$ $143\frac{7}{8}$.
Gain on 1 share = \$ $1\frac{1}{2}$.
Gain on 50 shares = \$ $1\frac{1}{2} \times 50$ = \$ 75.

EXERCISE 140

1. What should a person receive from the sale of 100 shares of Manhattan at the closing bid (given page 274)? from the sale of 100 shares at the lowest bid?

2. How much should a person receive from the sale of 100 shares of Atchison preferred at $97\frac{1}{2}$?

3. How many shares of Pennsylvania at $142\frac{1}{4}$ can be bought for $14,237.50?

4. What profit is made by buying 100 shares of Baltimore & Ohio at $91\frac{3}{8}$, and selling them at $92\frac{7}{8}$?

5. If 100 shares of St. Paul are bought at $167\frac{1}{8}$ and sold the same day at $168\frac{5}{8}$, what is the gain?

6. If 200 shares of New York Central are bought at 165 and sold the following day at $167\frac{1}{4}$, what is the gain?

7. A speculator bought Missouri Pacific at the lowest price quoted, page 274, and sold at the closing bid of the same day. If his gain was $300, how many shares did he buy?

8. If a speculator buys Metropolitan Street Railway at $131\frac{1}{2}$ and sells on the same day at 134, find the number of shares bought to realize a profit of $225.

9. How many shares of Atchison bought at $80\frac{5}{8}$ and sold at $81\frac{7}{8}$ realize a profit of $1000?

10. How many Chicago B. & Q. new 4s at $93\frac{1}{8}$ can I buy for $18,650?

Example **1.** From money invested in stock a man received a dividend amounting to 5% on the money invested. If this dividend is 4% on the par value of the stock, at what price per share was the stock purchased?

SOLUTION. The par value is $100 per share. A dividend of 4 % gives $4 per share. The $4 is equal to 5 % on the cost per share. If $4 is 5 %, 1 % is $\frac{1}{5}$ of $4, or $.80. If 1 % of the cost of one share is $.80, 100 % is 100 times $.80, or $80.

The stock cost $80 per share. The $80 includes the brokerage of $\$\frac{1}{8}$. Therefore the market price of the stock was $\$80 - \$\frac{1}{8} = \$79\frac{7}{8}$.

Example 2. How much money must be invested in United States old 4s registered at 103 to produce an annual income of $600?

SOLUTION. United States old 4s registered pay a dividend of $4 a share.

The number of 4s in 600 is 150.

One hundred and fifty shares produce $600.

The cost of 1 share is $103 + \frac{1}{8}$.

The cost of 150 shares is $103\frac{1}{8} \times 150 = \$15{,}462.50$.

EXERCISE 141

1. If stock paying a dividend of 8 % gives an income at the rate of 5 % on the money invested, what is the market value of the stock?

2. If 6 % stock produces a gain of 4 % on the money invested, find its market value.

3. How much money must I invest in Western Union at $92\frac{5}{8}$, paying a dividend of 5 %, to derive an annual income of $1000?

4. How much must be invested in Pullman Palace Car at 218, paying a yearly dividend of 8 %, to derive an income of $1200 a year?

5. If I buy $4\frac{1}{2}$ % stock at $97\frac{7}{8}$, what rate per cent do I get for my money?

6. What must I pay for 5 % stock so as to make a profit of 8 % on my investment?

7. How much must be invested in Northwestern, paying 6 %, at 206, to derive an income of $900?

8. Which is the more profitable investment, 5 % stock at $119\frac{7}{8}$ or 8 % stock at $219\frac{7}{8}$?

9. How much must be paid for a $1000 bond at $87\frac{1}{2}$?

10. City bonds bought at $89\frac{7}{8}$ pay 5 %. What rate of interest do they pay?

11. If I have $5000 of United States 4s, what annual dividend do I receive? If I sell at 102, what should it bring, after deducting brokerage?

12. At the highest prices paid March 11, 1903, how much should be realized by selling $10,000 of each of the stocks quoted in the list on page 274?

13. Which is the better investment, Illinois Central, paying 6 % dividend, at $137\frac{3}{8}$, or United States 4s at $109\frac{7}{8}$?

14. How many shares of stock must a broker sell to make $54.75 brokerage?

CUSTOMS AND DUTIES

Taxes levied on imported goods are called **duties.** Duties are of two kinds, **specific** and **ad valorem.** The latter is a percentage of the foreign value of the goods; the former is a definite amount of money on some standard quantity, such as a yard, gallon, pound, etc. Some articles are subjected to both specific and ad valorem duties. Duties are collected by United States customs officers at places known as **ports of entry.**

The United States revenue from duties for the year ending June 30, 1911, amounted to $314,497,071.

EXERCISE 142

1. Find the duty on 3200 lb. of refined sugar at 1.90¢ per pound.

2. Find the duty on 2000 bu. of potatoes at 25¢ per bushel.

3. Find the duty on 1800 lb. of salt at 7¢ per 100 lb.

4. Find the duty on 5 doz. parasols at 4*s*. 6*d*. each at 60% ad valorem.

5. Find the duty on 12 microscopes invoiced at £8 each, the rate of duty being 45% ad valorem.

6. The duty on books printed in England is 25% ad valorem. If I import the following books at the prices specified, how much duty in United States money will I have to pay?

Casey's Elements of Euclid	3*s*. 9*d*.
Casey's Analytical Geometry	10*s*. 0*d*.
Leatham's Spherical Trigonometry . .	4*s*. 0*d*.
Lamb's Infinitesimal Analysis . . .	12*s*. 0*d*.

7. Find the duty on the Clarendon Press edition of Shakespeare's plays, invoiced at 24*s*., at 25% ad valorem.

8. A hardware merchant imported 80 doz. razors at 2*s*. 6*d*. each, duty $1.75 per dozen and 20% ad valorem. Find the total duty paid.

9. What is the duty on 100 yd. of treble ingrain carpet valued at 90¢ a square yard, the duty being 22¢ a square yard and 40% ad valorem?

10. What is the duty on 40 clocks valued at \$4.50 each, at 40 % ad valorem? If the clocks are retailed at a profit of 30 %, find the selling price of these clocks.

11. Find the duty on 120 doz. linen cuffs at 7*s.* 6*d.* a dozen, the duty being 40¢ a dozen and 20 % ad valorem.

12. Find the duty on a fur rug valued at \$50, duty 35 % ad valorem. At what price should the rug be sold so as to make a profit of 20 %?

13. Calculate the duty levied on 5 doz. straw hats valued at \$30 a dozen, the duty being \$7 a dozen and 20 % ad valorem.

14. What is the duty on 800 yd. of silk valued at \$1.15 a yard at 60 % ad valorem? At what price per yard should the silk be sold to make a profit of 20 %?

15. Calculate the duty on 12 horses valued at \$250 each at 25 % ad valorem.

16. An Axminster carpet, 18 ft. by 12 ft., valued at \$1.50 per square yard, is imported. Find the duty at 90¢ per square yard and 40 % ad valorem.

17. What is the duty on 5 doz. opera glasses valued at \$3.50 each at 45 % ad valorem?

18. What is the duty on 500 lb. of glue valued at 50¢ per pound, the rate of duty being 15¢ per pound and 20 % ad valorem?

19. A suit of clothes imported from England costs a merchant \$38.80. The duty is 60 % ad valorem. Taking £1 = \$4.85, find the invoice price in pounds sterling.

20. Find the duty on 1200 yd. of linen invoiced at $7\frac{1}{2}d.$ per yard at 45 % ad valorem. At what price per yard should it be marked so that the dealer may give a discount of 10 % and make a profit of 20 %?

CHAPTER IV

INVOLUTION

In the language of mathematics $3x$ means three times the number x; x may stand for any number whatever.

$a+b$ stands for the sum of the two numbers, a and b. $a-b$ stands for the difference of two numbers, a and b. In other words, $a-b$ is the number which when added to b gives a for sum.

ab stands for the product of two numbers, a and b. $\frac{a}{b}$ stands for the quotient obtained by dividing a by b. In other words, $\frac{a}{b}$ is that number which when multiplied by b gives a for product.

a^2 stands for the square of a. a^3 stands for the cube of a, $(a+b)^2$ stands for the square of the sum of two numbers, a and b.

$(a-b)^2$ stands for the square of the difference of two numbers, a and b.

$7ab$ stands for seven times the number ab, which is itself the product of two numbers, a and b.

$\frac{a+b}{2}+\frac{a-b}{2}$ stands for one half the sum of two numbers plus one half their difference.

$(a+b)x$ stands for the product of a number, x, by the sum of two numbers, a and b.

$(a-b)x$ stands for the product of a number, x, by the difference of two numbers, a and b.

Add :

$$\begin{array}{r}4\,x\\7\,x\\5\,x\\\hline 16\,x\end{array}\qquad\begin{array}{c}14\,a\\6\,b\\7\,b\\\hline 14\,a+13\,b\end{array}\qquad\begin{array}{c}\\3\,ax\\4\,bx\\\hline (3\,a+4\,b)\,x\end{array}$$

EXERCISE 143

Add:

1. $\begin{array}{r}2\,x\\3\,x\\5\,x\\\hline\end{array}$
2. $\begin{array}{r}7\,y\\8\,y\\9\,y\\\hline\end{array}$
3. $\begin{array}{r}4\,a\\9\,a\\11\,a\\\hline\end{array}$
4. $\begin{array}{r}9\,b\\7\,b\\8\,b\\\hline\end{array}$
5. $\begin{array}{r}4\,z\\7\,z\\9\,z\\\hline\end{array}$
6. $\begin{array}{r}11\,a\\7\,a\\5\,a\\\hline\end{array}$
7. $\begin{array}{r}4\,ab\\5\,ab\\7\,ab\\\hline\end{array}$
8. $\begin{array}{r}9\,xy\\7\,xy\\8\,xy\\\hline\end{array}$
9. $\begin{array}{r}4\,bc\\7\,bc\\11\,bc\\\hline\end{array}$
10. $\begin{array}{r}12\,ab\\15\,ab\\10\,ab\\\hline\end{array}$
11. $\begin{array}{r}9\,ab\\4\,ab\\5\,ab\\\hline\end{array}$
12. $\begin{array}{r}7\,bc\\8\,bc\\9\,bc\\\hline\end{array}$
13. $\begin{array}{r}ax\\bx\\\hline\end{array}$
14. $\begin{array}{r}10\,x\\ax\\\hline\end{array}$
15. $\begin{array}{r}11\,b\\ab\\\hline\end{array}$
16. $\begin{array}{r}12\,m\\am\\\hline\end{array}$
17. $\begin{array}{r}11\,x\\bx\\\hline\end{array}$

Subtract :

$$\begin{array}{r}14\,a\\3\,a\\\hline 11\,a\end{array}\qquad\begin{array}{c}17\,a\\13\,b\\\hline 17\,a-13\,b\end{array}\qquad\begin{array}{c}9\,ax\\7\,bx\\\hline (9\,a-7\,b)\,x\end{array}$$

EXERCISE 144

1. $7\,x-4\,x=?$
2. $6\,x-x=?$
3. $11\,x-8\,x=?$
4. $12\,x-2\,x=?$
5. $5\,x-3\,x=?$
6. $12\,a-a=?$
7. $15\,a-11\,a=?$
8. $17\,a-12\,a=?$
9. $18\,a-13\,a=?$
10. $12\,ab-3\,ab=?$
11. $11\,ab-4\,ab=?$
12. $5\,ab-ab=?$
13. $2\,xy-xy=?$
14. $14\,xy-10\,xy=?$
15. $17\,xy-9\,xy=?$
16. $19\,xy-7\,xy=?$
17. $ax-bx=?$
18. $cx-dx=?$
19. $mx-nx=?$

Example 1. Multiply $4x$ by $5x$.

SOLUTION. $4x \times 5x = 4 \times x \times 5 \times x = 4 \times 5 \times x \times x = 20x^2$.

Example 2. Multiply $7x^2y$ by $3xy^2$.

SOLUTION. $7x^2y \times 3xy^2 = (7 \times x \times x \times y) \times (3 \times x \times y \times y) = 7 \times x \times x \times x \times 3 \times y \times y \times y = 21x^3y^3$.

Example 3. What is the square of $a + b$?

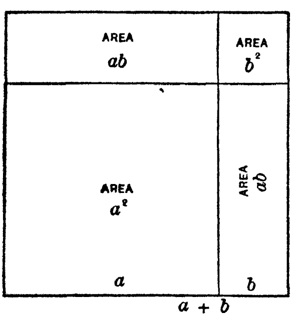
$$\begin{array}{r} a + b \\ \underline{a + b} \\ a^2 + ab \qquad \\ \underline{+ ab + b^2} \\ a^2 + 2ab + b^2 \end{array}$$

Multiply $a + b$ by a, and write the result, $a^2 + ab$. Next multiply $a + b$ by b, and write the result, $ab + b^2$. Then add.

In the language of mathematics it makes no difference in what order quantities are written, as the signs show the relations. Thus, $a^2 + 2ab + b^2 = a^2 + b^2 + 2ab$.

$\therefore (a + b)^2 = a^2 + b^2 + 2ab$. Hence, we have the following important conclusion:

The square of the sum of two numbers equals the square of the first number plus the square of the second number plus twice the product of the two numbers.



The process of finding the power of a number is called **involution.**

Draw a line to represent the number a. Extend this line a distance that may represent b. Then the whole line represents $a + b$. Draw a square on this line. This square is the square of $a + b$, *i.e.* its area in square units equals $(a + b)^2$. By drawing the square on

a and extending its sides we see that the large square is divided into two squares and two rectangles. The area of one square is a^2, the area of the other is b^2, and the area of each rectangle is ab, because its dimensions are a and b. Therefore, $(a + b)^2 = a^2 + 2\,ab + b^2$.

This equation expresses in mathematical language that the square of the sum of two numbers is equal to the square of the first, plus twice the product of the two, plus the square of the second number.

Example **1.** Square $x + 9$.

SOLUTION. $(x + 9)^2 = x^2 + 2 \times 9 \times x + 9^2 = x^2 + 18\,x + 81$.

Example **2.** What is the square of 43?

SOLUTION. $43^2 = (40 + 3)^2 = 40^2 + 2 \times 40 \times 3 + 3^2 = 1600 + 240 + 9 = 1849$.

EXERCISE 145

Find the square of:

1. $10 + x$.	**7.** $70 + b$.	**13.** 29.	**19.** 72.	**25.** 89.
2. $20 + x$.	**8.** $80 + c$.	**14.** 37.	**20.** 76.	**26.** 92.
3. $30 + x$.	**9.** $90 + y$.	**15.** 47.	**21.** 78.	**27.** 93.
4. $40 + x$.	**10.** 14.	**16.** 54.	**22.** 79.	**28.** 67.
5. $50 + x$.	**11.** 15.	**17.** 62.	**23.** 84.	**29.** 98.
6. $60 + a$.	**12.** 24.	**18.** 68.	**24.** 87.	**30.** 55.

The square of a fraction has for its numerator the square of the given numerator, and for its denominator the square of the given denominator.

EXERCISE 146

1. Find the square of .1, .2, .3, .4, .5, .6, .7, .8, .9.

2. Find the square of the reciprocals of the numbers from 1 to 20 inclusive.

3. Find the squares of $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$, $\frac{2}{7}$, $\frac{9}{10}$, $\frac{11}{12}$, $\frac{10}{13}$, $\frac{9}{16}$.

4. Find the squares of $\frac{5}{3}$, $\frac{7}{3}$, $\frac{9}{7}$, $1\frac{1}{4}$, $1\frac{3}{4}$, $1\frac{7}{8}$, $2\frac{1}{5}$, $6\frac{1}{4}$.

5. How does the square of a fraction compare in value with the fraction itself?

6. Find the cubes of the reciprocals of the numbers from 1 to 20 inclusive.

7. Find the value of $1^3 + 2^3 + 3^3 + 4^3 + 5^3 + 6^3 + 7^3 + 8^3 + 9^3 + 10^3$.

8. Square each of the following numbers, and give the product correct to four decimal figures: (*a*) 1.732, (*b*) 9.256, (*c*) 5.401, (*d*) 8.129, (*e*) .6834, (*f*) .7609, (*g*) 9.482, (*h*) .7071, (*i*) .7746, (*j*) .9487.

EVOLUTION — SQUARE ROOT

By the **square root** of a number is meant that number which when squared produces the given number. Thus, 4 is the square root of 16, since $4^2 = 16$. 7 is the square root of 49 for a similar reason.

The square root is defined also as one of the two equal factors of a number. Thus, $7 \times 7 = 49$. One of the factors is the square root of 49.

The symbol for square root, $\sqrt{\ }$, is called the *radical sign*. The exponent $\frac{1}{2}$ is also used as a sign for the square root. $\sqrt{49}$ is read: the square root of 49. $49^{\frac{1}{2}}$ indicates the same thing as $\sqrt{49}$; but usually it is read: 49, with the exponent one half.

Students should fix firmly in mind: —

$10^2 = 100$.	$40^2 = 1600$.	$70^2 = 4900$.
$20^2 = 400$.	$50^2 = 2500$.	$80^2 = 6400$.
$30^2 = 900$.	$60^2 = 3600$.	$90^2 = 8100$.

Example 1. What is the square root of 5329?

SOLUTION. 5329 is larger than 4900 and smaller than 6400. But 4900 is the square of 70, and 6400 is the square of 80. Therefore, 5329 is the square of a number between 70 and 80. So 5329 is the square of a number which may be represented as 70 plus some number. Represent this other number by x.

Then, $5329 = (70 + x)^2$.
But, $(70 + x)^2 = 4900 + 140\,x + x^2$.
Therefore, $5329 = 4900 + 140\,x + x^2$.
Subtract, $5329 - 4900 = 140\,x + x^2$.
$$429 = 140\,x + x^2.$$

We can see that $(140\,x + x^2)$ is the product of x and $(140 + x)$.

So we may write $429 = (140 + x)x$.

429 is x times $(140 + x)$.

429 divided by $(140 + x)$ will give x.

As we do not know the value of x, we try to find it by dividing 429 by 140. As 429 divided by 140 gives 3 and a remainder, we will try 3 as the value of x.

Then, $140 + x$ becomes $140 + 3 = 143$;
and $3 \times (140 + x) = 3 \times 143 = 429$.

Therefore, 3 is the value of x.

Then, $\sqrt{5329} = 70 + x = 70 + 3 = 73$.

The work is usually arranged as follows:

	7 3
	53′ 29
	49
143	4 29
	4 29

EXPLANATION. Begin at the decimal point and group the digits of the number into groups of two. This gives two groups in 5329. The largest perfect square smaller than the left hand group, 53, is 49. The square root of 49 is 7. Write the 7 above the 53 as part of the root. Write the 49 below the 53 and

subtract and bring down the next group. As the 49 is 49 hundreds, its square root is 7 tens. Now multiply 7 tens by 2. This gives 14 tens. Write 14 at the left of 429 as a trial divisor, remembering that it is tens. Divide 429 by 14 tens. Write the quotient, 3, above the 29, as the next figure of the root. Also write the 3 at the right of the 14. Multiply 143 by 3, write the product below the 429, and subtract. As there is no remainder, 73 is the root.

There will be a figure in the root for each group of figures in the number. The left-hand group may contain only one figure.

EXERCISE 147

Find the square root of :

1.	169.	9.	2809.	17.	7056.	25.	7921.
2.	441.	10.	3844.	18.	7569.	26.	6084.
3.	625.	11.	4225.	19.	8464.	27.	4761.
4.	961.	12.	5329.	20.	9216.	28.	3481.
5.	1024.	13.	5776.	21.	9604.	29.	2401.
6.	1849.	14.	6724.	22.	9801.	30.	3364.
7.	289.	15.	361.	23.	484.	31.	529.
8.	676.	16.	729.	24.	784.	32.	841.

When there are more than two groups in the number, the work is performed as before, with the two left-hand groups. After the last subtraction the next group is brought down to form, with the remainder, a new dividend. Then the part of the root already found is considered as tens. It is multiplied by 2 to obtain a trial divisor, and the work proceeds as before, this process being repeated until all figures of the given number have been brought down.

Example. Find the square root of 127,449.

	3 5 7
	12′ 74′ 49
	9
65	374
	325
707	49 49
	49 49

EXPLANATION. Point off in groups of two, beginning at the decimal point. 9 is the largest square less than 12. 3 is the square root of 9. Write 3 in the root, subtract 9 from 12, and bring down the 74. Multiply 3 tens by 2, and write the 6 tens as trial divisor. 6 tens will go 6 times in 374. Write 6 in the root and in the trial divisor. 6 times 66 is 396, which is larger than 374. This shows that 6 is not correct for the second figure of the root. Then try 5. Write 5 in the root and in the trial divisor. 5 times 65 gives 325. Subtract 325 from 374 and bring down the 49. Multiply 35 tens by 2 and place the 70 tens as trial divisor. 4949 divided by 70 tens gives 7. Write 7 in the root and also in the trial divisor. Then multiply 707 by 7 and subtract from 4949. There is no remainder.

$\therefore \sqrt{127449} = 357.$

EXERCISE 148

Extract the square root of :

1. 100,489.	**7.** 229,441.	**13.** 474,721.
2. 110,224.	**8.** 277,729.	**14.** 501,264.
3. 120,409.	**9.** 310,249.	**15.** 654,481.
4. 171,396.	**10.** 354,025.	**16.** 772,641.
5. 190,096.	**11.** 391,876.	**17.** 819,025.
6. 199,809.	**12.** 456,976.	**18.** 826,281.

Memorize :—

$(.1)^2 = .01.$	$(.5)^2 = .25.$	$(.9)^2 = .81.$
$(.2)^2 = .04.$	$(.6)^2 = .36.$	$(.11)^2 = .0121.$

$(.3)^2 = .09.$ $(.7)^2 = .49.$ $(.12)^2 = .0144.$

$(.4)^2 = .16.$ $(.8)^2 = .64.$ $(.01)^2 = .0001.$

To extract the square root of a decimal, begin *at the decimal point,* and proceeding to the right, point off the figures in periods of two; next proceed as if the number were an integer. Thus, in taking the square root of .0225, first point off in periods of two figures each. This gives .02 25. Next extract the root of the number denoted by the figures 225. The result is 15. Hence, the required root is .15.

To extract the square root of a number part integer and part decimal, begin at the decimal point, and proceeding to the left, point off the integral part in periods of two figures each; next point off the decimal part in periods of two figures each, beginning *at the decimal point.* If there are not enough figures in the decimal part to make an exact number of periods, annex a cipher or as many ciphers as are necessary to make the required number of periods.

Example. Extract the square root of 1.7.

$$
\begin{array}{r|l}
 & 1.\ 3\ \ 0\ \ 3\ \ 8\ \ 4 \\
 & 1.\ 70\ 00\ 00\ 00\ 00 \\
 & 1. \\
\hline
23 & 70 \\
 & 69 \\
\hline
2603 & 10000 \\
 & 7809 \\
\hline
26068 & 219100 \\
 & 208544 \\
\hline
260764 & 1055600 \\
 & 1043056 \\
\hline
\end{array}
$$

Double 1 for the first trial divisor. Double 13 for the next trial divisor. Then find the next figure of the root is 0. Write it in the root and in the trial divisor. Then annex two more ciphers, and find that the next figure of the root is 3, and so on.

$\therefore\ \sqrt{1.7} = 1.30384$, correct to five decimal places.

EXERCISE 149

Extract the square root of:

1. .150932.	4. .2909.	7. .5319.	10. .083.
2. .246016.	5. .2632.	8. .61575.	11. .062.
3. .3448.	6. .4616.	9. .784.	12. .0037.

To extract the square root of a fraction when its numerator and denominator are perfect squares is a simple matter. Thus, the square root of $\frac{25}{49}$ is $\frac{5}{7}$; the square root of $1\frac{17}{64}$, or $\frac{81}{64}$, is $\frac{9}{8}$, or $1\frac{1}{8}$.

To get the square root of a fraction, take the square root of the numerator, and the square root of the denominator, and then write the former result for numerator and the latter result for denominator. The fraction thus found is the required square root.

Example 1. What is the square root of $\frac{17}{36}$?

SOLUTION. $\sqrt{17} = 4.123$; $\sqrt{36} = 6$.

$$\therefore \sqrt{\tfrac{17}{36}} = \frac{4.123}{6} = .687.$$

Example 2. What is the square root of $\frac{23}{39}$?

SOLUTION. $\sqrt{23} = 4.796$; $\sqrt{39} = 6.245$.

$$\therefore \sqrt{\tfrac{23}{39}} = \frac{4.796}{6.245} = .768.$$

This is a roundabout way to take the square root of $\frac{23}{39}$. A shorter and better way is to reduce the fraction to an equivalent decimal, and then to extract the square root of this decimal.

To extract the square root of a fraction whose denominator is not a perfect square, reduce the fraction to an equivalent decimal and then extract the square root of this decimal.

EXERCISE 150

Extract, to three decimal figures, the square root of :

1. 1.2.	4. 5.2.	7. $3\frac{1}{8}$.	10. $4\frac{1}{2}$.	13. $\frac{2}{3}$.
2. 4.25.	5. 3.3.	8. $9\frac{1}{6}$.	11. $2\frac{7}{8}$.	14. $\frac{2}{5}$.
3. 1.1.	6. $5\frac{1}{4}$.	9. $1\frac{3}{8}$.	12. $\frac{7}{12}$.	15. $\frac{7}{11}$.

EXERCISE 151

PROBLEMS INVOLVING SQUARE ROOT

1. The area of a square field is 1 A. Find the length in yards of one of its sides.

2. The area of a square field is 12 A. Find the length in yards of one of its sides.

3. The dimensions of a rectangle are 289 yd. and 196 yd. Find the side of an equivalent square.

4. The dimensions of a rectangle are $1\frac{1}{2}$ mi. and .7 mi. Find, correct to four decimal figures, the side of an equivalent square.

5. Find in rods the perimeter of a square field whose area is $\frac{1}{3}$ of a square mile.

6. The area of a rectangle whose length is twice its breadth is 10 A. Find its dimensions in yards.

HINT. Draw a diagram; divide it into two equal parts by a line parallel to its width. Notice what each part is.

7. The area of a rectangle whose length is three times its width is 20 A. Find its dimensions in yards.

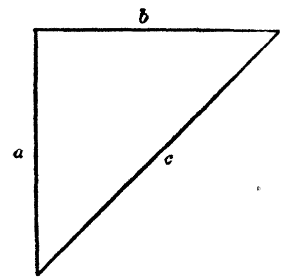
8. A square and a rectangle have the same area; namely, 40 A. If the length of the rectangle is twice its width, find in rods the difference between their perimeters.

The side of a right triangle opposite the right angle is called the **hypothenuse**. The other two sides are called the

legs of the right triangle. One of the legs is called the **base** of the right triangle, and the other leg is called the **altitude** of the right triangle.

In a right triangle the square on the hypothenuse is equal to the sum of the squares on the two legs.

Thus in the right triangle of the figure, if a is 6 ft., b is 8 ft., and c is 10 ft., $a^2+b^2=c^2$, or $6^2+8^2=10^2$, which we can see is true; because $36+64=100$.



This fact gives a method of finding one side of a right triangle if the other two are given.

Example. In a right triangle the legs are 7 and 24. Find the hypothenuse.

SOLUTION.
$$a^2+b^2=c^2.$$
$$\therefore 7^2+24^2=c^2.$$
$$\therefore 49+576=c^2.$$
$$\therefore c^2=625.$$
$$\therefore c=\sqrt{625}=25.$$

EXERCISE 152

1. In a right triangle, given $a=6$, $b=8$, find c.
2. In a right triangle, given $a=5$, $b=12$, find c.
3. In a right triangle, given $a=8$, $b=15$, find c.
4. In a right triangle, given $a=20$, $b=21$, find c.
5. In a right triangle, given $a=56$, $b=90$, find c.
6. In a right triangle, given $a=20$, $b=99$, find c.
7. In a right triangle, given $a=17$, $b=144$, find c.
8. In a right triangle, given $a=39$, $b=80$, find c.

9. In a right triangle, given $a = 51$, $b = 140$, find c.

10. In a right triangle, given $a = 44$, $b = 52.5$, find c.

11. In a right triangle, given $a = 87$, $b = 416$, find c.

12. In a right triangle, given $a = 136$, $b = 273$, find c.

13. In a right triangle, given $a = 145$, $b = 408$, find c.

14. In a right triangle, given $a = 207$, $b = 224$, find c.

15. A ladder is placed 14 ft. from a wall 48 ft. high. How long must the ladder be to reach to the top of the wall?

16. Find the length of the diagonal of a square if one side of the square is 10 rods.

17. Find the length of the diagonals of a rectangle, the dimensions of the rectangle being 17 rd. and 25 rd.

Example. If the hypothenuse of a right triangle is 493 and one leg is 468, find the other leg.

SOLUTION. Let the required leg be a.

Then, $$a^2 + 468^2 = 493^2.$$

$$\therefore a^2 + 219{,}024 = 243{,}049$$

Subtract 219,024 from each member of this equation.

$$\therefore a^2 = 24{,}025.$$

$$\therefore a = \sqrt{24{,}025} = 155.$$

EXERCISE 153

1. Hypothenuse = 377, base = 345, find the altitude.
2. Hypothenuse = 545, base = 513, find the altitude.
3. Hypothenuse = 449, base = 351, find the altitude.
4. Hypothenuse = 5.05, base = 4.56, find the altitude.
5. Hypothenuse = .461, base = .38, find the altitude.
6. Hypothenuse = .481, alt. = .36, find the base.
7. Hypothenuse = .641, alt. = .609, find the base.

8. Hypothenuse = .773, alt. = .195, find the base.

9. Hypothenuse = .697, alt. = .528, find the base.

AREAS OF PLANE TRIANGLES

The following rule gives the **area of any triangle**:

(1) Add the three sides and take half the sum.

(2) Subtract each side separately from the half sum.

(3) Find the continued product of the three remainders and the half sum.

(4) The square root of this product is the area.

This rule enables one to find the area of irregular tracts of land which can be divided into triangles whose sides can be measured.

Example. Find the area of a triangle whose sides are 34 ch., 65 ch., and 93 ch.

SOLUTION

$$\begin{array}{rrrrr} & 34 & 96 & 96 & 96 \\ & 65 & \underline{34} & \underline{65} & \underline{93} \\ & \underline{93} & 62 & 31 & 3 \\ 2) & 192 & & & \\ & 96 & & & \end{array}$$

$$\text{Area} = \sqrt{96 \times 62 \times 31 \times 3} = \sqrt{553535} = 744.$$

$$\therefore \text{area} = 744 \text{ sq. ch.} = 74.4 \text{ A.}$$

EXPLANATION. 192 is the sum of the sides. 96 is the half sum of the sides. 62, 31, and 3 are the remainders obtained by subtracting each side from the half sum of the sides. 553,535 is the continued product of the three remainders and the half sum. 744 is the square root of this product.

EXERCISE 154

Find the area of each of the following triangles:

1. Given the sides, 13, 20, 21.

2. Given the sides, 13, 30, 37.

3. Given the sides, 33, 34, 65.

4. Given the sides, 35, 52, 73.

5. Given the sides, 29, 60, 85.

6. Given the sides, 140, 143, 157.

7. Given the sides, 507, 603, 721.

8. Given the sides, 46 rd., 75 rd., 109 rd.

9. Given the sides, 40 rd., 51 rd., 77 rd.

10. Given the sides, 3.5 ch., 10 ch., 11.7 ch.

11. Given the sides, 5.6 ch., 6.1 ch., 7.5 ch.

12. Find area of a triangle, each side being 10 rd.

13. Find area of a triangle, each side being 50 rd.

14. Find the area of an isosceles right triangle if the hypothenuse is 27 inches.

15. Find the area of a square whose diagonal is 72 feet.

16. Find the side of a square equivalent to the difference of two squares whose sides are 89 feet and 68 feet.

MENSURATION OF THE CIRCLE, ETC.

Take a string, and find the length of the circumference of a circle. Take another string and find the length of the diameter of the circle. Divide the former result by the latter to get the ratio of the circumference of the circle to its diameter.

Let $r =$ radius of circle.
$c =$ circumference.
$d =$ diameter.

The ratio of the circumference of a circle to its diameter is approximately 3.1416. This ratio is denoted by the Greek letter π (Pi). In cases where the numbers involved are not very large, or where extreme accuracy is

not demanded, $3\frac{1}{7}$ is a sufficiently accurate approximation of π. In this chapter we shall consider $\pi = 3.1416$ unless otherwise stated.

Since $\frac{c}{d} = \pi$, $\therefore\ c = \pi d = \pi(2\,r) = 2\,\pi r$.

Express in words the relation $c = \pi d$.

Express in words the relation $c = 2\,\pi r$.

EXERCISE 155

Find the circumference when:

1. The diameter is 22.
2. The diameter is 46.
3. The diameter is 150.
4. The diameter is 164.
5. The diameter is 196.
6. The radius is 67.
7. The radius is 86.
8. The radius is 3.6.
9. The radius is 5.9.
10. The radius is 7.3.
11. Find d when $c = 320.44$.
12. Find d when $c = 477.52$.
13. Find d when $c = 24.50$.
14. Find d when $c = 41.7$.
15. Find r when $c = 377$.
16. Find r when $c = 53.41$.
17. Find r when $c = 60.319$.
18. Find r when $c = 42.097$.
19. The diameter of the front wheel of a carriage is 3 ft. 6 in. How many times does the wheel revolve in going 1 mi.? Take $3\frac{1}{7}$ as the value of π.

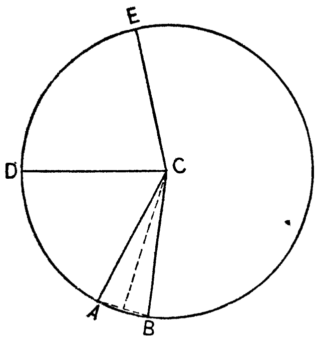
AREA OF A CIRCLE

A **sector of a circle** is a portion of a circle bounded by two radii and their included arc.

Thus, in Fig. 1, the figure bounded by EC, DC, and arc DE, is a sector.

Notice that the area of the sector ABC is not much larger than the area of the triangle ABC. Also that the altitude of triangle ABC is not much shorter than the

radius AC. Now imagine AB to become very much shorter. Then the radius of the circle may be considered as the altitude of the triangle, the arc AB may be considered as the base of the triangle, and the area of the sector may be considered to be the same as the area of the triangle. But the area of a triangle equals half the product of its base and altitude. Hence, the area of a sector equals half the product of its arc by the radius.



Let C stand for the area of the circle. Then C can be considered as the sum of a large number of sectors whose arcs added together make the circumference.

$$C = \tfrac{1}{2}cr, \text{ but } c = 2\pi r.$$
$$\therefore\ C = \tfrac{1}{2} \times 2\pi r \times r = \pi r^2.$$

Also, since $\quad r^2 = (\tfrac{1}{2}d)^2 = \tfrac{1}{4}d^2,$

$$\therefore\ \pi r^2 = \frac{\pi}{4}d^2.$$

Hence, the **three rules for finding the area of a circle:**

(1) Multiply one half the circumference by the radius.

(2) Multiply the square of the radius by π.

(3) Multiply the square of the diameter by $\tfrac{1}{4}\pi$.

There is a **fourth rule** for finding the area of a circle.

$$c = 2\pi r. \quad \therefore\ r = \frac{c}{2\pi}.$$

Squaring both sides of this equation gives

$$r^2 = \frac{c^2}{4\pi^2}.$$

Multiplying both sides by π gives

$$\pi r^2 = \pi \times \frac{c^2}{4\pi^2},$$

which reduces to

$$\pi r^2 = \frac{c^2}{4\pi}.$$

The left-hand member equals the area of the circle. Therefore, the right-hand member equals the area of the circle.

$\frac{c^2}{4\pi}$ may be written $\frac{1}{4\pi} \times c^2$. $\frac{1}{4\pi}$ reduces to .07958, which makes $\frac{c^2}{4\pi}$ reduce to $.07958\,c^2$.

That is, *the area of a circle equals the square of its circumference multiplied by* .07958.

EXERCISE 156

1. Given $r = 14$, find C.
2. Given $r = 22$, find C.
3. Given $r = 36$, find C.
4. Given $r = 4.7$, find C.
5. Given $r = 6.5$, find C.
6. Given $r = 8.6$, find C.
7. Given $r = 9.7$, find C.
8. Given $d = 78$, find C.
9. Given $d = 64$, find C.
10. Given $d = 96$, find C.
11. Given $d = 75$, find C.
12. Given $d = 56$, find C.
13. Given $d = 74$, find C.
14. Given $d = 92$, find C.
15. Given $c = 100$, find C.
16. Given $c = 78$, find C.
17. Given $c = 83$, find C.
18. Given $c = 93$, find C.
19. Given $c = 8.7$, find C.
20. Given $c = 6.9$, find C.
21. Given $c = 9.8$, find C.
22. Given $c = 10.8$, find C.
23. Given $c = 500$, find C.
24. Given $c = 25$, find C.

Example 1. Given the area of a circle equal to 535.08, find the radius of the circle.

SOLUTION.
$$\pi r^2 = C.$$
$$\therefore 3.1416\, r^2 = 535.08.$$
$$\therefore r^2 = \frac{535.08}{3.1416} = 170.31.$$
$$\therefore r = \sqrt{170.31} = 13.05, \text{ nearly.}$$

Example 2. Given the area of a circle equal to 658.98, find the circumference of the circle.

SOLUTION.
$$.07958\, c^2 = C.$$
$$.07958\, c^2 = 658.98.$$
$$\therefore c^2 = \frac{658.98}{.07958} = 8281, \text{ nearly.}$$
$$\therefore c = \sqrt{8281} = 91, \text{ nearly.}$$

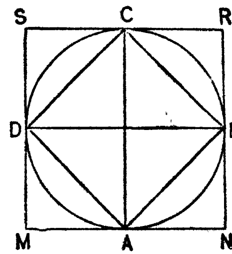
EXERCISE 157

1. Given $C = 3019.1$, find r.
2. Given $C = 907.9$, find r.
3. Given $C = 3421.2$, find r.
4. Given $C = 5541.8$, find r.
5. Given $C = 21.24$, find r.
6. Given $C = 40.715$, find d.
7. Given $C = 66.476$, find d.
8. Given $C = 75.43$, find d.
9. Given $C = 109.36$, find d.
10. Given $C = 141.03$, find d.
11. Given $C = 4.5964$, find c.
12. Given $C = 6.883$, find c.
13. Given $C = 779.94$, find c.

14. Given $C = 57,495$, find c.

15. Given $C = 33,621$, find c.

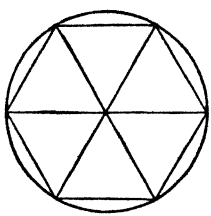
16. If a cow tied to a stake by a rope can graze over 1 A., how long is the rope?



17. *ABCD* is a square described in a circle; *MNRS* is a square described about the circle. If the radius of the circle is 12, find the areas of *ABCD*, *MNRS*, and of the circle.

18. If the diameter of a circle is 34 in., find the difference between the area of the circle and that of the square described in it.

19. If the diameter of a circle is 88 in., find the difference between the area of the square described about the circle and the area of the circle.



20. A regular hexagon is a figure of six equal sides and six equal angles. A hexagon may be broken up into six equilateral triangles.

If the side of a regular hexagon is 10, find its area.

21. If a regular hexagon is described in a circle, its side equals the radius of the circle. If the radius of a circle is 16 in., find the difference between the area of the circle and that of the regular hexagon described in it.

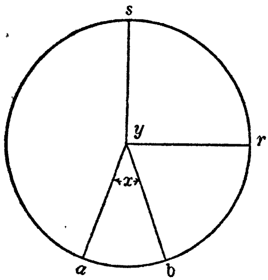
22. A regular hexagon and a square have each a perimeter of 60 in. Find their areas.

23. The circumference of a circle equals the perimeter of a square. Find which has the larger area.

24. What is the largest area that can be enclosed by a piece of woven wire fence 100 ft. long? What shape will it have?

ANGLES AND SUBTENDED ARCS

It is proved in geometry that angles at the center of a circle are proportional to the arcs which they subtend.



Angle x : angle y = arc ab : arc rs.

Also $x° : 360°$ = arc ab : circumference.

$y° : 360°$ = arc rs : circumference.

Example. The radius of a circle is 15 in. Find the length of an arc 37° 30′ of this circle.

SOLUTION. Circumference $= \pi \times$ diameter.

$\therefore$ circumference $= 3.1416 \times 30$ in. $= 94.248$ in.

$\therefore$ Arc $ab : 94.248 = 37.5 : 360$.

$\therefore 360 \times$ arc $ab = 94.248 \times 37.5$.

Arc $ab = 9.82$. *Ans.* 9.82 in.

EXERCISE 158

1. The radius of a circle is 37 in. Find the length of an arc of 72° of this circle.

2. The radius of a circle is 94 in. Find the length of an arc of 30° of this circle.

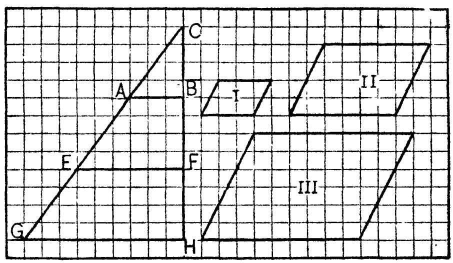
3. What is the length of an arc of 1° on a circle whose radius is 58 ft.?

4. What angle does an arc of 40.212 ft. subtend at the center of a circle whose radius is 64 ft.?

5. What angle does an arc of 6.032 ft. subtend at the center of a circle whose radius is 96 ft.?

6. The distance of the moon from the earth is 239,000 mi., and the diameter of the moon is 2170 mi. To an observer on the earth, what angle does the moon's diameter subtend?

Using for a measure the side of a small square as ruled, how long is CB? CF? CH? AB? EF? GH?



What is the value of the ratio $CB : CF$? of $CB : CH$? of $CF : CH$? of $AB : EF$? of $AB : GH$? of $EF : GH$?

Are any of these ratios equal? Which ones?

Is this a true proportion, — $AB : EF = CB : CF$?

Is this a true proportion, — $EF : GH = CF : CH$?

What kind of figures are ABC, EFC, GHC?

Name their bases. Their altitudes.

What is the length of the base of I? of II? of III?

What is the length of the altitude of I? of II? of III?

Are the following proportions true?

Base of I : base of II = altitude of I : altitude of II;

base of I : base of III = altitude of I : altitude of III;

base of II : base of III = altitude of II : altitude of III.

SIMILAR FIGURES

Similar figures are figures having the same form.

Triangles ABC, EFC, and GHC are similar triangles.

Parallelograms I, II, and III are similar parallelograms.

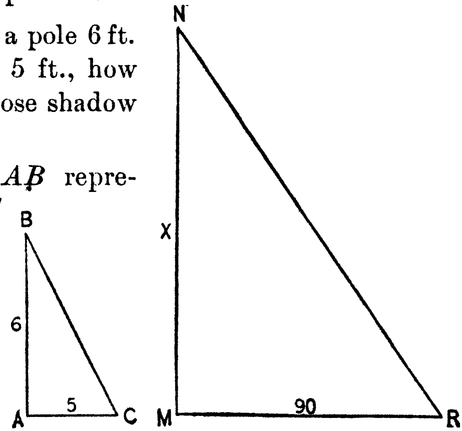
All regular polygons of the same number of sides are similar figures. The drawing which a surveyor makes of a tract of land is similar to the tract of land.

It is shown in geometry that **corresponding dimensions of similar figures have the same ratio.**

We noticed this to be true in the above figures. This fact is useful in computation.

Example. When a pole 6 ft. high casts a shadow 5 ft., how high is a steeple whose shadow is 90 ft.?

SOLUTION. Let AB represent the pole, AC its shadow, x the steeple, and MR its shadow. Then
$5 : 90 = 6 : x$.
$\therefore x = 108$.
Ans. 108 ft.



EXERCISE 159

1. When a tree 90 ft. high casts a shadow 75 ft. long, find the length of the shadow cast by a pole 24 ft. high.

2. How high is an object which casts a shadow 110 ft. when a pole 8 ft. high casts a shadow 5 ft.?

3. A map is drawn to a scale of 40 mi. to 1 in. On this map two cities are $2\frac{3}{4}$ in. apart. How many miles are there between these cities?

4. In a map of a city two public buildings are $9\frac{1}{2}$ in. distant. If the map is drawn to the scale of 1 in. to $\frac{3}{4}$ of a mile, how far is it from one of these buildings to the other?

Refer again to the illustrations.

Compute the area of triangle ABC. Of EFC. Of GHC. Of parallelogram I. Of II. Of III.

Find the values of the following ratios. Area ABC;

area EFC; area ABC : area GHC; area EFC : area GHC; area I : area II ; area I : area III ; area II : area III.

Are the following proportions true?

Area ABC : area $EFC = \overline{AB}^2 : \overline{EF}^2$.

Area EFC : area $GHC = \overline{FC}^2 : \overline{HC}^2$.

Area I : area II = square of base I : square of base II.

Area II : area III = square of altitude II : square of altitude III.

The areas of similar figures are to each other as the squares of their corresponding dimensions.

This fact is also useful in computations.

Example. The area of a triangle, one of whose sides is 5 rd., is 11 sq. rd. Find the corresponding side of a similar triangle whose area is three times as great.

SOLUTION. Let X equal the required side.

The area of the first triangle : the area of the second triangle = square of the side of the first triangle : square of the side of the second triangle.

$$i.e. \quad 11 : 33 = 5^2 : X^2.$$

$$\therefore 11\, X^2 = 33 \times 5^2.$$

$$\therefore \quad X^2 = \frac{\cancel{33} \times 5^2}{\cancel{11}} = 3 \times 5^2 = 75.$$

$$\therefore \quad X = \sqrt{75} = 8.662. \quad \textit{Ans.}\ 8.662 \text{ rd.}$$

5. The area of a triangle is 15 sq. ft., and one of its sides is 10 ft. Find the corresponding side of a similar triangle five times as large.

6. The altitude of a triangle is 10 ft. If the triangle is divided into two equal parts by a line parallel to its base, how far from the vertex must this line be drawn?

7. Corresponding sides of two similar quadrilaterals are in the ratio of 4 to 11. Find the ratio of their areas.

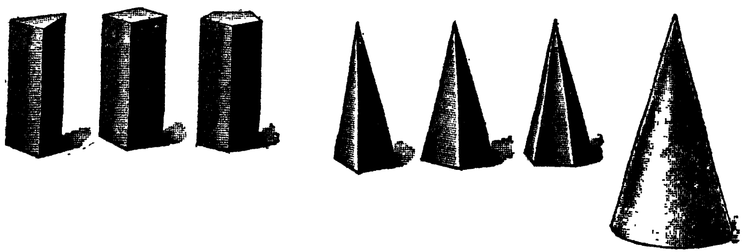
8. The diameters of two circles are 12 and 18 in. Find the ratio of their areas. Find the ratio of their radii. Find the ratio of their circumferences.

9. The distance between two cities is 90 mi., and on a map containing both cities their positions are distant $5\frac{5}{6}$ in. What area is represented by a circle of $\frac{1}{2}$ in. radius on this map?

SURFACES OF THE PRISM, PYRAMID, CYLINDER, CONE, AND SPHERE

A **right prism** is a solid, two of whose faces are equal and parallel polygons, and whose other faces are rectangles.

The upper and lower faces are called the *bases*, and the other faces are called *lateral faces*.



PRISMS PYRAMIDS CONE

A prism is named according to the shape of its bases. Thus, a prism whose bases are triangles is called a **triangular** prism. The area of the surface of a prism can be computed by adding together the areas of the separate faces.

The following rule may easily be established experimentally by paper cutting or other devices:

The **lateral surface of a right prism** equals the product of the perimeter of its base by the height of the prism.

EXERCISE 160

1. Find the lateral surface of a quadrangular prism, the dimensions of whose base are 16 ft. and 8 ft., and whose height is 12 ft.

2. Find the area of the walls of a room, having given the dimensions of the floor as 18 ft. and 16 ft., and the height as 10 ft.

3. Find the height of a triangular prism, the sides of its base being 5, 6, and 7 ft., and its lateral area being 190 sq. ft.

4. Find the height of a pentagonal prism whose lateral area is 300 sq. ft., and each side of whose base is 8 ft.

A plane figure bounded by straight lines is called a **polygon.** A polygon having its sides equal and its angles equal is called a **regular** polygon. Equilateral triangles and squares are examples of regular polygons.

A pyramid is named according to the shape of its base. Thus, a pyramid which has a hexagon for its base is called a hexagonal pyramid.

The lateral surface of a pyramid equals the sum of the triangles which form its faces.

How do you find the area of a triangle?

A **right pyramid** is a solid whose base is a regular polygon, and whose other faces are triangles equal in area.

The **lateral surface of a right pyramid** equals one half the perimeter of its base by the altitude of one of its lateral faces.

5. The base of a square pyramid is 40 ft. long, and the altitude of each of its triangular faces is 26 ft. Find its lateral area. Find the cost of painting its lateral surface at $2\frac{1}{2}$ ¢ per square foot.

6. Each side of a hexagonal pyramid is 14 ft., and its slant height is 15 ft. Find the area of its lateral surface.

7. A pyramidal tent whose base is a square 22 ft. on a side has a slant height of 30 ft. Find the cost of the canvas for the tent, at 18¢ per square yard.

8. Find the number of square yards in the lateral surface of a triangular pyramid, each side of the base being 21 ft. and the slant height being 42 ft.

A **cone** is a solid that we may imagine to be made by revolution of a right triangle about one of its legs.

A cone may be considered as a pyramid with a great number of very narrow faces. Then the base may be considered as a polygon of a great number of very short sides.

From this imaginary pyramid we obtain the rule for the lateral surface of a cone.

The **convex surface of a cone** equals one half the circumference of its base by its slant height. Why?

9. The radius of the base of a right cone is 49 in., and the slant height is 50 in. Find its convex surface in square feet.

10. Find the lateral surface of a right circular cone, the diameter of whose base is 64 in., and whose slant height is 40 in.

11. How many yards of canvas are required to make a conical tent, the diameter of the base being 24 ft. and the slant height 16 ft.?

A **cylinder** is a solid that we may imagine to be made by the revolution of a rectangle about one of its sides.

Make a cylinder of paper. Cut it open along a perpendicular line and spread the curved surface out flat. You have a rectangle whose base is the circumference of

the cylinder and whose altitude is the altitude of the cylinder.

The convex surface of a cylinder equals the product of its height by the circumference of its base.

12. Find the convex surface of a cylinder the diameter of whose base is 19 ft. and whose height is 50 ft.

13. Find the convex surface of a cylinder, the radius of the base being 41 in., and the height 60 in.

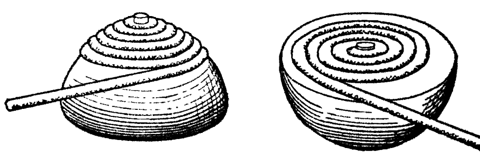
14. A standpipe has a diameter of 30 ft. and is 150 ft. high. Find the cost of painting it at 25¢ per square yard.

A **sphere** is a solid that is round like a ball. It is defined as a solid all points of whose surface are at equal distances from a point within called the center.

A **great circle** of a sphere is a circle formed by cutting the sphere into hemispheres.

It is proved by geometry that, if r is the radius of a sphere, the surface of the sphere equals $4\pi r^2$.

This can be shown readily by the use of model hemispheres as shown in the figure. Wind a cord on the flat base of the hemisphere until the surface is completely covered. Then wind a cord of the same size



on the curved surface of the hemisphere until that is completely covered. The length of cord required to cover the curved surface will be twice as long as that required on the flat base. Then the area of half the surface of a sphere is equal to twice the area of a great circle of the sphere. Therefore the area of the whole sphere is four times the area of a great circle. But the

area of a circle is πr^2. See page 298. Therefore, the surface of a sphere is equal to $4 \pi r^2$.

15. Find the surface of a sphere whose radius is 98 in.

16. Find the surface of a sphere if its diameter is 42 in.

17. The diameter of the planet Mercury is 3030 mi.; find the area of the planet.

18. The diameter of the planet Venus is 7700 mi.; find the area of the planet.

VOLUMES OF SOLIDS

The volumes of right prisms and cylinders are found by a method similar to that used in the case of rectangular solids.

The volume of a right prism is equal to the area of its base multiplied by its altitude.

The volume of a cylinder is equal to the area of its base multiplied by its altitude.

EXERCISE 161

1. Find the volume of a triangular prism, the sides of the base being 11, 25, 30 in., respectively, and the height of the prism being 40 in.

2. The surface of a cube contains 84 sq. ft. 54 sq. in. Find its volume.

3. How many gallons does a cylindrical cistern hold, the diameter of its base being 9 ft. 4 in. and its height 8 ft.?

4. How many gallons does a cylindrical cistern contain, if the diameter of its base is 11 ft. and its height is 6 ft. 5 in.?

VOLUME OF PYRAMID AND CONE

Make of paper a hollow prism and a hollow pyramid with equal bases and equal altitudes. Compare their volumes by filling the pyramid with sand and pouring the sand into the prism.

In a similar way compare the volumes of a cone and of a cylinder with equal bases and equal heights.

In this way it may be learned that the volume of a pyramid is one third the volume of a prism having the same base and altitude; and the volume of a cone is equal to one third the volume of a cylinder having the same base and altitude.

Therefore, the volume of a pyramid, or of a cone, is equal to one third the product of the area of the base by the height.

5. Find the volume of a square pyramid, if the sides of the base are each 10 in. and the height is 21 in.

6. Find the volume of a cone, the radius of its base being 12 in. and its height being 27 in.

7. Find the volume of a cone, if the radius of the base is 25 in. and the height is 24 in.

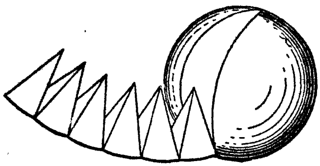
8. Find the volume of a hexagonal pyramid, each side of its base being 10 in., and its height being 30 in.

9. The base of a pyramid is a triangle whose sides are 1 ft. 1 in., 3 ft. 1 in., 3 ft. 4 in., and its volume is 1 cu. ft. 1152 cu. in. Find its height.

VOLUME OF SPHERE

A sphere may be considered as cut into a great number of figures, which are like pyramids, except for the curved base. The diagram shows how this may be done. If these

figures are cut so that their bases are very small, they may be considered to be pyramids and their volume can be found by the method used for pyramids.



The sum of all these pyramids is the sphere. The sum of the bases of these pyramids is the surface of the sphere, and the altitude is the radius of the sphere.

Therefore, the sum of the volumes of these pyramids is one third the product of the surface of the sphere by its radius.

This gives the volume of a sphere as $\frac{1}{3} \times 4\,\pi r^2 \times r$, which is given more briefly as $\frac{4}{3}\,\pi r^3$; and since $r = \frac{1}{2}\,d$, $\therefore\ r^3 = \frac{1}{8}\,d^3$;

$$\therefore\ \tfrac{4}{3}\,\pi r^3 = \tfrac{4}{3}\,\pi \times \tfrac{1}{8}\,d^3 = \frac{\pi}{6} d^3 = .5236\ d^3.$$

10. Find the volume of a sphere whose radius is 20 in.

11. Find the volume of a sphere, the radius being 8 ft.

12. The surface of a sphere equals 1257 sq. in. Find its volume.

13. The surface of a hemispherical dome is 2513.5 sq. ft. Find its diameter.

14. The volumes of similar solids are to each other as the cubes of their corresponding dimensions. How many times as large as the earth is the sun? The diameter of the sun is nearly 888,000 mi., and the diameter of the earth is nearly 8000 mi.

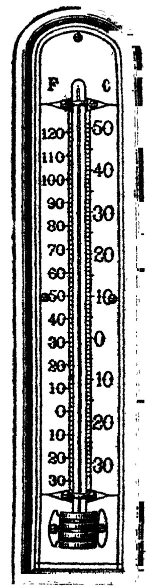
15. How many times as large as the moon is the earth? The moon's diameter is 2200 mi., nearly.

16. How many times as large as the earth is Saturn? The diameter of Saturn is 73,000 mi.

17. How many times as large as the earth is Jupiter? The diameter of Jupiter is 88,000 mi., nearly.

MEASURE OF TEMPERATURE

A **thermometer** is an instrument for measuring heat. The principle of the thermometer is that substances expand with heat according to a natural law.



There are two different styles of thermometers in general use,—the Centigrade and the Fahrenheit. The Centigrade is used on the continent of Europe for all purposes and in this country for scientific purposes. The Fahrenheit is commonly used in America as applied to "weather." The Centigrade thermometer marks the melting point of ice 0°, and the boiling point of water 100°. The interval between these points is divided into 100 parts, or degrees, so that the change in the volume of the mercury between any two consecutive marks is $\frac{1}{100}$ of the change from 0° to 100°.

The Fahrenheit thermometer divides the space between the melting point of ice and the boiling point of water into 180°. It marks the melting point of ice 32°, and the boiling point of water 212°. 0° is therefore 32° below freezing.

Notation. 92° C. means 92 degrees on the Centigrade thermometer.

45° Fahr. means 45° on Fahrenheit's thermometer.

+ 10° means 10° above zero.

− 10° means 10 degrees below zero.

Verify the following by counting 32° backward.

$$20^\circ - 32^\circ = -12^\circ.$$
$$10^\circ - 32^\circ = -22^\circ.$$
$$-2^\circ - 32^\circ = -34^\circ, \text{ etc.}$$

(1) **To change from degrees Fahrenheit to degrees Centigrade, subtract 32° and multiply the remainder by $\frac{5}{9}$.**

(2) **To change from degrees Centigrade to degrees Fahrenheit, multiply the number of degrees Centigrade by $\frac{9}{5}$ and add 32 to the product.**

Explanation of the rules:

(1) Suppose the temperature on a Fahrenheit thermometer is n degrees. Subtract 32° to get the number of degrees from 0°. A difference of 180° Fahrenheit = a difference of 100° Centigrade. Therefore, a difference of 1° Fahrenheit = a difference of $\frac{5}{9}$° Centigrade. Therefore, a difference of $(n - 32^\circ)$ Fahrenheit $= \frac{5}{9}(n - 32^\circ)$ Centigrade, which symbolizes the first of the above rules.

(2) A difference of n° C. = a difference of $\frac{9}{5}n^\circ$ Fahrenheit. Hence, n° C. $= (\frac{9}{5}n^\circ + 32^\circ)$ Fahr.

EXERCISE 162

Express the following Fahrenheit temperatures on the C. scale:

1. 86°.	**4.** 248°.	**7.** 38°.	**10.** − 13°.
2. 77°.	**5.** 68°.	**8.** 23°.	**11.** − 40°.
3. 203°.	**6.** 54°.	**9.** 15°.	**12.** − 90°.

Express the following C. temperatures on the Fahr. scale:

13. 55°.	**15.** 18°.	**17.** − 20°.	**19.** − 24°.
14. 25°.	**16.** 8°.	**18.** − 14°.	**20.** − 273°.

21. Give the following table in the Fahrenheit scale.

TABLE OF MELTING POINTS

Mercury	– 40° C.	Lead	326° C.	Gold	1035° C.
Sulphur	113° C.	Zinc	415° C.	Cast iron,	1100° to 1200° C.

25. Water attains its maximum density at 4° C. Express this temperature on Fahrenheit's scale.

SPECIFIC GRAVITY

If the weight of a substance is divided by the weight of an equal volume of water, the quotient is the **specific gravity** of the substance. **Specific gravity** is a statement of how many times as heavy as water a substance is. Thus, cast iron is 7.21 times as heavy as water, and hence its specific gravity is 7.21. Cork is about one fourth as heavy as water, and hence its specific gravity is $\frac{1}{4}$. **1 cu. ft. of water weighs 1000 oz.**

TABLE OF SPECIFIC GRAVITIES

Ash	.84	Ebony	1.33	Steel	7.83	Clay	1.2
Beech	.85	Glass	2.89	Copper	8.95	Mercury	13.57
Brass	8.40	Gold	19.26	Silver	10.47	Bar iron	7.79
Butter	.94	Granite	2.78	Lead	11.95	Platinum	21.5
				Ice	.92		

Example. Find the weight of 28 cu. in. of mercury.

SOLUTION. 1728 cu. in. of water weigh 1000 oz.

$$\frac{28}{1728} \times 1000 \text{ oz.} = 13.57 \text{ oz., weight of 28 cu. in. of water.}$$

$$13.57 \times \frac{28}{1728} \times 1000 \text{ oz.} = \text{weight of 28 cu. in. of mercury.}$$

$$\frac{13.57 \times \overset{7}{\cancel{28}} \times \overset{125}{\cancel{1000}}}{\underset{\underset{54}{\cancel{432}}}{\cancel{1728}}} = \frac{11873.75}{54} = 219.88. = 13 \text{ lb. } 11.88 \text{ oz.}$$

EXERCISE 163

1. Find the weight of 1 cu. ft. of steel; 1 cu. ft. of glass; 1 cu. ft. of clay.

2. Find the weight of 1 cubic inch of water. Find the weight of 1 gal. of water.

3. A cubic foot of marble weighs 2700 oz. Find the specific gravity of marble.

4. A cubic foot of sea water weighs $64\frac{3}{4}$ lb. Find the specific gravity of sea water.

5. A cubic foot of goat's milk weighs 65 lb. Find how many times as heavy as water goat's milk is.

6. The mercury in the barometer exactly counterbalances the pressure of the atmosphere. If the barometer is 30 in. high, find the pressure of atmosphere upon every square inch of surface.

7. A swimming pool of fresh water is 25 ft. by 16 ft. by 5 ft. Find the weight of the water it contains.

8. A block of granite is 6 ft. by 4 ft. by 2 ft. thick. Find its weight in tons.

9. Find the weight of 1 cu. in. of gold; of 1 cu. in. of silver; of 1 cu. in. of platinum; of 1 cu. in. of lead.

10. A block of ice 3 ft. by 2 ft. by 1 ft. thick weighs how many pounds?

11. What is the weight in tons of 1 cu. yd. of clay?

12. Find the weight of the butter required to fill a box 16 in. by 9 in. by 8 in.

13. Find the weight of the air in a hall 27 ft. by 24 ft. by 15 ft. 6 in. 1 cu. ft. of air weighs .08073 lb.

14. A cubic foot of coal weighs $81\frac{1}{4}$ lb. Find the specific gravity of coal.

15. How many cubic inches of copper weigh just as much as 1 cu. in. of platinum?

16. Find the weight of a block of ebony 4 ft. long, 9 in. wide, and 8 in. thick.

17. Find the weight of a block of ash 12 in. long, 8 in. wide, and 6 in. thick.

18. How many times as heavy as glass is mercury?

19. What is the weight of a bar of iron 2 in. by 2 in. by 8 ft.?

20. Find the weight of a beam of beech wood 8 in. by 6 in. by 12 ft.

THE METRIC SYSTEM OF WEIGHTS AND MEASURES

The metric system is now used by more than forty countries, and it is generally used in textbooks of science. Upward of twenty nations contribute to the support of the International Bureau of Weights and Measures in Paris. For these reasons the Metric System deserves to be called the International System.

The only great nations that have not adopted the Metric System are the United States and Great Britain. In the United States the system is legalized, and it is used in the Philippines and Porto Rico.

The Metric System is so called because the meter is the basis of the system. The **meter** is the standard unit of linear measure. Its length is the ten-millionth part of the distance from the equator to the north pole measured on the meridian of Paris. Its length in this country is 39.37 inches. In the United Kingdom the legal equivalent of the meter is 39.370113 inches, and on the continent of Europe 39.370432 inches.

The standard meter from which all others are derived is a bar made of an alloy of platinum and iridium, kept in the International Bureau of Weights and Measures in Paris. Duplicates of this standard meter have been furnished to all the nations of the world.

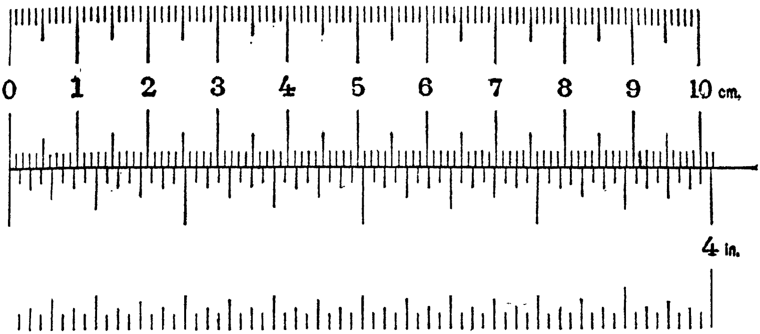
The Metric System is a decimal system. In it there are no compound rules. It is the simplest and the best system known.

The names for the multiples of the standard unit in the Metric System are formed from the names of the standard unit by means of the prefixes, deca-, hecto-, kilo-, and myria-, derived from the Greek words meaning ten, one hundred, one thousand, and ten thousand. The names for the submultiples of the standard units are formed in a similar manner by the prefixes deci-, centi-, and milli-, from the Latin words meaning ten, one hundred, one thousand. Thus:

Decameter means ten meters.
Hectometer means one hundred meters.
Kilometer means one thousand meters.
Myriameter means ten thousand meters.
Decimeter means one tenth of a meter.
Centimeter means one hundredth of a meter.
Millimeter means one thousandth of a meter.

The **standard units** are the *meter*, the *liter*, and the *gram*. The gram being a very small weight, the kilogram is most used in ordinary affairs. Where the Metric System is used, the meter, the liter, and the kilogram serve for everyday trade in exactly the same manner as the yard, the quart measure, and the pound Avoirdupois in our system of weights and measures. The international meter and the kilogram are, since 1893, legal standards of length and mass in the United States.

The **gram** is the weight of a cubic centimeter of distilled water at 4° Centigrade.



COMPARISON SCALE: 10 CENTIMETERS AND 4 INCHES. (ACTUAL SIZE.)

LINEAR MEASURE

10 millimeters (mm.)	= 1 centimeter
10 centimeters (cm.)	= 1 decimeter
10 decimeters	= 1 meter
10 meters (m.)	= 1 decameter
10 decameters	= 1 hectometer
10 hectometers	= 1 kilometer
10 kilometers (km.)	= 1 myriameter

The units in common use are the centimeter, meter, and kilometer.

SURFACE MEASURE

100 square millimeters (qmm.)	= 1 square centimeter
100 square centimeters (qcm.)	= 1 square decimeter
100 square decimeters	= 1 square meter
100 square meters (qm.)	= 1 are
100 ares	= 1 hectare
100 hectares (ha.)	= 1 square kilometer (qkm.)

An are is the area of a square whose side is ten meters. 1 qm. is therefore equal to a centare (ca.). The areas of

small tracts of land, such as gardens, are expressed in ares.

The areas of larger tracts of land, such as farms, are expressed in hectares. The areas of still larger tracts, such as countries, are expressed in square kilometers.

Cubic Measure

1000 cubic millimeters (cmm.)	=	1 cubic centimeter
1000 cubic centimeters (ccm.)	=	1 cubic decimeter, or liter
1000 liters (l.)	=	1 cubic meter (cbm.)

The liter corresponds in common use to our quart. The hectoliter (hl.) corresponds to our bushel. The cubic meter is also called a *stere*.

Weight

1000 milligrams (mg.)	=	1 gram
1000 grams (g.)	=	1 kilogram, or kilo
1000 kilograms (kg.)	=	1 tonneau (T.), or ton

A gram represents the weight of 1 ccm. of distilled water at 4° C.

A kilogram is the weight of 1 cu. dm. of distilled water at 4° C.

A tonneau, or ton, is the weight of 1 cbm. of distilled water at 4° C.

A quintal is 100 kg.

REDUCTION

Example 1. Reduce 75.623 m. to centimeters, and also to millimeters.

Solution. (*a*) Since there are 100 cm. in 1 m., reduce meters to centimeters by multiplying the number of meters by 100. This is done by moving the decimal point two places to the right.

(*b*) Since there are 1000 mm. in 1 m., therefore multiply the number of meters by 1000. This is done by moving the decimal point three places to the right.

Ans. (*a*) 7562.3 cm.
(*b*) 75623 mm.

Example 2. Reduce 85679 mm. to meters.

SOLUTION. This is done by dividing by 1000, *i.e.* by moving the decimal point three places to the left.

Ans. 85.679 m.

EXERCISE 164

1. With the aid of a metric ruler, draw lines whose lengths are 3.7 cm., 8.9 cm., 1.73 dm., 845 mm.

2. Draw lines 3, 7, and 11 in. long, and then measure them in centimeters.

3. Draw a line 7.5 in. long and measure it in decimeters, in centimeters, and in millimeters.

4. A man walks, on four successive days. The first day he walks 11.7 km.; the second day, 984 m.; the third day, 2950 m.; the fourth day, 12.8 km. How far does he travel? Give the answer in meters.

5. From 25.724 km. take 6270 m.

6. Multiply 11.732 m. by 12.

7. How many times is 25 mm. contained in 24 m.?

8. How many times is 7.03 m. contained in .0494209 km.?

9. How many times is 12 ca. contained in 6 ha.?

10. How many farms of 8 ha. each can be made out of a square whose side is 3 km.?

11. A vessel contains 250 ccm. How many such vessels would hold 5 cbm.?

12. How many times is 800 ccm. contained in 50 l.?

13. How many times is 450 ccm. contained in 22.5 hl.?

14. A dime weighs 2.5 g. How many dimes can be coined from 3 kg. standard silver? how many 25¢ pieces, weights being in proportion to values?

15. How many times is 1521 mg. contained in 5.9319 kg.?

Example. Find the area of a rectangle, if its length is 425.8 m. and its breadth is .3256 km.

SOLUTION. First, reduce .3256 km. to meters.

.3256 km. = 325.6 m.

Second, multiply in the usual manner and get for product 138,640.48 qm. Reduce this to ares by dividing by 100, and reduce the ares to hectares by dividing by 100. Both operations can be performed at once by moving the decimal point four places to the left. *Ans.* 13.864048 ha.

```
    425.8
    325.6
    25548
   21290
  8516
12774
138640.48 qm.
```

EXERCISE 165

Find the area of each of the following rectangles:

	LENGTH	WIDTH		LENGTH	WIDTH
1.	625 m.	125 m.	**4.**	369.4 m.	184.7 m.
2.	305 m.	61 m.	**5.**	488.9 m.	244.45 m.
3.	338 m.	169 m.	**6.**	5767 m.	11.534 m.

7. Find the volume of a rectangular solid whose dimensions are 1.2 m., 9 m., and .75 m.

8. Find the area of a triangle whose sides are 14 m., 48 m., and 50 m.

9. Find the area of a circle whose radius is .78 m.

10. Find the volume of a sphere whose radius is 18 cm.

11. The legs of a right triangle are 56 cm. and 105 cm. Find the hypothenuse.

12. Find the volume of a rectangular prism, the base being 14 m. by 12 m., and height 9.5 m.

13. A boiler has 300 tubes 2.4 m. long, 7.5 cm. diameter. What is the area of the tube heating surface?

14. Find the weight of a spherical cast iron shell 32.5 cm. outside and 27.5 cm. inside diameter.

15. An iron plate 3 mm. thick weighs 1 kg. What is its area?

The following approximations should be fixed in mind:

1 meter = 39 inches; 1 kilometer = $\frac{5}{8}$ of 1 mile.
1 centimeter = $\frac{2}{5}$ of an inch; 1 hectare = $2\frac{1}{2}$ acres.
1 liter = 1 quart; 1 kilogram = $2\frac{1}{5}$ pounds Avoirdupois.

Equivalents of Common Units in Metric Units

1 inch	= 25.4001 mm.	1 sq. foot	= .0929 qm.
1 foot	= .304801 m.	1 sq. yard	= .8361 qm.
1 yard	= .914402 m.	1 A.	= .4047 ha.
1 mile	= 1.60935 km.	1 sq. mi.	= 2.59 qkm.
1 quart	= .94636 l.	1 cu. inch	= 16.3872 ccm.
1 gallon	= 3.78543 l.	1 cu. foot	= .02832 cbm.
1 bushel	= .35239 hl.	1 cu. yard	= .7646 cbm.
1 sq. inch	= 6.452 qcm.	1 lb. Avoir.	= .45359 kg.

Equivalents of Metric Units in Common Units

1 meter	= 39.37 in.	1 sq. centimeter	= .155 sq. in.
1 kilometer	= .62137 mi.	1 sq. meter	= 1.196 sq. yd.
1 hectare	= 2.471 A.		
1 qkm.	= .3861 sq. mi.		
1 cu. centimeter	= .061 cu. in.		
1 cu. meter	= 35.314 cu. ft.		
1 liter	= 1.0567 qt.*	1 kilogram	= 2.20462 lb.
1 hectoliter	= 2.83774 bu.	1 tonneau, or ton	= 2204.6 lb.

* Liquid quarts, or 0.9081 dry quarts.

PERSONAL AND FAMILY SAVINGS

One of the most important uses of Arithmetic is to help people to manage their affairs in such a manner as to produce the most satisfactory results. Such management is called thrift.

Thrifty management of one's affairs requires that accurate record be kept of business transactions and that plans be made in advance for the wise expenditure of income.

An accurate written record of money received, spent, and saved is called a **cash account.**

A plan made in advance for the expenditure of one's income for a week, or month, or a year is called a **budget.**

EXERCISE 166

The method of keeping the personal cash account with which we are most familiar is the following:

PERSONAL CASH ACCOUNT

			Received			Paid Out	
Sept.	19—						
	2	Balance on hand	$3	75			
		Received weekly allowance	1	00			
		Received for errand		10			
	3	Paid for entertainment					25
		Deposited in Savings Bank				2	00
	4	Received for labor		40			
	6	Paid for books				1	75
	7	Received for labor		25			
		Balance (red ink)			(red ink)	1	50
			5	50		5	50
	9	Balance on hand	1	50			

1. How many of the following questions related to the preceding account can you answer?

(*a*) What items in this account represent earnings? What is the total amount earned?

(*b*) What items represent spending? What is the total amount?

(*c*) What items represent savings? What is the amount?

(*d*) If one were examining this account for signs of thrift, what items would appear to be satisfactory?

(*e*) In order to be thrifty is it necessary that one avoid all expense for entertainment, books, etc.? To what extent may one spend his money and still be thrifty? May one reasonably spend to maintain health? To secure education? For patriotic purposes?

(*f*) May one be saving in his personal habits but wasteful of the family income? Of the material of the school? Of the advantages offered by the community? Explain how?

(*g*) How may one help his family or his community to save? Will watchful care of the family supplies help? Will care of public property help the community? How?

2. Notice the word "Balance" and the item, $1.50 in the paid out column. Can you find out from this account how this balance is found? Why is it written in the paid out column? Why in keeping accounts does the accountant write this item in red ink?

3. Rule a sheet of your notebook for a cash account and write in such items as a physician might have in his account for a week. Find the balance and close the account as indicated in the preceding account.

4. Make a cash account for a week for a farmer, a carpenter, a salesman, a housekeeper.

5. Copy and use the following form of budget and personal cash account, and fill in the amounts:

Amount to be Received			Set Aside for Savings			Allowed for Wise Expenditure		
1. On hand			1. For______			1. For______		
2. Allowance						2. For______		
3. Earnings						3. For______		

6. Make a form for your personal budget and your cash account for one month. Keep your account as required by the budget.

7. An important item in any budget is "money received." How are you to get money to save and to spend? Can your parents afford to give you an allowance? In war times parents are under heavy burdens for taxes, for the purchase of bonds, for contributions to the Red Cross, etc. Is there any other source that you can think of from which you can secure money? Can you earn money? How? Should one expect to be paid for helping in his own home? Why not?

8. Sometimes cash accounts, especially when they contain a large number of items, are arranged as follows:

Receipted					Paid Out				
19—					19—				
Nov.	1	Balance forwarded	$110	29	Nov.	1	Board and room	$7	50
	7	Salary	25	00		4	Laundry		60
						5	Clothing	2	25
						6	Concert ticket	1	00
						7	City fare and lunch	1	12
							Balance (red ink)	(?)	
		Balance forwarded	(?)						

The left-hand side is called the *debit* side, and the right-hand side the *credit* side, in accounts. Explain how the balance is made out. What suggestion for thrift may be made in this account?

EXERCISE 167

5. It has been suggested as a patriotic service that every boy and girl in the upper grades of the elementary school keep in a notebook a record showing a personal budget and cash account like the following:

RECORD OF SAVINGS By ______________ ______________

Of school ______________ Home address ______________

During week beginning (dated) ______________

(1) I Expect to Receive —	(2) Expect to Save —	(3) Expect to Spend —
On hand ________ $ ______		For school ________ $ ______
From earnings ________ $ ______	For ________ $ ______	For war relief ________ $ ______
From gifts ________ $ ______	For ________ $ ______	For ________ $ ______
	Total ______ $ ______	
From ________ $ ______		Total ________ $ ______
Total ______		

Month and Day	Balance on Hand	Amount Received	Received From	Amount Saved	Amount Spent	Spent For
	Totals for week .					

1. Copy the form of the budget and cash account and carry through an imaginary account of a week's receipts, savings, and expenditures.

2. Examine the items in the imaginary account that you have made to see how the amounts received compare from day to day with the amounts spent.

3. Suppose that the total for the week of money spent equals the total amount received, how much have you saved? If an account shows such a condition, would it not seem to indicate a lack of thrift?

4. Explain as fully as you can why it is necessary to be thrifty, and how a budget and personal cash account will help one to be thrifty.

5. In order to carry on the war in which we are now engaged, our government has been obliged to raise great sums of money. Plans have been made in advance not only for the spending of this money but for the raising of it. One of the sources from which the government plans to raise this money is through the sale of War Savings Stamps, little government bonds that every one should save to buy. How many kinds of War Savings Stamps are there? What is the maturity value of each? Which one bears interest? Where in the budget and expense account should record be made of the plan to purchase these little bonds?

6. Explain how interest on War Savings Certificates is paid. Can you get this information at your bank?

FAMILY ACCOUNTS AND BUDGETS

Next in importance to the personal cash account and budget is the family account and budget. It is the duty of every member of the family to help the family save.

To-day it is not only a duty but a patriotic service. How may one help his family to save? Keep a careful record of the family expenses, savings, and earnings. Study this record to find ways to economize without injury to health or ability to serve. By pointing out these opportunities one may help the family to save.

EXERCISE 168

1. What are the chief expenses of a family? One classification of family expenses consists of the following:

(*a*) For food that has to be purchased.

(*b*) For rent of the house or apartment where the family lives, or for interest on mortgage, taxes, insurance, and repairs if the house be owned by the family.

(*c*) For housekeeping expenses, such as heat and light, help, repairs to furniture, etc.

(*d*) For clothing for the family.

(*e*) For personal expenses of members of the family, such as insurance, vacation, medicine, newspapers, car fare, education, charity, religion, etc.

(*f*) For savings, such as investments in Liberty Bonds, War Savings Stamps, Savings Bank Accounts, purchase of property.

What are the chief expenses of your family? Does your family keep a record of its expenses?

2. How much may the family's expense be? This depends upon many conditions. No two families can be found whose expenses will be the same. The important conditions that affect the expenditure of a family are:

(*a*) The number of people in the family and the ages of the children.

(*b*) Whether the family lives on a farm, in a small town, or in a large city.

(*c*) Whether the family rents or owns its own home.

(*d*) Whether the family income is small or large.

How do these conditions affect the expenditures of families in your neighborhood?

3. What is a convenient form for keeping the family account? The family account, like the personal account, should consist of two parts, viz: the family budget for the day, week, or month; and the family expense account. In a clear and simple way these accounts may be kept in a notebook ruled as follows:

WEEK BEGINNING __________ NAME ____________________

On hand at beginning of week __________	$ __________
Total __________	

Budget, or plan to spend:

For food __________	
House (rent or expense) __________	
Housekeeping __________	
Clothing __________	
Personal __________	
Savings __________	
Total __________	

4. Copy the form and fill in an imaginary budget for a family of three children and their parents with an income of $30 a week.

5. Make a budget for the expenditures of the income of your family for one week.

6. The weekly family expense account may be kept in a notebook ruled as follows:

Family____________________ Week of____________________

Date	Article or Expense	Cost	
	Total expenditures		

7. Rule a sheet of paper in your notebook and fill in an imaginary family expense account for one week for a family of three children and their parents with an income of $40 a week.

8. Rule a sheet of your notebook and classify the following items as suggested in Exercise 3.

WEEKLY SUMMARY OF EXPENSES

Family____________________ Week____________________

Money on hand at the beginning of the week			Money spent for food . .		
			Money spent for house .		
			Money spent for clothing		
Total money received during the week . .			Money spent for personal use		
Total money for use during the week . .			Money deposited in savings bank or invested in safe investments .		
Total money spent . .					
Balance on hand for next week			Total money spent . .		

9. The income and expenses of a family of five who live in a small city were for the month of May as follows:

Income . . .	$110.00	Fruit	$2.20
Rent	25.00	Vegetables . . .	2.45
Groceries	16.50	Light and Gas . .	1.70
Theater	2.50	Life Insurance . .	1.20
Church	1.75	Clothing	7.75
Laundry	.75	Shoes	3.50
Meat	5.70	Milk	2.10
Butter	2.10	Papers and Postage	.80
Miscellaneous Items including payment on Liberty Bonds of $5.00			11.00

Classify the expense items under the general heads mentioned in Exercise 8. Find the total expenses and the balance on hand at the end of the month.

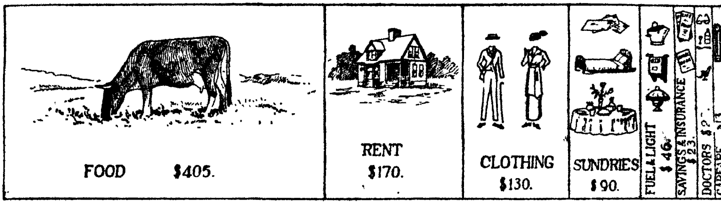
10. In Exercise 9 what per cent of the total income is spent for rent? For food? For clothing? For operating expenses? For personal use? What per cent of the income is left for savings at the end of the month?

11. Examine the items of expense in Exercise 9 to find out whether any of them should be changed for a family of 5 in your neighborhood with an income of $110.00 per month.

12. It has been estimated that a family, living in a city, which has an income of $1000 a year should divide the income as follows:

Rent	20 %
Food	30 %
Clothing	15 %
Operating expenses . .	10 %
Personal use	25 %

How much of the $1000 should be allowed for each division?



TYPICAL DIVISION OF A SMALL INCOME

13. In the illustration of the typical division of a small income what is the total income? About what part of the income is given to rent? Food? Operating expenses? Clothing? Personal use? Under what headings is savings? How much does this budget allow for savings? What per cent of the whole income is this? Since this is a typical division of a small income, are families with this income saving as much as they should? What suggestions can you make to provide for savings here?

14. What items in the division of income can be reduced by food from the farm? By the food from the home garden? Estimate the amount in each case.

15. If possible estimate the income of a family in your community and decide how it is divided.

INCOME PROBLEMS

How much of the income shall be spent for food? How much for the house; how much for housekeeping; how much for clothing; and how much for personal expense and savings? For incomes between $20 and

$30 a week the following table on the basis of $1 has been worked out:

Chief Expense Items	Of Each Dollar Spent —	For an Income of	
		$20 per Week	$30 per Week
1. For food	5 dimes	?	?
2. For house (rent or expense)	2 dimes	?	?
3. For housekeeping	1 dime	?	?
4. For clothing	1 dime	?	?
5. For personal expense and savings	1 dime	?	?

EXERCISE 169

1. From an income of $20 per week what expenditure may be allowed for each of the chief items of family expense?

2. From an income of $30 per week what expenditure may be allowed for each of the chief items of family expense?

3. In this plan the expenditure for food is what per cent of the income? Is that too large an amount? How may this be reduced without injury?

4. What is the second most important item of expenditure? How may it be reduced?

5. In the plan what expenditure is allowed for rent? What two items should be watched closely in this account? Why?

6. Rule your note book and copy the form of spending. Make a satisfactory distribution of an income of $1000.

7. In spending the income how may the money for food be spent for foods of different kinds to advantage?

A distribution of the money among different foods may be made to advantage according to the following table:

The Chief Kinds of Food Bought by a Family with a $20–$30 Income	Of $1, Spent for —	For a Food Expenditure of $10 per Week Spent —
1. Grain foods, breads, cereal, meal, spaghetti, rice, etc.	2½ dimes	$2.50
2. Meat	2 dimes	2.00
3. Milk	2 dimes	2.00
4. Fruits and vegetables	2 dimes	2.00
5. Miscellaneous (fats, butter, oil, lard, sugar, beverages) . . .	1½ dimes	1.50

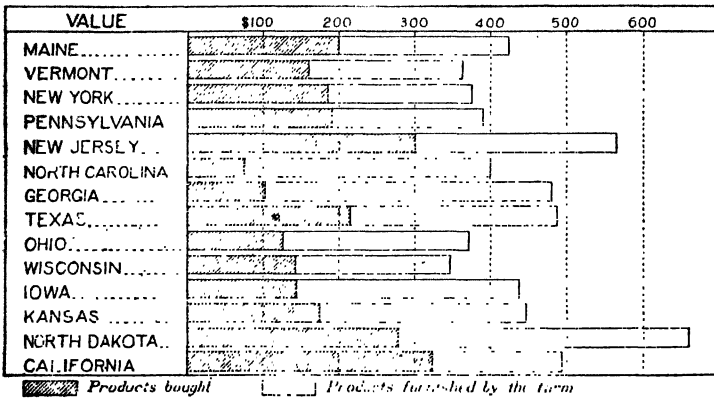
8. According to the preceding table what are the chief kinds of food? How much of the weekly food allowance for a family with a $20 to $30 per week income may according to the preceding plan be spent for each kind of food?

9. How much of this food can be produced in the home garden? What can be secured from the farm?

INCOME FROM THE FARM

In any study of the expenditure of income the cost of food is a large item. What per cent of income is usually spent for it? For those who live on farms how large a percentage of the food of the family is furnished by the farm? From the information gathered in a study made by the United States Department of Agriculture of 950 families in various sections of the United States to find out the average annual value of food used per family, the amount of this food that the families bought, and the

amount that was furnished by the farm, the following graphical illustration has been made.



THE VALUE OF THE FOOD USED BY THE AVERAGE FAMILY THAT LIVES ON A FARM, THE VALUE OF THE FOOD PRODUCTS BOUGHT AND OF THOSE FURNISHED BY THE FARM.

EXERCISE 170

1. Study the foregoing illustration to answer the following questions. Estimate the average annual value of food used per family as shown in this illustration. How much is it for Texas, California, North Carolina, and Georgia?

2. For Texas what approximately is the annual value of this food furnished by the farm? What per cent of the whole value is this?

3. What is the approximate annual value of the food bought by a Texas family? What per cent is it of the whole value?

4. If there are on the average 4.8 persons in each family, how much is the total annual cost or value of food per person for these Texas families?

5. If the families in North Dakota average 6.2 persons in each family, what is the total annual value of food per person in that state?

6. For families who own and live on farms, in addition to the food the use of the house is usually provided. Estimate the saving in rent that this use of the house saves the farmer.

7. The average value of farms in the United States is about \$5000. Suppose that on an average the live stock and equipment are worth \$3000 additional. On this total investment how much yearly at 8% ought the farmer to receive? Because the farmer assumes some risk in putting his money into this enterprise should he receive more than the usual rate of interest?

8. The income in cash of a farmer is \$1000. The income on his farm investment of \$8000 is at the rate of 5%, food from the farm and the use of the house in which he lives saves him \$1200. To what is his total income from these sources equivalent?

9. The most important items of farm investment are as follows: (*a*) land and buildings, (*b*) live stock, teams, etc., (*c*) machinery and tools, (*d*) farm products on hand, (*e*) household goods. A complete list of items with their values computed, usually at the cost price, is called an *inventory*.

10. An inventory of a farm investment on Jan. 1, 1900, shows items as follows: land and buildings, \$10,000; live stock, teams, etc., \$2500; machinery and tools, \$2500; farm products on hand, \$850; household goods and equipment, \$3000. What would be the yearly income on this investment if the money were invested in securities that paid 5% per annum?

SAVINGS BANK ACCOUNTS

There are many ways of saving money. Personal budget and cash account help to reduce unnecessary expenses, and the family budget is a guide, for that helps the family to save. There are ways of saving, such as good judgment in buying, of which no record can be kept.

One of the best ways of saving money is to put it in the savings bank. In many schools, school savings banks, branches of the local city banks, have been opened in which deposits may be made and business carried on as it would be with the bank itself. Have you such a bank in your school?

In order to put money in the savings bank one must open a savings account. Do you know how to do this? One must go to the savings bank; give his name, address, and business references; place his signature on a card for use in comparison and identification; make his deposit of money, and receive his bank book in which his savings account will be kept.

The usual method of making a deposit after the savings account is opened is to make out what is called a *deposit ticket* showing the amount in bills, coin, and checks deposited, and hand it to the receiving teller at the bank together with the deposit and the bank book. The receiving teller will check up the deposits, find the total, enter the total in the savings bank book, return the book, and see that credit in your name is given on the bank's ledgers for the deposit.

If you can arrange to do so, visit the savings bank or company in your home city and find out how to open a savings account.

EXERCISE 171

1. If we were to reproduce a page from a depositor's savings bank book, often called a pass book or call book, we would find items upon it as follows:

HOME SAVINGS BANK

Account of Mr. Samuel Snow

19—		Deposited		Withdrawn		Interest		Balance	
Jan.	2	40	00					40	
Feb.	1	50	00					90	
Mar.	4	45	00					135	
Apr.	1	100	50					235	50
May	1	25	25					260	75
June	3	40	00					300	75
July	1			25	00	2	75	278	50

2. In the preceding account what is the total amount deposited? What is the sum withdrawn? What interest was paid on the deposits? What is the balance in the bank on July 1, 19—?

3. At what dates was interest paid on the deposits? What will be the next date at which interest will be paid? How many times yearly will interest be paid on this account? How often is the interest compounded?

4. What sum of money has been on deposit for six months preceding July 1? Suppose that the bank allows 3 days at the first of the month, in which deposits on the savings account may be made and draw interest, for how many months will the deposit of Jan. 2 draw interest?

5. If the bank pays semi-annual interest on savings accounts at the rate of 4 per cent, what interest will the deposit of Jan. 2 earn by July 1? (See solution of examples pp. 161, 239.)

6. Although savings banks usually compute interest semiannually on Jan. 1 and July 1, it is customary for them to pay interest on balances that have been on deposit for three months preceding these dates. In Exercise 1, what deposits made after Jan. 2 were on deposit three months previous to July 1? Was any money withdrawn during this period?

7. On or before the beginning of what month should deposits be made in order to draw interest on July 1? What deposit in Exercise 1, in addition to the deposit of Jan. 2, will draw interest on July 1?

8. What is the interest on $195.50 for three mo. at 4%? Find the interest on $195.00. What is the total interest on these deposits on July 1? (Interest on fractions of a dollar is generally not counted.)

9. The practice of paying interest on savings deposits varies in different banks and savings institutions. What are the dates upon which your local savings bank pays interest on deposits? How long must money be on deposit before it will begin to draw interest? What are the rules of the bank for withdrawing money from the savings account?

10. Rule a page for a savings account. Fill in the deposits and withdrawals for the six months from Jan. 1 to July 1. Fill in the items from July 1 to Jan. 1. Compute the interest. Find the balance in the bank at the end of one year. Find out from your bank officers whether the account is correctly kept.

11. What reasons can you give for depositing money on hand in the savings bank? What use does the bank make of the money deposited?

In business practice it is customary to use interest tables in figuring interest. The following forms are among the most simple:

INTEREST TABLE

Four Per Cent

	$1	$2	$3	$4	$5	$6	$7	$8	$9	$10	$100	$1000
1 da. . .	$.00	$.00	$.00	$.00	$.00	$.00	$.00	$.00	$.00	$.00	$.01	$ .11
3 da. . .	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00½	.03½	.33
5 da. . .	.00	.00	.00	.00	.00	.00	.00	.00½	.00½	.00½	.05½	.56
10 da. . .	.00	.00	.00	.00½	.00½	.00½	.01	.01	.01	.01	.11	1.11
1 mo. . .	.00	.00½	.01	.01½	.01½	.02	.02½	.02½	.03	.03½	.33	3.33
2 mo. . .	.00½	.01½	.02	.02½	.03½	.04	.04½	.05½	.06	.06½	.67	6.67
3 mo. . .	.01	.02	.03	.04	.05	.06	.07	.08	.09	.10	1.00	10.00
4 mo. . .	.01½	.02½	.04	.05½	.06½	.08	.09½	.10½	.12	.13½	1.33	13.33
6 mo. . .	.02	.04	.06	.08	.10	.12	.14	.16	.18	.20	2 00	20.00
9 mo. . .	.03	.06	.09	.12	.15	.18	.21	.24	.27	.30	3.00	30.00
1 yr. . .	.04	.08	.12	.16	.20	.24	.28	.32	.36	.40	4.00	40.00

Five Per Cent

	$1	$2	$3	$4	$5	$6	$7	$8	$9	$10	$100	$1000
1 da. . .	$.00	$.00	$.00	$.00	$.00	$.00	$.00	$.00	$.00	$.00	$.01	$ .14
3 da. . .	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.04	.42
5 da. . .	.00	.00	.00	.00	.00	.00	.00	.01	.01	.01	.07	.69
10 da. . .	.00	.00	.00	.00	.01	.01	.01	.01	.01	.01½	.14	1.39
1 mo. . .	.00½	01	.01	.02	.02	.03	.03	.03	.04	.04	.42	4.17
2 mo. . .	.01	.01½	.03	.03	.04	.05	.06	.07	.08	.08	.83	8.33
3 mo. . .	.01	.02½	.04	.05	.06	.08	.09	.10	.11	.13	1.25	12.50
4 mo. . .	.01½	.03	.05	.07	.08	.10	.12	.13	.15	.17	1.67	16.67
6 mo. . .	.02½	.05	.08	.10	.13	.15	.18	.20	.23	.25	2.50	25.00
9 mo. . .	.03¾	.07½	.11	.15	.19	.23	.26	.30	.34	.38	3.75	37.50
1 yr. . .	.05	.10	.15	.20	.25	.30	.35	.40	.45	.50	5.00	50.00

SIX PER CENT

	$1	$2	$3	$4	$5	$6	$7	$8	$9	$10	$100	$1000
1 da. . .	$.00	$.00	$.00	$.00	$.00	$.00	$.00	$.00	$.00	$.00	$.02	$ .17
3 da. . .	00	.00	.00	.00	.00	.00	.00	.00	.00	.01	.05	.50
5 da. . .	.00	.00	.00	.00	.00	.01	.01	.01	.01	.01	.08	.83
10 da. . .	.00	.00	.01	.01	.01	.01	.01	.01	.02	.02	.17	1.67
1 mo. . .	.00½	.01	.02	.02	.03	.03	.04	.04	.05	.05	.50	5.00
2 mo. . .	.01	.02	.03	.04	.05	.06	.07	.08	.09	.10	1.00	10.00
3 mo . .	.01½	.03	.05	.06	.08	.09	.11	.12	.14	.15	1.50	15 00
4 mo. . .	.02	.04	.06	.08	.10	.12	.14	.16	.18	.20	2.00	20.00
6 mo. . .	.03	.06	.09	.12	.15	.18	.21	.24	.27	.30	3.00	30.00
9 mo. . .	.04½	.09	.14	.18	.23	.27	.32	.36	.41	.45	4.50	45.00
1 yr. . .	.06	.12	.18	.24	.30	.36	.42	.48	.54	.60	6.00	60.00

EXERCISE 172

1. Using the interest table, find the interest on $100 for one month at 6%. At 5%. At 4%.

2. Find the interest on $200 for 1 year at 6%. 2 years at 5%. 3 years at 4%.

3. Find the interest at 6% on $500 for 1 year. For 6 months. For 3 months. For 1 month. For 2 months. For 1 day. For 10 days.

4. Find the interest at 4% on $600 for 1 year. For 2 years. For 6 years. For 6 months. For 4 months. For 15 days.

5. From the four per cent table make a new table at eight per cent on $100. On $10.

6. From the six per cent table make a new table for $1000 at 3%.

7. From the four per cent table make a new table at 1% on $100. ½% on $100.

8. Having an interest table for 4 % and one for 3 % on \$100, how can an interest table at 7 % be made?

9. Explain how to make a $4\frac{1}{2}$ % interest table. A $4\frac{1}{4}$ % interest table.

10. Compare the simple interest table, p. 341, with the compound interest table on p. 261. Explain the use of each.

EXERCISE 173

Discuss the following topics. Consider what are the important points under each topic. Tell what is to be said under each point. Write briefly the main facts under each point.

1. A good plan to follow in personal saving.
2. A good plan to follow in helping the family to save.
3. A good plan to follow in investing one's savings.
4. The ways and means of earning money that are open to you.
5. The principal items in the household expenses of your family and the ways to reduce them. The five principal items in a family.
6. How the family dollar may be spent to advantage.
7. The best way to spend a dollar for food.
8. How to open a savings bank account.
9. How the savings bank helps one to save.
10. How an individual may help a community to save.
11. How a person may do his part in helping the state to save.
12. What may a person do to-day to help the National Government provide the labor, the food, and the money that it needs to meet the obligations it is undertaking?

MISCELLANEOUS EXAMPLES

(ARRANGED BY TOPICS)

NOTATION

1. Write in figures ten million ten.

2. Write in figures seven million two hundred five thousand.

3. Write in figures one billion one million one.

4. Write in figures five million fifty thousand.

5. Write in figures ten billion ten million one hundred.

6. Write decimally: Twenty-five tenths. One hundred twenty hundredths. One hundred and fifty-five hundredths. Ten thousand ten, hundred thousandths. One million one, ten-millionths. Fifty-five thousand two hundred eighteen ten-thousandths. Twelve millionths.

ADDITION AND SUBTRACTION

7. The following table of railroad mileage in the United States is taken from the report of the Interstate Commerce Commission:

YEAR	MILEAGE	YEAR	MILEAGE
1890	163,597.05	1898	186,396.32
1891	168,402.74	1899	189,294.66
1892	171,563.52	1900	193,345.78
1893	176,461.07	1901	197,237.44
1894	178,708.55	1902	202,471.85
1895	180,657.47	1903	207,977.22
1896	182,776.63	1904	213,904.34
1897	184,428.47	1905	218,101.04

Find the number of miles built during each year, beginning with 1890, up to 1905.

8. From 9000 take .009.

9. From 275 take .000275.

10. Add:

657,987,324,011
119,008,675,987
199,887,564,999
999,555,777,888
987,345,234,876
985,234,678,100
923,524,896,987
987,567,342,959
725,926,846,368
929,935,829,349
929,563,768,968

MULTIPLICATION

11. Multiply 10,500 by 60,600.

12. Multiply 15,010 by 50,080.

13. Multiply 1.01 by 7.07.

14. Multiply 9010.9 by 90.4.

15. A train travels at the rate of 30.25 mi. an hour. How far will it go in $5\frac{1}{2}$ hr?

16. Find the price of $87\frac{1}{2}$ A. of land at $43.75 an acre.

17. Find the price of 4225 bu. of wheat at $84\frac{1}{2}$¢ per bushel.

18. Find the value of $.1 \times .2 \times .3 \times .4 \times .5 \times .6 \times .7$.

19. Find the value of $(1.03)^4$.

20. The number of bales of cotton produced in Texas in 1901–2 was 2,993,000, and in 1900–01, 3,550,000. Allowing 500 lb. to a bale, how many more pounds of cotton were produced in the latter year than in the former?

DIVISION

21. A steamer's cargo consisted of 120,000 bu. of corn, valued at \$57,000; 13,541 bbl. of flour, valued at \$47,499; 3050 bales of cotton, valued at \$158,224. Find the value of a bushel of corn, a barrel of flour, and a bale of cotton.

22. Divide 26.78508 by .072.

23. The annual consumption of sugar in a certain state was, in 1890, 702,201 T., which was found to be 49.93 lb. per head of population. Find the population.

24. Make a column of eight numbers, the first of which is 73,214, the second is $\frac{2}{3}$ of the first, the third is $\frac{2}{3}$ of the second, and so on for the other numbers.

25. How many miles in 278,784,000 ft.?

26. Divide $1.125^2 - (.784)^2$ by $1.125 - .784$.

27. Divide $(.75)^3 - (.26)^3$ by $.75 - .26$.

28. Divide 14.302 by 83.92, correct to four places.

29. Divide 24.619 by 56,000.

30. The length of a degree on the earth's surface is approximately 69.15 mi. Two places are on the same meridian and 1000 mi. apart. Find, in degrees, the difference in latitude.

31. Two places on the 60th parallel of latitude are 300 mi. apart. Find the difference of their longitudes. ($1° =$ 183,085. ft.)

32. A bankrupt's liabilities are \$47,875; his assets are \$38,650. How many cents on the dollar can he pay?

33. The product of two numbers is 642,978, and one of the numbers is 5.67. Find the other number.

34. If the quotient is 24,400, the remainder is 15, and the dividend is 6,100,015, find the divisor.

35. The total amount of money in circulation in the United States on March 1, 1903, was \$2,353,738,834. The per capita circulation in the United States on the same day was \$29.41. Find the population of the United States.

36. Divide the square of 1001 by 77×169.

37. When 450 lb. of sugar cost \$20.25, find the price of 84 lb.

38. Find the value of a rectangular plot of ground 726 yd. long and 240 yd. wide, at \$50 an acre.

39. Find in United States currency the value of £79.

40. When 1.75 yd. of silk cost \$3.85, find the cost of 14 yd.

41. Divide 39.328 by .0032.

42. If .6 of a yard of cloth cost 27¢, find the cost of 45 yd.

43. Divide 1 by 1.732.

44. Divide the cube of 11.1 by 27 times 1369.

45. What is the ratio of 25 A. to 640 A.?

G. C. D. AND L. C. M.

46. Find the G. C. D. of 288 and 432.

47. Find all the common factors of 36 and 54.

48. Find the common divisors of 288 and 360.

49. Express 1110, 777, and 1001 as the products of prime numbers. Find their L. C. M.

50. Find the G. C. D. of 208, 572, and 1326.

51. Find the L. C. M. of 26, 28, 48, 70, and 117.

52. Find the G. C. D. of 625 and 2525.

53. Find the G. C. D. and L. C. M. of 209, 304, and 380.

54. Find the prime factors of 80,850.

55. Two numbers have for their G. C. D. 101, and for L. C. M. 27,573. Find the product of these numbers.

56. Resolve 61,776 into its prime factors.

57. Two tracts of land, containing 1225 acres and 1675 acres, are divided into farms each containing the same number of acres. What is the largest possible acreage of each farm?

58. Telephone poles are 231 ft. apart. What is the smallest number of poles which will correspond to an exact number of half miles?

FRACTIONS, DECIMALS, AND DENOMINATE NUMBERS

59. Arrange in order of magnitude, $\frac{3}{4}$, $\frac{7}{9}$, $\frac{11}{15}$.

60. Find the difference between the greatest and the least of the fractions $\frac{2}{3}$, $\frac{5}{8}$, $\frac{17}{21}$, and $\frac{19}{24}$.

61. Add: $2\frac{1}{2}$, $3\frac{1}{4}$, $5\frac{5}{8}$, $3\frac{7}{12}$.

62. Reduce to its lowest terms $\frac{999}{1110}$

63. Express as decimals $\frac{35}{32}$, $\frac{7}{64}$, $\frac{16}{125}$.

64. Reduce to common fractions .0375, .0175, .03125

65. Simplify $\dfrac{1\frac{3}{4} - \frac{2}{3}}{8\frac{3}{4} - 3\frac{1}{3}} \times 15\frac{3}{5}$.

66. Simplify $2\frac{5}{6}$ of $\dfrac{4\frac{1}{5}}{4\frac{2}{3}} - (\frac{1}{7}$ of $17\frac{1}{2}$ of $\frac{2}{5}$ of $1\frac{1}{2})$.

67. Reduce 198 ft. to the decimal of $1\frac{1}{2}$ mi.

68. Reduce 2° 30′ to the decimal of 90°.

69. Reduce 3 pt. to the decimal of 5 gal.

70. Reduce .375 of 16*s*. 8*d*. + $\frac{2}{3}$ of 15*s*. 6*d*., to the decimal of £ 5.

71. The length of the true year is 365 da. 5 h. 48 min. What difference in length between 33 true years and 33 ordinary years, of which every fourth is a leap year?

72. Suppose the calendar so arranged that there would be 31 leap years in a period of 128 years. How much would such a period of 128 years differ from 128 true years?

73. The following distances have been run by trains in the times indicated. Find in each case the rate per hour.

Route	Distance in Miles	Time
Jersey City to Oakland . . .	3311	83 hr. 45 min.
New York to Chicago	964	19 hr. 57 min.
Chicago to New York	962	17 hr. 45 min.
London to Aberdeen	539.75	8 hr. 32 min.
Chicago to Buffalo	510.1	8 hr. 1 min. 7 sec.
Albany to Syracuse	147.84	2 hr. 10 min.
Erie to Buffalo Creek	86	1 hr. 10 min. 45 sec.
Camden to Atlantic City . . .	58.3	45 min. 45 sec.
Liberty Park to Absecon . . .	49.8	37 min. 30 sec.
Berlin to Absecon	35.6	25 min. 45 sec.
New York to Philadelphia . .	90	1 hr. 17 min.

74. Find the value of 25,000 bu. of oats at $46\frac{3}{4}$¢ per bushel.

75. The price of oats in June, 1900, was $26\frac{1}{4}$¢ per bushel, and in August it was 21¢ per bushel. If a speculator lost $1050 by buying oats at the former price and selling at the latter, how many bushels did he buy?

76. A speculator in Chicago bought 10,000 bu. of corn in February, 1901, at $38\frac{3}{8}$¢ per bushel, and sold it in December, 1901, at $69\frac{1}{4}$¢ per bushel. Find his profit.*

77. The total number of bales of cotton exported from the United States for the season of 1901–1902 was

* Allow $\frac{1}{8}$¢ per bushel brokerage for buying and for selling.

6,715,793, valued at $284,779,190. Find the average price per bale, correct to the cent.

78. The total number of farms in Alabama is 223,220; the total acreage of these is 20,685,427. Find, correct to two decimal places, the average number of acres in a farm.

79. The total sugar production of California was, in 1902, 356,500 T., valued at $15,500,000. Find the average price per 100 lb.

80. According to the census of 1900, the number of persons employed in manufacturing industries in Florida was 1778, and the salaries paid amounted to $1,295,139. Find the average salary received by each person, correct to the cent.

81. The total enrollment in the elementary and secondary schools in the United States in 1901 was 15,603,451, and the total number of teachers was 430,004. Find the average number of pupils to a teacher.

82. The total expenditure for higher education in Canada in a recent year was $1,014,254. This expenditure was 19.5¢ per capita of the total population. Find the population of Canada.

83. The total expenditure for higher education in Germany in a recent year was $7,450,366. The per capita expenditure was 14.3¢. Find the population of Germany.

The cost of higher education in Great Britain and Ireland for a recent year was given as $8,353,655. The per capita expenditure was 21.7¢. Find the population of Great Britain and Ireland.

84. The total amount of money in circulation in the German Empire is, estimated in our currency, $1,080,100,000. The per capita circulation is $19.53. Find the population of Germany.

85. Express in feet .002357 of a mile.

86. Find the value of $\frac{5}{16}$ of a ton + $\frac{7}{8}$ of a hundredweight. Give your answer in pounds.

87. How many times is 12 lb. 8 oz. contained in 2 T.?

88. Light travels at the rate of 185,000 mi. a second. How long does it take a ray of light to pass from the earth to the moon, a distance of 239,000 mi.?

89. How many cubic yards of sand are required to fill a street $1\frac{1}{2}$ mi. long, 40 ft. wide, to the depth of 5 in.?

90. Express $\frac{3}{4}$ of a day as a decimal of a common year.

91. If $\frac{5}{16}$ of an acre of land is worth $23, find the value of 85 A. of land.

92. If .375 of an acre of land is worth $22, find the value of 57 A.

93. Multiply 68.4 by .0027, and divide the product by $\frac{3}{4}$ of .96.

94. Find the value of .1875 of a guinea + $\frac{2}{3}$ of £1 + .25 of 7*s.* 8*d.* Give your answer in pounds, shillings, and pence.

95. Reduce 12*s.* 6*d.* to the decimal of £4 sterling.

96. (*a*) Divide $1293.46 by .00$\frac{7}{8}$. (*b*) Divide $147.32 by .00$\frac{2}{3}$. (*c*) Divide $3473.85 by .00$\frac{3}{4}$. (*d*) Divide $3295 by .00$\frac{2}{5}$.

97. (*a*) Divide $1456.77 by .00$\frac{7}{8}$. (*b*) Divide $3947.85 by .00$\frac{3}{4}$.

98. How many acres in a field 160 ch. long, 40 ch. wide?

99. A wheel is $12\frac{1}{2}$ ft. in circumference. How many revolutions will it make in going 6 mi. 80 rd.?

100. How many bushels will a bin 7 ft. by 5 ft. and 4 ft. deep hold?

101. Reduce $\frac{11}{16}$ to a fraction having 12 for denominator.

102. .08 of a boy's money is \$6. How much money has the boy?

103. .875 of a man's property is valued at \$21,700. What is the value of the man's property?

104. How many acres in a square field whose side is 40 rd.?

105. A and B can mow a field in 7 da. A, B, and C mow the same field in 5 da. for \$50. What should C receive?

106. Write decimally three-eighths of one hundredth, and reduce it to a simple decimal.

107. Reduce $\frac{1}{8}$, $\frac{1}{5}$, $\frac{1}{12}$, and $\frac{1}{16}$ to equivalent fractions having 100 for denominator.

108. Find the difference between $\frac{.16}{16}$ and $\frac{16}{.16}$.

109. Reduce $16\frac{3}{4}$ to an improper fraction having 16 for a denominator.

110. A rectangular field which is 18 rd. wide contains 6 A. How much will it cost to fence it at 75¢ a rod?

111. What decimal of 4 ft. 2 in. is 9 ft. 6 in.?

112. Find the least fraction which added to the sum of $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, and $\frac{1}{6}$ will make the sum an integer.

113. Divide 27.8 of a yard by .00125 of a foot.

114. 8 cwt. 20 lb. of sugar cost \$41.42. What will 1 T. cost at the same rate?

115. Find the least length which is a multiple of 1 ft. 3 in., 1 ft. 8 in., 2 ft. 1 in., and 2 ft. 6 in.

116. Twelve tenths of a number equals 42. Find the number.

117. Divide 54,218 by 64, using the factors of 64.

118. A train 165 yd. long passes a telegraph pole in 12 sec. Find the rate of the train in miles per hour.

119. A city lot 42 ft. by 120 ft. is sold for $840. At this rate, find the value of 1 A. of land in that city.

120. Find the greatest number which, when divided into 1958 and 2741, will give for remainders 8 and 11 respectively.

121. By buying eggs at 25 ¢ per dozen and selling them at 60 ¢ a score, a dealer makes a profit of $10.01. How many eggs does he sell?

122. Reduce $\frac{8568}{10296}$ to its lowest terms.

123. If a sum of money which will pay A's wages for $41\frac{3}{4}$ da. will pay B's wages for $55\frac{2}{3}$ da., for how long will it pay both?

124. If gold weighs 19.3 times as much as water, and copper 8.9 times as much as water, how much heavier than water is an alloy consisting of 16 parts of gold and 3 of copper?

125. A rectangular tank is 18 ft. $8\frac{3}{4}$ in. long, 11 ft. $3\frac{5}{6}$ in. wide, and contains 41 cu. yd., 6 cu. ft., and $34\frac{1}{8}$ cu. in. Find its depth. Find the area of each of its faces.

126. A tennis court is 42 yd. long and 20 yd. wide. It has a walk around it 6 ft. wide. Find the cost of paving the walk at $1.25 per square yard.

127. Telegraph poles along a certain railroad are 132 ft. apart. Find the rate of a train, in miles per hour, which passes 18 poles in 24 sec.

LONGITUDE AND TIME

128. A train leaves New York City at 9 A.M., Apr. 1, 1903, and arrives in Carson City, Nev., in 109 hr. 15 min. Find the hour of the day, and day of the month, Standard time, that it reaches its destination.

129. The time of mail transit between Chicago and Santa Fé, N. M., is 60 hr. 55 min. "The California Limited" leaves Chicago at 10 P.M. At what time, by the clocks in Santa Fé, should "The California Limited" pass Santa Fé?

130. The longitude of Cairo, Egypt, is 31° 21′ E., and the longitude of Savannah, Ga., is 81° 5′ 30″ W. Find the difference in local time.

131. The longitude of Toulon is 5° 56′ E. The time difference between Toulon and Halifax, N. S., is 4 hr. 38 min. 4 sec. Find the longitude of Halifax.

132. The time difference between Toulon and Point Barrow, Alaska, is 10 hr. 48 min. 44 sec. Find the longitude of Point Barrow.

133. The time difference between Osaka and Point Barrow is 19 hr. 26 min. 48 sec. Find the longitude of Osaka. (See previous problem.)

MISCELLANEOUS EXAMPLES

134. (*a*) Portland, Me., uses the Standard time of the meridian 75° west of Greenwich. If Portland's local time is 19 min. ahead of its Standard time, find the longitude of Portland.

(*b*) Fort Wayne, Ind., and Cleveland, O., use the Standard time of the meridian 90° west of Greenwich. If the local time at Fort Wayne is 20 min. ahead of Standard time, what is the longitude of Fort Wayne?

(*c*) If the local time at Cleveland is 33 minutes ahead of Standard time, what is the longitude of Cleveland?

PERCENTAGE

135. The total sugar production of the world for the year 1902 was 9,635,000 T. The amount of sugar consumed in the United States the same year was 2,372,000 T. What per cent of the world's production was the amount consumed in the United States?

136. The foreign-born population of New Orleans, according to the census of 1900, was 30,325; of this number, 1262 came from England, 4428 from France, 8733 from Germany, 5398 from Ireland. What per cent of the foreign-born population of New Orleans came from England? from France? from Germany? from Ireland?

137. The number of Canadians in Detroit, according to the census of 1900, was 25,400; this number was 26.3 % of the foreign-born population. Find, correct to 100, the number of foreign-born population of Detroit.

138. A horse is sold, at a loss of 15 %, for $127.50. Find the cost of the horse.

139. By selling silk at $1.60 per yard, a dealer makes a profit of 25 %. What would the selling price be if he made a profit of $12\frac{1}{2}$ %?

140. When cloth is sold for $1.04 per yard, a clothier makes a profit of 30 %. What would his profit be if he sold the cloth at 96¢ per yard?

141. A wholesale dealer makes a profit of 10 % on canned goods. The retail dealer makes a profit of 25 %. Find the original cost of canned goods which cost the consumer $11.

142. A coal merchant buys coal by the long ton at \$4.50 a ton, and sells it at the rate of \$5 a short ton. Find his gain per cent.

143. How much water must be added to a 25 % wine mixture to make it a 20 % mixture?

144. A sells goods to B at a profit of 20 %; B sells them to C at a profit of 20 % on his outlay; C sells them to D for \$180, thereby losing $16\frac{2}{3}$ %. How much did the goods cost A?

145. A merchant buys goods at 20 %, and 10 % off list price, and sells them at the list price. Find his per cent of gain.

146. When 20 lb. of tea are sold for what $22\frac{1}{2}$ lb. cost, what is the gain per cent?

147. (*a*) A vessel contains 31 gal. of wine and 17 gal. of water. What per cent of the mixture is wine and what per cent is water? (*b*) How many gallons of water must be added to this mixture to make a mixture containing 60 % wine?

148. The following table gives the distances from Atlantic to Pacific ports by the present routes:

New York to San Francisco	13,244 mi., nautical
New York to Sydney	14,560 mi., nautical
Charleston to San Francisco	13,180 mi., nautical
Charleston to Valparaiso	8,296 mi., nautical
New Orleans to San Francisco	13,644 mi., nautical
New Orleans to Melbourne	15,535 mi., nautical
Galveston to San Francisco	13,826 mi., nautical
Galveston to Wellington	14,182 mi., nautical
Liverpool to San Francisco	13,844 mi., nautical
Hamburg to Callao	10,702 mi., nautical
Bordeaux to San Francisco	13,691 mi., nautical

The following table gives the distances from Atlantic to Pacific ports via the Panama Canal route:

New York to San Francisco . . .	5299 mi., nautical
New York to Sydney	9852 mi., nautical
Charleston to San Francisco . . .	4898 mi., nautical
Charleston to Valparaiso	4229 mi., nautical
New Orleans to San Francisco . .	4698 mi., nautical
New Orleans to Melbourne	9826 mi., nautical
Galveston to San Francisco . . .	4800 mi., nautical
Galveston to Wellington	8392 mi., nautical
Liverpool to San Francisco	8038 mi., nautical
Hamburg to Callao	6527 mi., nautical
Bordeaux to San Francisco	7938 mi., nautical

What per cents of the distances by the old routes are saved by the Panama Canal route?

INTEREST

149. Find the simple interest on $78 for 93 da. at 8 %.

150. Find the simple interest on $98 for 63 da. at 7 %.

151. Find the amount of $179 for 123 da. at 6 %.

152. Find the simple interest on £324 7*s.* 9*d.* from June 12 to Dec. 7 following at 5 %.

153. Find the simple interest on £1169 6*s.* 8*d.* from Jan. 25 to June 18 following at 9 %.

154. What principal will produce $19.50 in 1 yr. at $6\frac{1}{2}$ %?

155. What principal will produce $180 interest in 3 mo. at 5 %?

156. What principal will amount to $412.50 in $7\frac{1}{2}$ mo. at 5 %?

157. What principal will amount to $1219 in 3 mo. 5 da. at 6 %?

158. What principal will produce $29.17 in 5 mo. at 7 %?

159. At what rate will $1000 produce $23.33 interest in 4 mo.?

160. What principal will produce 75¢ interest in 9 da. at 6 %?

161. Find the exact interest on $73.15 from June 18 to Aug. 1 at 7 %.

162. A note for $3500 bearing interest at 8 % and dated Jan. 2, 1900, was indorsed as follows: June 7, 1900, $450; Aug. 2, 1900, $208; Jan. 2, 1901, $500; July 7, 1901, $800; Oct. 4, 1901, $500; Jan. 11, 1902, $300; Aug. 4, 1902, $700. Calculate, by the United States Rule for partial payments, the amount due on this note on Jan. 1, 1903.

163. A demand note dated Jan. 5, 1902, and drawn for $575 was paid 6 mo. 18 da. later. Find the date of payment and the amount of the note, the rate of interest being 7 %.

BANK DISCOUNT

164. A note for 60 da. is drawn on Jan. 10, 1903. Find the proceeds of this note, if its face is $150, the date of discount Feb. 5, and the rate 6 %. (Neglect days of grace.)

165. A note for $900, dated Mobile, Ala., Jan. 8, 1903, and drawn for 90 da., is discounted March 1. Find the proceeds.

166. A 60-day note bearing interest at 8 %, drawn Feb. 1, 1903, for $1000, is discounted Feb. 27 at 9 %. Find the proceeds of this note.

167. A demand note was drawn Oct. 1, 1902, for $800, and paid 5 mo. 10 da. later. Find the date of payment and the amount of the note; rate of interest, 7%.

168. The proceeds of a note is $450 when the term of discount is 93 da., and the rate of interest is 8%. What is the maturity value of the note?

MENSURATION

169. Find the area of a parallelogram if its base is 100 yd. and its altitude is 75 yd.

170. Find the area of a trapezoid if its parallel sides are 60 yd. and 80 yd., and its altitude is 50 yd.

171. A tract of land is sold for $3943.84; the land cost as many dollars per acre as there were acres in the tract. Find the cost per acre.

172. Find the number of square yards in the walls and ceiling of a room 36 by 23 by 16 ft.

173. Find the perimeter of a square which contains 40 A.

174. A tract of land in the shape of a rectangle contains 320 A.; its length is twice its width. Find its dimensions.

175. Find the area of an equilateral triangle one side of which is 100 ft.

176. Find the area of a regular hexagon each side of which is 50 ft.

177. Find the circumference of a circle whose radius is 56.5 in.

178. Find the area of a circle if its diameter is 20 in.

179. Find the surface of a sphere whose diameter is 20 in.

PERSONAL AND FAMILY SAVINGS

180. Describe the simplest form of personal cash account. Rule a page of notebook for such an account.

181. Explain how to balance a cash account.

182. Define account, budget.

183. Explain how a budget of expenditures and an expense account can be made to help one to save.

184. Explain what is meant by thrift.

185. Describe the method of keeping a family budget.

186. What are the chief expenses of a family? What are the two most important?

187. What are the chief foods purchased by the family? What proportion of a dollar of a moderate income goes to each?

188. The National War-Savings Committee, Washington, D. C., has prepared a bulletin entitled "Learning How to Save." A copy of this bulletin can be obtained free by addressing the Committee.

MISCELLANEOUS EXAMPLES (B)

(TAKEN FROM VARIOUS EXAMINATION PAPERS)

1. What fractional part of $\frac{5}{7}$ of a gallon is $\frac{3}{28}$ of a pint?

2. The difference in time between two places is 2 hr. 33 min. Find the difference in longitude.

3. A bicycle wheel measuring 88 in. in circumference must make how many revolutions a minute to run eighteen miles an hour?

4. A coal bin $16\frac{1}{2}$ ft. long and 8 ft. 9 in. wide must be how deep to contain 10 T. of coal, if one ton of coal occupy 40 cu. ft. of space?

5. Reduce 2 yr. 21 da. to years and decimals of a year.

6. Reduce .09625 bbl. to integers of lower denominations.

7. Find the value of a piece of land 64 ch. by $13\frac{1}{2}$ ch. at $\$48\frac{1}{3}$ an acre.

8. Required the cost of 18 $2\frac{1}{2}$ in. plank 16 ft. long and 10 in. wide, and 33 pieces of scantling 2 in. by 4 in. by 16 ft. at $22 per M, board measure.

9. The average yield per bushel of wheat seed is 14 bu. 1 pk. What will 7 bu. 3 pk. 2 qt. of seed yield?

10. What is the difference in weight, expressed in Avoirdupois pounds, between 100 lb. Troy and 100 lb. Avoirdupois?

11. Reduce to simplest form $\dfrac{\frac{2}{3}+\frac{3}{7}}{7-\frac{4}{5}} \div \dfrac{\frac{2}{9}}{2+\frac{1}{2}}$.

12. If it cost $510 to fence a rectangular field 98 rd. by 72 rd., what will it cost to fence a square field of the same area?

13. Express $\frac{9}{7}$ as a decimal fraction.

14. What is the ratio of 32 ft. to 6 yd.? Express the result decimally.

15. What is the length of a plank $1\frac{1}{2}$ in. thick, 1 ft. 6 in. wide, containing 36 board feet?

16. When it is 12 M. in New York City (74° W.), what is the time in Manila (120° E.)?

17. If the value of $\frac{1}{7}$ of $\frac{3}{8}$ of an estate is $4500, what is the value of $\frac{1}{14}$ of $\frac{1}{12}$ of it?

18. At $16 per M, board measure, find the cost of 20 plank 2 in. by 8 in. by 18 ft. long, and 30 plank $1\frac{1}{2}$ in. by 6 in. by 10 ft. long.

19. A can do a piece of work in 6 da. and B can do the same work in 8 da. How long will it take B to finish alone after they have worked together two days?

20. $20\frac{3}{4}$ is the product of three factors. Two of these factors are $1\frac{3}{5}$ and $4\frac{5}{8}$. Find the other factor.

21. How many bushels of wheat will a box 6 ft. by $3\frac{1}{2}$ ft. by 2 ft. 8 in. hold?

22. How many yards in .04675 mi.?

23. If the dividend is 807 and the quotient $34\frac{1}{3}$, what is the divisor?

24. How many rods of fence will inclose a square field whose area is 20 acres?

25. Coal sells at $5.75 per ton. What will be the cost of 2315 lb. at this rate?

26. How many gallons of water will a tank 5 ft. by 2 ft. by 2 ft. hold?

27. What is the length of one side of a square piece of land whose area is 538,756 sq. rd.?

28. A room is 27 ft. by 22 ft. 6 in. How many yards of carpet 27 in. wide will be required to carpet this room?

29. A man is hired to dig a cellar 20 ft. by 15 ft. by 5 ft. How much money will he receive at 30¢ per cu. yd.?

30. How many days between Aug. 14 and Dec. 29?

31. Find the value of a car load of wheat, estimated at 21,643 lb., at 92¢ per bushel.

32. Two persons travel in opposite directions from the same point at the rate of $4\frac{1}{3}$ and $7\frac{3}{4}$ mi. per hour, respectively. How far apart are they after traveling $37\frac{1}{2}$ hrs.?

33. A man was born Nov. 22, 1861. What is his age to-day?

34. Factor the following numbers and from these factors determine the G. C. D.: 42, 112, 140, 308.

35. What will 75 boards 2 in. by 4 in. by 16 ft. long cost at $12 per M board measure?

36. 160 rd. of fence will inclose how many acres in the form of a square?

37. The difference in longitude between two places is $7° 42' 30''$. Find the difference in time.

38. How wide is a rectangular field containing 5 A., the length of the field being 7 ch. 25 l.?

39. A pavement is $5\frac{1}{3}$ rd. long and 8 ft. 6 in. wide. What did it cost at $1.40 per sq. yd.?

40. Three men, A, B, and C, do a piece of work; A works 3 da. of 5 hr. each, B, 2 da. of 6 hr. each, and C, 7 da. of 3 hr. each. At the same rate of wages, how should they divide $120, the total amount received for doing the work?

41. A miller charges $\frac{1}{16}$ for toll. How many bushels of wheat must one take to mill to get 12 bbl. of flour, each containing 196 lb., if a bushel of wheat makes 40 lb. of flour?

42. The annual rainfall in a certain locality is 30 in. How many tons of water fall on an acre of land in this locality, if a cubic foot of water weighs 1000 oz.?

43. How much does a man gain or lose on the sale of two houses at $1200 each, if he gains $\frac{1}{3}$ of the cost price on one, and loses $\frac{1}{5}$ of the cost price on the other?

44. What is the ratio of 7 lb. Troy weight to 10 oz. Avoirdupois?

45. The divisor is 357, the quotient is $6\frac{3}{7}$; what is the dividend?

46. A farmer had 28 A. of land left after selling $\frac{1}{4}$ of his farm to one neighbor, $\frac{2}{5}$ of it to another, and $\frac{3}{7}$ of the remainder to another. How large was his farm?

47. Multiply 8.035 by .0035, add 3, and divide the sum by .000625.

48. Divide \$459.25 into three parts that shall be to one another as $\frac{3}{4}$, $\frac{5}{6}$, and 3 respectively.

49. When it is two o'clock P.M. in Jerusalem, what is the time in Cincinnati? The longitude of Jerusalem is 35° 12′ E., and of Cincinnati 84° 26′ W.

50. Find the exact number of days between Dec. 23, 1902, and to-day.

51. A man's farm is mortgaged for $\frac{2}{3}$ of its cost; he sells it for \$6000, which is 25% above its cost. How much money will he have after paying the mortgage?

52. A note for \$600, dated Oct. 24, 1902, and due in 8 mo., with interest at 6% per annum, is discounted at bank Dec. 20, 1902. Find the proceeds.

53. A man sold two lots each for \$600, gaining 20% on one, and losing 20% on the other. What was his gain or loss?

54. A man buys a book the list price of which is \$7.20, at a discount of $16\frac{2}{3}$%, and sells it for \$7.50. What is his gain per cent?

55. What principal at interest for 1 yr. 3 mo. will amount to \$506, the rate of interest being 8% per annum?

56. Twelve per cent of 90 is what per cent of 100?

57. At the following rates per annum of simple interest, *what time* is *required* for the accruing interest to equal the principal: 6%, 8%, $9\frac{1}{11}$%?

58. What is the exact interest on \$10,000 from Jan. 18, 1903, to May 6, 1904, at $3\frac{1}{2}$ %?

59. A 30-day note, without interest, is discounted at a bank at 8 % for \$350. What is the face of the note?

60. Bonds bearing 5 % interest are bought at 120. What is the rate of income on these bonds?

61. An agent buys sugar at $4\frac{3}{4}$¢ per pound; his commission at $\frac{1}{4}$ % is \$25. How many pounds of sugar does he buy?

62. The discount of a note, discounted at bank, for 3 mo. 18 da. at 5 % is \$4.20. Find the proceeds.

63. What single discount is equivalent to trade discounts of 25 %, 10 %, and 5 %, on the list price of an article?

64. The property in a school district is assessed at \$196,000. What rate of taxation would be required to provide \$800 annually for the improved maintenance of the schools? What annual tax would a man pay on this account whose property is assessed at \$1200?

65. A man sold two horses at \$80 each. On one he gained 20 %, on the other he lost 20 %. Find the gain or loss.

66. What must I ask for an article worth \$36 that, after giving a discount of 20 %, I may gain 25 % on the value?

67. A school district advertised for bids to build a schoolhouse, the lowest bid being \$21,049. If it costs 3 % to collect the money, how large a levy should be made, supposing 29 % of it to be non-collectible?

68. What must a man pay for 4 % stock to get 5 % on his investment?

69. If you buy United States 3's at 110, what per cent per annum would your investment pay?

70. A merchant's expenses average 10 % of his sales. At what per cent advance on cost must he sell his goods to clear 20 % profit?

71. A merchant sold goods to the amount of \$760.95, thereby losing 11 %. What did he pay for the goods?

72. A ship is insured for half its value for \$374. If the rate is $2\frac{3}{4}$ %, what is the value of the ship?

73. A carriage dealer sold 16 buggies at \$200 each; on one half he gained 10 %, and on the other half he lost 10 %. Find his net gain or loss.

74. What principal will amount to \$1253.86 in 2 yr. 11 mo. 13 da., interest at 5 %?

75. How do you find the rate per cent per annum when the principal, interest, and time are given?

76. How do you find the principal when the rate per cent per annum, time, and interest are given?

77. How do you find the time when the principal, rate per cent per annum, and interest are given?

78. The list price of office desks is \$15, but 12 desks are sold for \$126. What rate of discount is allowed?

79. In a certain time \$650 will amount to \$713.05 at 6 % simple interest. Find the time.

80. A note for \$500, due in 3 months, is discounted at a bank at 6 %. Find the proceeds.

81. $236\frac{1}{2}$ is what per cent of $78\frac{5}{6}$?

82. A man insures his life, paying a premium of \$28, which is at the rate of $\frac{7}{8}$ % on the amount of his insurance. Find the face of the policy.

83. If 25 % of the selling price of an article is profit, what is the per cent of gain on its cost?

84. A man fails in business; his assets amount to $2100, his liabilities to $6000. What per cent will his creditors receive?

85. What is the interest on $475 for 1 yr. 3 mo. 24 da. at 6 %?

86. A man bought four loads of hay, each weighing 2750 lb., at $20 per ton; he gave in payment his note, without interest, at 60 da. What are the proceeds of this note, discounted at a bank at 6 %?

87. What per cent of $\frac{1}{2}$ is $\frac{1}{8}$?

88. An agent's commissions at 5 % amount to $37.65. Find the amount of his sales.

89. The tax on property assessed at $8500 is $48.37. What is the rate on $1000?

90. Find the date of maturity of a note made and dated Sept. 11, 1902, and payable 90 da. after date.

91. Find the cost of 87 shares of stock at $76\frac{1}{4}$, brokerage $\frac{1}{8}$ per cent.

92. A New York sight draft was sold in Atlanta, Ga., for $3542, exchange being at $\frac{3}{4}$ % premium. What was the face of the draft?

93. What per cent of 5 lb. is 3 oz. Avoirdupois?

94. An agent's commission for renting a house is $13.25; his rate of commission is $2\frac{1}{2}$ %. What is the yearly rent of the house?

95. A man pays a premium of $150 for insuring his house for $\frac{2}{3}$ of its value; the rate of premium is $1\frac{1}{2}$ per cent per annum. What is the value of the house?

96. A building worth \$6000 is insured for $\frac{3}{4}$ of its value at 75¢ on the \$100. In case of the destruction of the building by fire, what will be the owner's loss, including premium?

97. What per cent of 1 bu. is 3 qt.?

98. A merchant can buy flour on six months' credit at \$8 per barrel, or for cash at \$7.50 per barrel. He buys 100 bbl., paying cash, but borrows the money at 8 % to pay for it. Is this better than to buy on credit, and how much better?

99. A man sells 16 shares of bank stock at $127\frac{3}{4}$, brokerage $\frac{1}{8}$ %. How much does he receive for his stock?

100. A speculator buys stocks at 20 % premium, and sells the same stocks at 10 % discount. What per cent of his investment does he lose?

101. A hardware merchant's books show sales during the month of March amounting to \$1000. One half of his sales are at a profit of 25 % on the cost, and the other half a loss of $16\frac{2}{3}$ % on the cost. Find the cost of the goods sold during the month.

102. A merchant failing in business paid his creditors \$3874.75, which was at the rate of 55¢ on every dollar of his indebtedness. Find his indebtedness.

103. The list price of a mower is \$38; the retail dealer is allowed discounts of 20 %, 5 %, and 3 %. What does he pay for mowers? If the retailer sells these mowers at a profit of 50 %, what does the farmer pay for these mowers?

104. A certain stock, selling at $121\frac{3}{4}$, pays a semiannual dividend of 4 %. What is the rate per cent per annum on an investment in this stock?

TABLES

Apothecaries' Weight

20 grains (gr.)	= 1 scruple (℈)
3 scruples	= 1 dram (ʒ)
8 drams	= 1 ounce (℥)
12 ounces	= 1 pound (℔)

Liquid Measure

4 gills (gi.)	= 1 pint (pt.)
2 pints	= 1 quart (qt.)
4 quarts	= 1 gallon (gal.)
31½ gallons	= 1 barrel (bbl.)
2 barrels	= 1 hogshead (hhd.)

Dry Measure

2 pints (pt.)	= 1 quart (qt.)
8 quarts	= 1 peck (pk.)
4 pecks	= 1 bushel (bu.)

Numerical Measure

12 articles	= 1 dozen
12 dozen	= 1 gross
12 gross	= 1 great gross
20 articles	= 1 score

Avoirdupois Weight

16 drams (dr.)	= 1 ounce (oz.)
16 ounces	= 1 pound (lb.)
25 pounds	= 1 quarter
100 pounds	= 1 hundredweight (cwt.)
20 cwt.	= 1 ton (T.)
2240 pounds	= 1 long ton

Long Measure

12 inches (in.)	= 1 foot (ft.)
3 feet	= 1 yard (yd.)
5½ yards	= 1 rod (rd.), or pole
40 rods	= 1 furlong
8 furlongs	= 1 mile (mi.)

Troy Weight

24 grains (gr.)	= 1 pennyweight (pwt.)
20 pennyweights	= 1 ounce (oz.)
12 ounces	= 1 pound (lb.)

Paper Measure

24 sheets	= 1 quire
20 quires	= 1 ream
2 reams	= 1 bundle
5 bundles	= 1 bale

Circular Measure

60 seconds (″)	= 1 minute (′)
60 minutes	= 1 degree (°)
30 degrees	= 1 sign (S.)
12 signs	= 1 circle (C.) or circumference
360 degrees	= 1 circumference

Nautical Measure

6 feet	= 1 fathom
608 feet	= 1 cable length
10 cable lengths	= 1 nautical mile (6080 feet)

The following denominations are also used:

1.152 statute miles	= 1 geographic mile, or knot
3 geographic miles	= 1 league
60 geographic miles, or 69.1 statute miles	= 1 degree of latitude on a meridian
360 degrees	= the circumference of the earth

4 inches	=	1 hand
9 inches	=	1 span
21.888 inches	=	1 sacred cubit
3 feet	=	1 pace

Time Measure

60 seconds (sec.)	=	1 minute (min.)
60 minutes	=	1 hour (hr.)
24 hours	=	1 day (da.)
7 days	=	1 week (wk.)
4 weeks	=	1 lunar month
30 days	=	1 commercial month
12 months	=	1 year
365 days	=	1 common year
366 days	=	1 leap year

Surveyors' and Land Measure

7.92 inches	=	1 link (l.)
25 links	=	1 rod
4 rods	=	1 chain (ch.)
10 square chains	=	1 acre
640 acres	=	1 square mile

625 square links (sq. l.)	=	1 pole (P.)
16 poles	=	1 square chain

Apothecaries' Fluid Measure

60 minims (m.)	=	1 fluidrachm (f ʒ)
8 fluidrachms	=	1 fluidounce (f ℥)
16 fluidounces	=	1 pint (O)
8 pints	=	1 gallon (Cong.)

Cubic Measure

1728 cubic inches (cu. in.)	=	1 cubic foot (cu. ft.)
27 cubic feet	=	1 cubic yard (cu. yd.)
128 cubic feet	=	1 cord of wood (cd.)
$24\frac{3}{4}$ cubic feet	=	1 perch of stone or masonry (pch.)

Square Measure

144 square inches (sq. in.)	=	1 square foot (sq. ft.)
9 square feet	=	1 square yard (sq. yd.)
$30\frac{1}{4}$ square yards	=	1 square rod or perch (sq. rd. or sq. pch.)
40 square rods	=	1 square rood (sq. R.)
4 roods	=	1 acre (A.)
640 acres	=	1 square mile (sq. mi

Spanish Land Measure

In Texas, California, New Mexico, and other parts of this country which were formerly parts of the Spanish empire, the vara, the unit of linear measure, is still used in connection with original grants of land. In Texas the value of the vara is $33\frac{1}{3}$ in. In California and New Mexico it is usually considered 33 in.

1,000,000 square varas	=	1 labor	=	177.136 acres
25,000,000 square varas	=	1 league	=	4428.4 acres
3,612,800 square varas	=	1 square mile	=	640 acres
1,806,400 square varas	=	$\frac{1}{2}$ square mile	=	320 acres
903,200 square varas	=	$\frac{1}{4}$ square mile	=	160 acres
451,600 square varas	=	$\frac{1}{8}$ square mile	=	80 acres
225,800 square varas	=	$\frac{1}{16}$ square mile	=	40 acres
5645 square varas	=	1 acre		

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